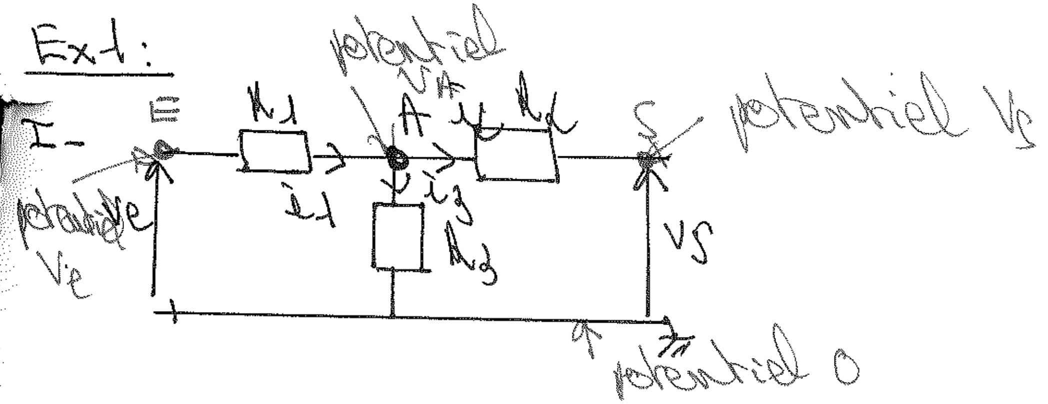
(1)

Ex 1 :



La ligne de masse est considérée au potentiel 0 V.
De ce fait le potentiel du point E s'identifie à la tension d'entrée c.a.d V_E.
De même le potentiel du point S vaut V_S.

La loi des noeuds au point A donne :

$i_1 = i_2 + i_3$

$$\underbrace{\frac{V_e - V_A}{R_1}}_{i_1} = \underbrace{\frac{V_A - V_S}{R_2}}_{i_2} + \frac{V_A - 0}{R_3}$$

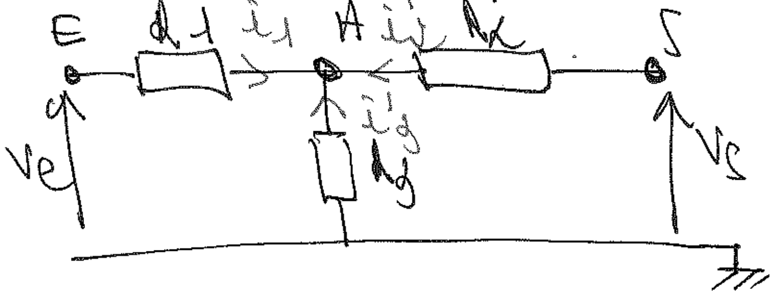
$$\Rightarrow \boxed{\frac{V_e - V_A}{R_1} = \frac{V_A - V_S}{R_2} + \frac{V_A}{R_3}}$$

(2)

Technique : pour déterminer l'expression de i_1 par exemple, on suit la flèche de courant :

$$i_1 = \frac{\text{potentiel de départ} - \text{pot d'arrivée}}{R_1}$$

Si on change des sens d'intensité par ex :



Loi des noeuds :

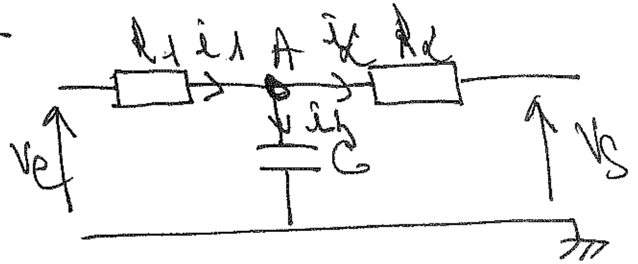
$i_1 + i_2' + i_3' = 0$

$$\frac{V_E - V_A}{R_1} + \frac{0 - V_A}{R_3} + \frac{V_S - V_A}{R_2} = 0$$

$$\Rightarrow \boxed{\frac{V_E - V_A}{R_1} - \frac{V_A}{R_3} + \frac{V_S - V_A}{R_2} = 0}$$

On obtient bien ici, la même relation entre les potentiels.

