Signaux sinusoïdaux et images complexes

Loi des mailles: $u_R + u_L = e$ or $u_R = R.i$ et $u_L = L\frac{di}{dt}$ donc

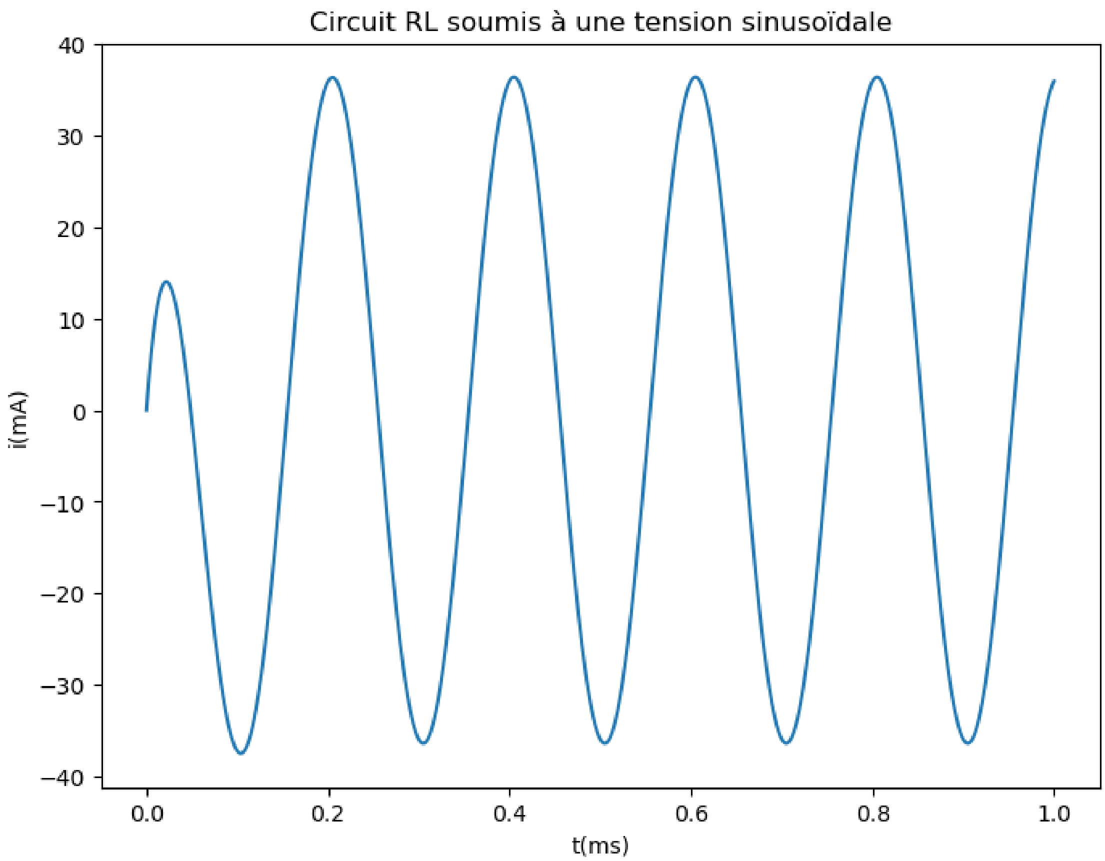
$$L\frac{di}{dt} + Ri = e = E\cos(2\pi f t)$$

[21]:
```python
import numpy as np
import matplotlib.pyplot as plt

R,L=100,3e-3
E,f=5,5000
w=2*np.pi*f
def e(t):
    return E*np.cos(w*t+0.6)
```

[22]:
```python
T=1/f
pas=T/10000
tmax=5*T
N=int(tmax/pas)
def didt(i,t):
    return (e(t)-R*i)/L
tab_t=np.zeros(N+1)
tab_i=np.zeros(N+1)
tab_t[0],tab_i[0]=0,0
for n in range(N):
    tab_t[n+1]=tab_t[n]+pas
    tab_i[n+1]=tab_i[n]+pas*didt(tab_i[n],tab_t[n])
```

