FORMULAIRE D' ÉLECTROMAGNÉTISME

1 Expressions des opérateurs en coordonnées cartésiennes

Les expressions des opérateurs en coordonnées cartésiennes doivent être connues. Elles ne sont jamais rappelées.

$$\begin{aligned}
\overrightarrow{grad} f &= \frac{\partial f}{\partial x}\,\vec{u}_x + \frac{\partial f}{\partial y}\,\vec{u}_y + \frac{\partial f}{\partial z}\,\vec{u}_z \\
div\,\vec{a} &= \vec{\nabla}\,.\,\vec{a} = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z} \\
\overrightarrow{rot}\,\vec{a} &= \vec{\nabla}\wedge\vec{a} = \left(\frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z}\right)\vec{u}_x + \left(\frac{\partial a_x}{\partial z} - \frac{\partial a_z}{\partial x}\right)\vec{u}_y + \left(\frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y}\right)\vec{u}_z \\
\triangle f &= div\,\overrightarrow{grad}\,f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \\
\triangle\vec{a} &= \triangle a_x\,\vec{u}_x + \triangle a_y\,\vec{u}_y + \triangle a_z\,\vec{u}_z
\end{aligned}$$

2 Identités remarquables

Si $f : (M,t) \mapsto f(M,t)$ désigne un champ scalaire et $\vec{a} : (M,t) \mapsto \vec{a}(M,t)$ un champ vectoriel, définis en tout point M de l'espace et à chaque instant t, alors on a les identités suivantes :

$$\overrightarrow{rot}\left[\overrightarrow{rot}\,(\vec{a})\right] = \overrightarrow{grad}\,(\,div\,(\vec{a}\,)) - \triangle\,\vec{a} \qquad (F_1)$$

$$div\,(f\,\vec{a}\,) = f\,div\,(\vec{a}) + \vec{a}.\overrightarrow{grad}(f) \qquad (F_2)$$

$$\overrightarrow{rot}\,(f\,\vec{a}\,) = f\,\overrightarrow{rot}\,(\vec{a}) + \overrightarrow{grad}\,(f)\wedge\vec{a} \qquad (F_3)$$

$$div\left(\vec{a}\wedge\vec{b}\right) = \vec{b}.\overrightarrow{rot}(\vec{a}) - \vec{a}.\overrightarrow{rot}(\vec{b}) \qquad (F4)$$

$$\overrightarrow{rot}\left[\overrightarrow{grad}(f)\right] = \vec{0} \qquad (F_5)$$

$$div\left[\overrightarrow{rot}\,(\vec{a})\right] = 0 \qquad (F_6)$$

3 Opérateurs dans les coordonnées cylindriques et sphériques

Coordonnées cylindriques

$$\begin{aligned}
\overrightarrow{grad}\;f &= \frac{\partial f}{\partial r}\,\vec{u}_r + \frac{1}{r}\frac{\partial f}{\partial\theta}\,\vec{u}_\theta + \frac{\partial f}{\partial z}\,\vec{u}_z \\
div\;\vec{A} &= \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\theta}{\partial\theta} + \frac{\partial A_z}{\partial z} \\
\overrightarrow{rot}\;\vec{A} &= \left(\frac{1}{r}\frac{\partial A_z}{\partial\theta} - \frac{\partial A_\theta}{\partial z}\right)\vec{u}_r + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\vec{u}_\theta + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right)\vec{u}_z \\
\Delta\;f &= div\;\overrightarrow{grad}\;f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial\theta^2} + \frac{\partial^2 f}{\partial z^2}
\end{aligned}$$

Coordonnées sphériques

$$\begin{aligned}
\overrightarrow{grad}\;f &= \frac{\partial f}{\partial r}\,\vec{u}_r + \frac{1}{r}\frac{\partial f}{\partial\theta}\,\vec{u}_\theta + \frac{1}{r\sin\theta}\,\frac{\partial f}{\partial\varphi}\,\vec{u}_\varphi \\
div\;\vec{A} &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2A_r\right) + \frac{1}{r\sin\theta}\,\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\,\frac{\partial A_\varphi}{\partial\varphi} \\
\overrightarrow{rot}\;\vec{A} &= \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(\sin\theta A_\varphi) - \frac{\partial A_\theta}{\partial\varphi}\right)\vec{u}_r + \left(\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{1}{r}\,\frac{\partial}{\partial r}(rA_\varphi)\right)\vec{u}_\theta \\
&\quad + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right)\vec{u}_\varphi \\
\Delta\;f &= div\;\overrightarrow{grad}\;f = \frac{1}{r^2}\,\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\,\frac{\partial^2 f}{\partial\varphi^2}
\end{aligned}$$