II –

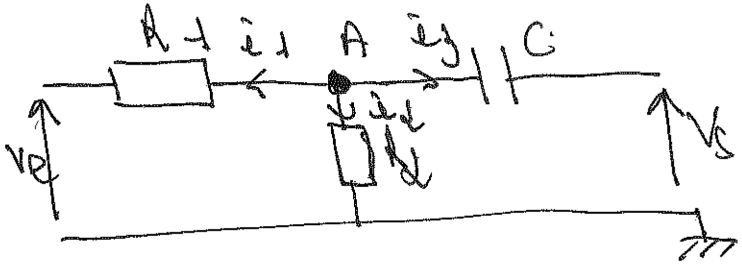


$$i_1 = i_2 + i_3$$

$$\frac{V_e - V_A}{R_1} = \frac{V_A - V_S}{R_2} + C\frac{d(V_A - 0)}{dt}$$

$$\boxed{\frac{V_e - V_A}{R_1} = \frac{V_A - V_S}{R_2} + C\frac{dV_A}{dt}}$$

III –

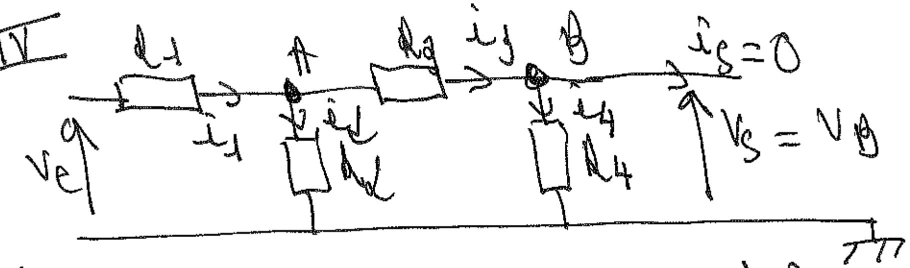


$$i_1 + i_2 + i_3 = 0$$

$$\frac{V_A - V_e}{R_1} + \frac{V_A - 0}{R_2} + C\frac{d(V_A - V_S)}{dt} = 0$$

$$\boxed{\frac{V_A - V_e}{R_1} + \frac{V_A}{R_2} + C\frac{d(V_A - V_S)}{dt} = 0}$$

IV



Loi des noeuds au noeud A :

$$i_1 = i_2 + i_3$$

$$\frac{V_e - V_A}{R_1} = \frac{V_A - 0}{R_2} + \frac{V_A - V_S}{R_3}$$

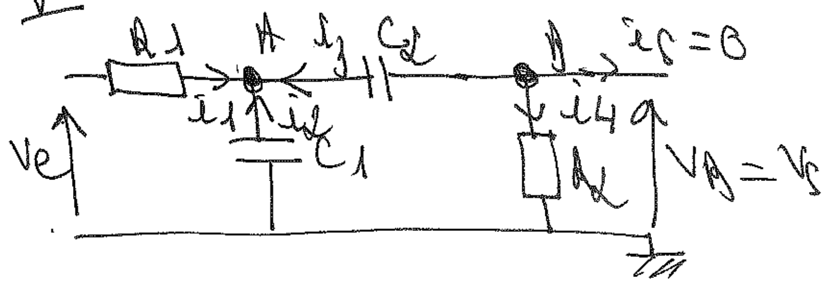
$$\boxed{\frac{V_e - V_A}{R_1} = \frac{V_A}{R_2} + \frac{V_A - V_S}{R_3}}$$

Loi des noeuds au noeud B :

$$\frac{V_A - V_S}{R_3} = \frac{V_S - 0}{R_4}$$

$$\boxed{\frac{V_A - V_S}{R_3} = \frac{V_S}{R_4}}$$

V



Au noeud A : $i_1 + i_2 + i_3 = 0$

$$\frac{V_e - V_A}{R_1} + C_1 \frac{d(0 - V_A)}{dt} + C_2 \frac{d(V_S - V_A)}{dt} = 0$$

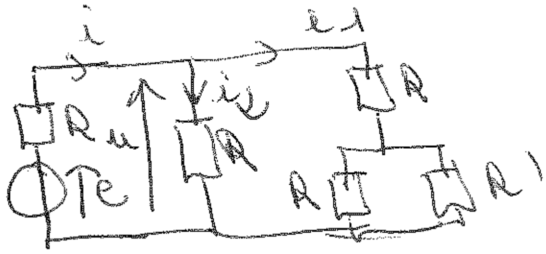
$$\boxed{\frac{V_e - V_A}{R_1} - C_1 \frac{dV_A}{dt} + C_2 \frac{d(V_S - V_A)}{dt} = 0}$$