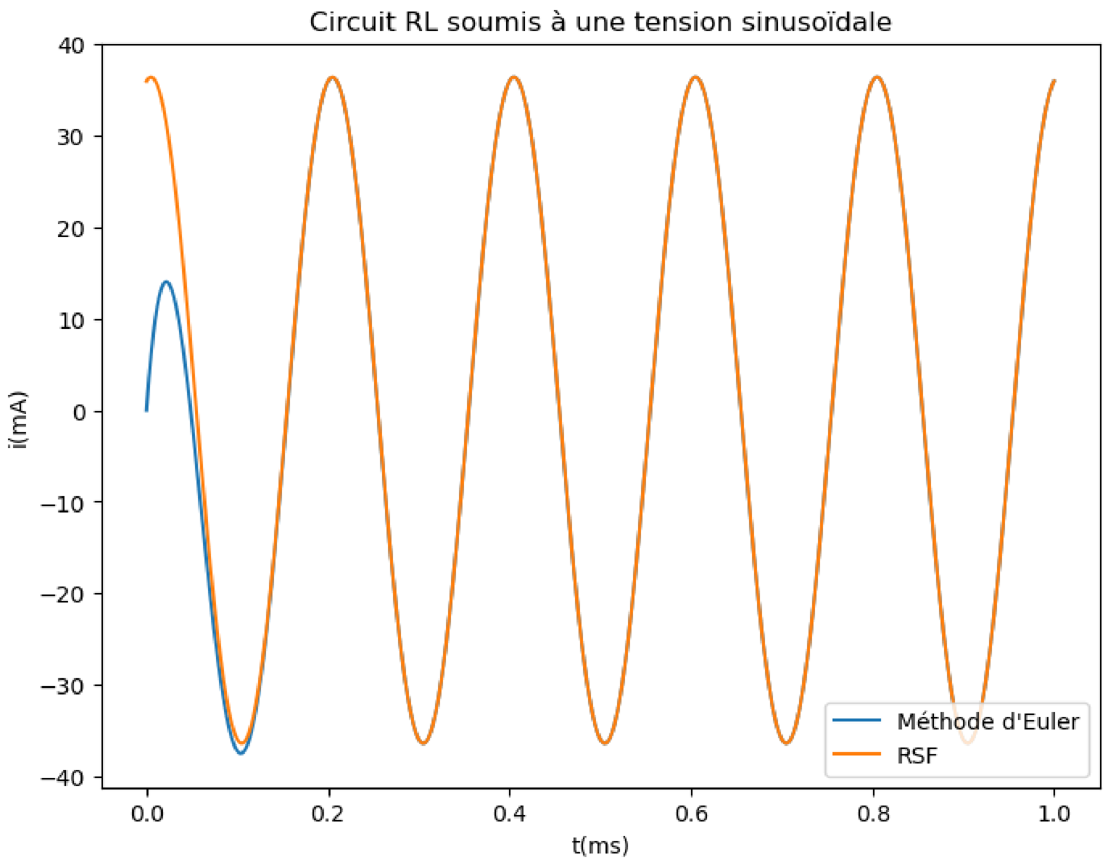
[23]:
```python
plt.figure(figsize=(8,6))
plt.plot(tab_t*1e3,tab_i*1e3,label="Méthode d'Euler")
plt.xlabel("t(ms)")
plt.ylabel("i(mA)")
plt.title("Circuit RL soumis à une tension sinusoïdale")
plt.show()
```



