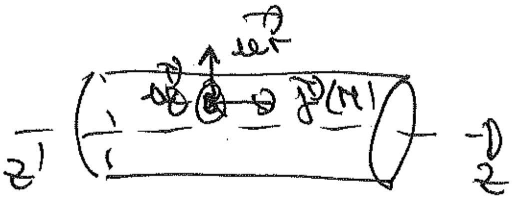
Ex :

$\vec{j} = j_0 \left(\frac{r^2}{a^2}\right)\vec{e_z}$



(M ; $\vec{u_r}$, $\vec{u_z}$) plan de symétrie, $\vec{B}$ $\perp$ à ce plan $\vec{B} = B(M)\,\vec{u_\theta}$.

1- Th. d'Ampère $\oint_{(C)} \vec{B}\cdot d\vec{l} = \mu_0 I_{enlacé}$

(C) = cercle de rayon r

$\oint_{(C)} \vec{B}\cdot d\vec{l} = B(r)\,2\pi r$.

$r < a$: $I_{enlacée} = \int_0^r j_0 \left(\frac{u^2}{a^2}\right) 2\pi u\, du$

$$I_{enlacée} = \frac{2\pi j_0}{a^2}\left(\frac{u^4}{4}\right)_0^r$$

$$\boxed{\vec{B} = \frac{\mu_0 j_0}{4a^2}\, r^3\, \vec{u_\theta}}$$

$r > a$: $I_{enlacée} = \int_0^a j_0 \left(\frac{u^2}{a^2}\right) 2\pi u\, du$

$$I_{enlacée} = \frac{2\pi j_0}{a^2}\,\frac{a^4}{4} = \frac{2\pi j_0 a^2}{4}$$

$$\boxed{\vec{B} = \frac{\mu_0 j_0 a^2}{4r}\, \vec{u_\theta}}$$

2- Éq Maxwell-Ampère .

$$\vec{rot}\,\vec{B}(M) = \mu_0 \vec{j}(M)$$

$$\frac{1}{r}\frac{d(rB)}{dr}\vec{u}_z = \mu_0 \vec{j}(M)$$

$r < a$ $\quad \vec{j}(M) = j_0 \frac{r^2}{a^2}\vec{e}_z$

$$\frac{1}{r}\frac{d(rB)}{dr} = \mu_0 j_0 \frac{r^2}{a^2}$$

$$\frac{d(rB)}{dr} = \mu_0 j_0 \frac{r^3}{a^2}$$

$$rB = \frac{\mu_0 j_0 r^4}{4a^2} + K_1$$

$$B(r) = \frac{\mu_0 j_0 r^3}{4a^2} + \frac{K_1}{r}$$

$r \to 0$ $\quad B(r)$ ne diverge pas $\Rightarrow K_1 = 0$

$$\boxed{\vec{B} = \frac{\mu_0 j_0 r^3}{4a^2}\vec{u}_\theta}$$

$r > a$ $\quad \vec{j} = \vec{0}$

$$\frac{1}{r}\frac{d(rB)}{dr} = 0$$

$$rB(r) = K_2$$

$$B(r) = \frac{K_2}{r}$$

Continuité en $r = a$

$$K_2 = a \times \frac{\mu_0 j_0 a}{4}$$

$$\boxed{\vec{B} = \frac{\mu_0 j_0 a^2}{4r}\vec{u}_\theta}$$