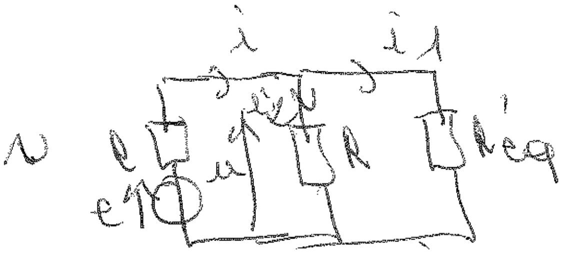
Au noeud B :

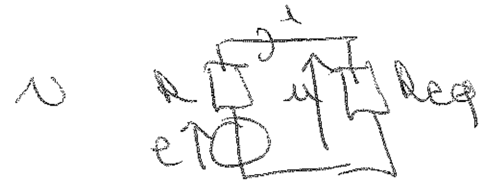
$$i_3 + i_4 = 0$$

$$\boxed{C_2 \frac{d(V_S - V_A)}{dt} + \frac{V_S}{R_2} = 0}$$

Exercice 1 :







1 -

1 - $R'_{eq} = R + \frac{RR'}{R+R'} = \frac{R^2 + 2RR'}{R+R'}$

$$R_{eq} = \frac{R R'_{eq}}{R + R'_{eq}} = \frac{R(R^2+2RR')}{R(R+R') + R^2 + 2RR'}$$

$$R_{eq} = \frac{R^2(R+2R')}{2R^2 + 3RR'}$$

$$\boxed{R_{eq} = \frac{R(R+2R')}{2R+3R'}}$$

2 - D'après le diviseur de tension.

$$u = \frac{R_{eq}}{R+R_{eq}} e = \frac{R(R+2R')}{R(2R+3R') + R(R+2R')} e$$

$$\boxed{u = \frac{R(R+2R')}{R(3R+5R')} e}$$

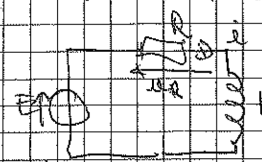
3 - $i = \frac{e}{R+R_{eq}} = \frac{2R+3R'}{R(2R+3R')+R(R+2R')} e$

$$\boxed{i = \frac{2R+3R'}{R(3R+5R')} e}$$

$$\boxed{i_2 = \frac{u}{R} = \frac{R+2R'}{R(3R+5R')} e}$$

$$\boxed{i_1 = i - i_2 = \frac{R+R'}{R(3R+5R')} e}$$

Ex : Établissement courant dans une inductance.



1 - $E = L\frac{di}{dt} + Ri$.

$\frac{L}{R}\frac{di}{dt} + i = \frac{E}{R}$ $\quad \tau = \frac{L}{R}$

$i(t) = \frac{E}{R}\left(1 - e^{-t/\tau}\right)$

$u_R = Ri = E\left(1 - e^{-t/\tau}\right)$

$t_0 = 2\,ms$ $\quad u_R = 2,7\,V = U_0$

$\frac{U_0}{E} = 1 - e^{-t_0/\tau}$

$e^{-t_0/\tau} = 1 - \frac{U_0}{E}$

$t_0 = -\frac{L}{R}\ln\left(1 - \frac{U_0}{E}\right)$

$$\boxed{L = \frac{R t_0}{-\ln\left(1 - \frac{U_0}{E}\right)} = 50\,mH}$$

2 - $\boxed{W_m = \frac{1}{2} L i^2(t_0) = \frac{1}{2} L \times \frac{U_0^2}{R^2} = 0,81\,mJ}$

3 - $P = Ri^2$

$W_J = \int_0^{t_0} Ri^2\,dt$

+ simple bilan d'énergie

$Ei = Ri^2 + Li\frac{di}{dt}$

énergie fournie entre 0 et t_0 par le géné $= W_J + W_m$.

$W_{géné} = \int_0^{t_0} E\,i(t)\,dt$

$= \int_0^{t_0} \frac{E^2}{R}\left(1 - e^{-t/\tau}\right)dt$

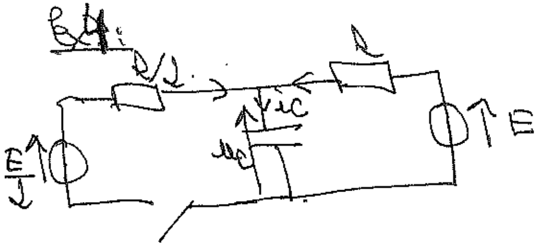
$= \frac{E^2}{R}\left[t_0 + \tau\left(e^{-t_0/\tau} - 1\right)\right]$

$W_{géné} = \frac{E^2}{R}\left[t_0 + \frac{L}{R}\left(e^{-\frac{Rt_0}{L}} - 1\right)\right]$

$W_{géné} = 1,19\,mJ$

$\underline{W_J = 0,38\,mJ}$

Ex4:



1- $t \geq 0$ $\quad i_C = C\dfrac{du_C}{dt} = \dfrac{\frac{E}{2} - u_C}{R/2} + \dfrac{E - u_C}{R}$

$$RC\frac{du_C}{dt} = E - 2u_C + E - u_C$$

$$RC\frac{du_C}{dt} + 3u_C = 2E$$

$$\boxed{\frac{RC}{3}\frac{du_C}{dt} + u_C = \frac{2E}{3}}$$

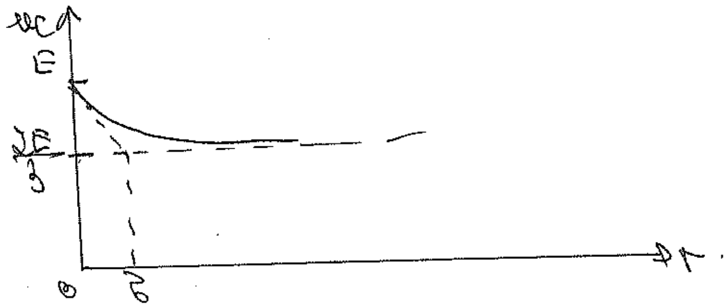
2- $\tau = \dfrac{RC}{3}$

$$u_C(t) = \frac{2E}{3} + A\,e^{-t/\tau}$$

$t = 0^-$ $\quad u_C = E \Rightarrow u_C(0^+) = E$

$$A = E - \frac{2E}{3} = \frac{E}{3}$$

$$\boxed{u_C(t) = \frac{E}{3}\left(2 + e^{-t/\tau}\right)}$$



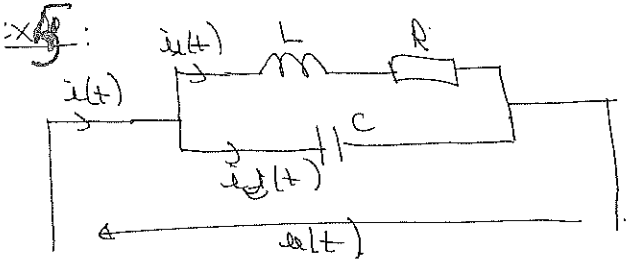
3- t_1 à 1% $\quad u_C(t_1) = 0{,}99 \times \dfrac{2E}{3} = \dfrac{2E}{3} + \dfrac{E}{3}e^{-t_1/\tau}$

$$0{,}01 \times \frac{2E}{3} = \frac{E}{3}e^{-t_1/\tau}$$

$$e^{-t_1/\tau} = 0{,}02$$

$$\boxed{t_1 = -\tau \ln 0{,}02} = \boxed{\tau \ln(50)} = 3{,}9\,\tau \;(\simeq 4\tau)$$

Ex 5 :



$u(t) = U_{eff}\sqrt{2}\cos(2\pi\nu t)$ $\quad \omega = 2\pi\nu = 1000\pi \text{ rad s}^{-1}$

$\underline{i_1}(t) = \dfrac{\underline{u}(t)}{\underline{Z_1}} = \underline{u}(t)\dfrac{1}{R + jL\omega}$

$I_{1eff} = \dfrac{U_{eff}}{\sqrt{R^2 + L^2\omega^2}}$ $\quad I_{1eff} = 77 \text{ mA}$

$\varphi_1 = -\arctan\left(\dfrac{L\omega}{R}\right)$ $\quad \varphi_1 = -1{,}26$

$\underline{i_1(t) = 77\sqrt{2}\cos(1000\pi t - 1{,}26)}$ en mA

$\underline{i_2}(t) = jC\omega\,\underline{u}(t)$ $\quad \varphi_2 = \dfrac{\pi}{2}$

$I_{2eff} = C\omega U_{eff} = 40 \text{ mA}$

$i_2(t) = 40\sqrt{2}\cos\left(1000\pi t + \dfrac{\pi}{2}\right)$

$\underline{i_2(t) = -40\sqrt{2}\sin(1000\pi t)}$ en mA

$\underline{i}(t) = \underline{Y}\,\underline{u}(t)$

$= \left(jC\omega + \dfrac{1}{R + jL\omega}\right)\underline{u}(t)$

$\underline{i}(t) = \left(jC\omega + \dfrac{R - jL\omega}{(R^2 + L^2\omega^2)}\right)\underline{u}(t)$

$= \left(\dfrac{R}{(R^2 + L^2\omega^2)} + j\left(C\omega - \dfrac{L\omega}{R^2 + L^2\omega^2}\right)\right)\underline{u}(t)$

$= \dfrac{1 - LC\omega^2 + jRC\omega}{R + jL\omega}\underline{u}(t)$

$I_{eff} = U_{eff}\sqrt{\dfrac{(1 - LC\omega^2)^2 + R^2C^2\omega^2}{R^2 + L^2\omega^2}}$ $\quad 1 - LC\omega^2 = 0{,}5 > 0$

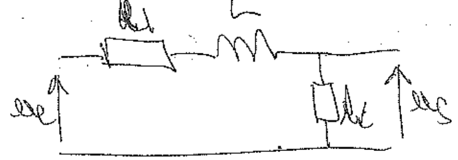
$\varphi = \arctan\left(\dfrac{RC\omega}{1 - LC\omega^2}\right) - \arctan\left(\dfrac{L\omega}{R}\right)$

$I_{eff} = 41 \text{ mA}$

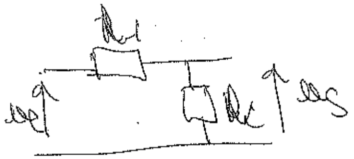
$\varphi = -0{,}9621 \text{ rad}$

$\underline{i(t) = 41\cos(1000\pi t - 0{,}96)}$

Ex 6 :



1 - $\omega \to 0$ L ~ —



$$\boxed{u_s = \frac{R_2}{R_1 + R_2} u_e}$$

(la relation entre u_s et u_e est attendue ne pas se contenter de $u_s \neq 0$)

$\omega \to \infty$: L ~ — / — $\Rightarrow$ pas de courant dans $R_2 \Rightarrow \boxed{u_s \to 0}$

Filtre passe-bas d'ordre 1

2 - Diviseur de tension :

$$\underline{H}(j\omega) = \frac{u_s}{u_e} = \frac{R_2}{R_1 + R_2 + jL\omega}$$

$$\underline{H}(j\omega) = \frac{\frac{R_2}{R_1+R_2}}{1 + j\frac{L}{R_1+R_2}\omega}$$

$$H_0 = \frac{R_2}{R_1 + R_2} \qquad \omega_0 = \frac{R_1 + R_2}{L}$$

D'où $\boxed{\underline{H}(j\omega) = \dfrac{H_0}{1 + j\frac{\omega}{\omega_0}}}$

3 - $\omega \ll \omega_0$: $\underline{H}(j\omega) \simeq H_0 > 0$

$G_{dB} \simeq 20\log(H_0) < 0$

$\varphi \simeq 0$

$\omega \gg \omega_0$: $\underline{H}(j\omega) \simeq \dfrac{H_0}{j\frac{\omega}{\omega_0}}$

$G_{dB} \simeq 20\log(H_0) - 20\log\left(\frac{\omega}{\omega_0}\right)$

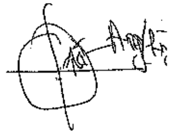
$\varphi \simeq -\frac{\pi}{2}$

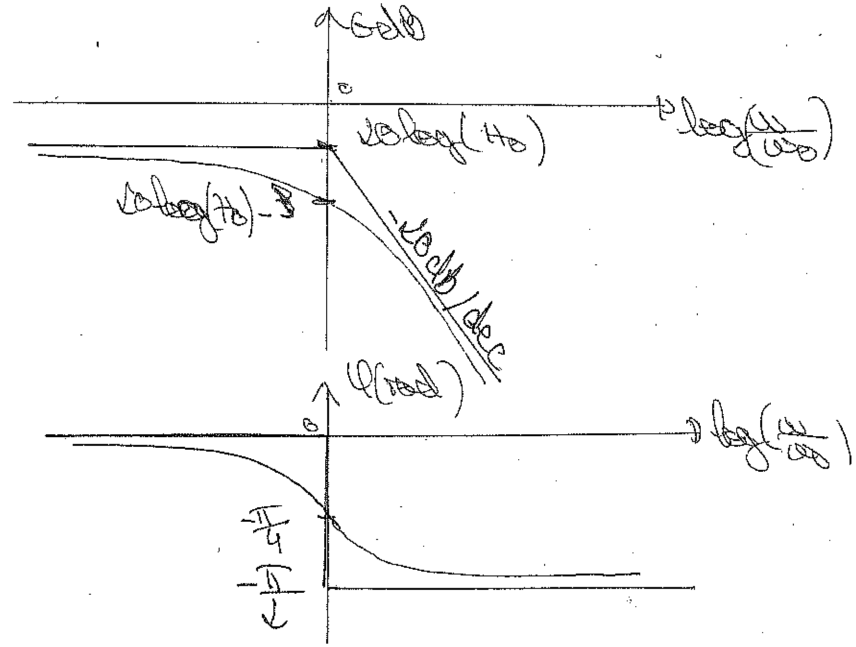
$\omega = \omega_0$: $\underline{H}(j\omega_0) = \dfrac{H_0}{1+j}$ $\quad |\underline{H}(j\omega_0)| = \dfrac{H_0}{\sqrt{2}}$

$G_{dB} = 20\log(H_0) - 3\ (dB)$

ω_0 est la pulsation de coupure à $-3\,dB$

$\varphi = -\mathrm{Arg}(1+j) = -\frac{\pi}{4}$





(3)

4 – a – $f_1 = 2f_0$

$$u_S(t) = V_1 \underbrace{|\underline{H}(jf_1)|}_{\text{amplitude } u_S \;=\; \text{amplitude } u_E \times |\underline{H}(jf_1)|} \cos\left(2\pi f_1 t + \underbrace{\varphi(f_1)}_{\text{phase de } u_S}\right)$$

fréquence de u_E

phase de u_S = phase de u_E + arg de $\underline{H}$ calculé pour $f = f_1$

$$|\underline{H}(jf_1)| = \frac{H_0}{\sqrt{1 + \left(\frac{f_1}{f_0}\right)^2}} = \frac{H_0}{\sqrt{1+4}} = \frac{H_0}{\sqrt{5}} = 0{,}15$$

$$\varphi(f_1) = -\text{Arctan}\left(\frac{f_1}{f_0}\right) = -\text{Arctan}(2)$$

$$\varphi(f_1) = -1{,}11 \text{ rad}$$

$$u_S(t) = 10 \times 0{,}15 \cos(2\pi \times 1600\, t - 1{,}11) \text{ en V}$$

$$\underline{u_S(t) = 1{,}5 \cos(3200\pi t - 1{,}11) \text{ en V}}$$

b – $u_E(t) = E_L + V_L \cos(2\pi f_0 t)$

Filtre linéaire ⇒ u_S = somme des sorties associées à E_L et $V_L \cos(2\pi f_0 t)$.

Pour chaque signal, on calcule $\underline{H}$.

→ Signal E_L : signal continu, sa fréquence est nulle

$f = 0 \quad \underline{H}(j0) = H_0$ réel

$$E_L \xrightarrow{\text{Filtre}} H_0 E_L$$

(4)

→ Signal $V_L \cos(2\pi f_0 t)$:

$$f = f_0 \quad \underline{H}(jf_0) = \frac{H_0}{1+j}$$

$$|\underline{H}(jf_0)| = \frac{H_0}{\sqrt{2}} \quad \varphi(f_0) = -\frac{\pi}{4}$$

$$V_L \cos(2\pi f_0 t) \xrightarrow{\text{Filtre}} \frac{H_0 V_L}{\sqrt{2}} \cos\left(2\pi f_0 t - \frac{\pi}{4}\right)$$

$$u_S(t) = H_0 E_L + \frac{H_0}{\sqrt{2}} V_L \cos\left(2\pi f_0 t - \frac{\pi}{4}\right)$$

$$u_S(t) = 2 + 1{,}9 \cos\left(1600\pi t - \frac{\pi}{4}\right) \text{ en V}$$