$\underline{i} = \frac{e}{R+jL\omega} = \frac{5e^{j(10000\pi t+0,6)}}{100+j.3.10^{-3}.10000\pi}$ donc on trouve après calculs $|\underline{i}(0)| = 36,4mA$ et $\arg \underline{i(0)} = -0,156rad$; donc

$$u_C(t) = 36,4\cos(10000\pi t - 0,156)mA$$

[24]:
```
i_=5*np.exp(0.6j)/(100+1j*w*3e-3)
iamp=np.abs(i_)
iph=np.angle(i_)
plt.figure(figsize=(8,6))
plt.plot(tab_t*1e3,tab_i*1e3,label="Méthode d'Euler")
plt.plot(tab_t*1e3,iamp*1e3*np.cos(w*tab_t+iph),label="RSF")
plt.xlabel("t(ms)")
plt.ylabel("i(mA)")
plt.title("Circuit RL soumis à une tension sinusoïdale")
plt.legend(loc=4)
plt.show()
```

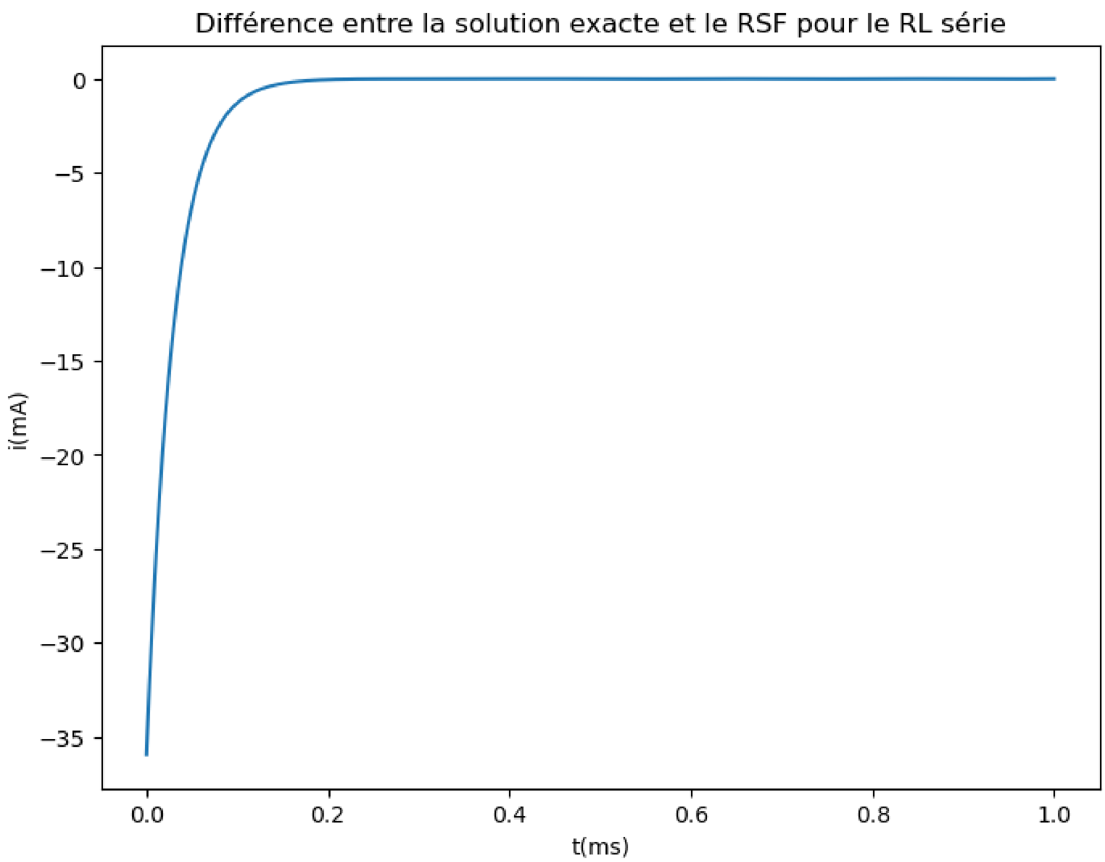


[ ]:

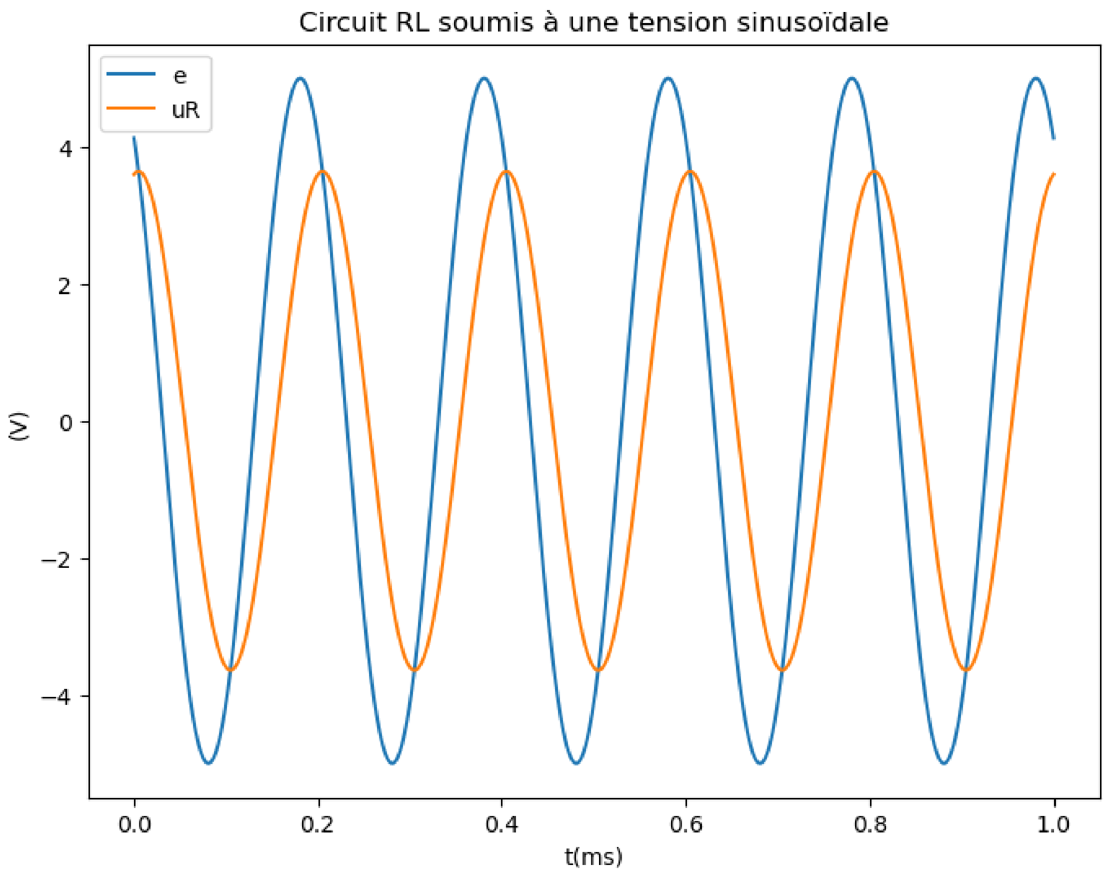
```
```

[25]:

```
i_=5*np.exp(0.6j)/(R+1j*L*w)
iamp=np.abs(i_)
iph=np.angle(i_)
plt.figure(figsize=(8,6))
plt.plot(tab_t*1e3,tab_i*1e3-iamp*1e3*np.cos(w*tab_t+iph))
plt.xlabel("t(ms)")
plt.ylabel("i(mA)")
plt.title("Différence entre la solution exacte et le RSF pour le RL série")
plt.show()
```



[26]:
```
plt.figure(figsize=(8,6))
plt.plot(tab_t*1e3,5*np.cos(10000*np.pi*tab_t+0.6),label="e")
plt.plot(tab_t*1e3,R*0.0364*np.cos(10000*np.pi*tab_t-0.156),label="uR")
plt.xlabel("t(ms)")
plt.ylabel("(V)")
plt.title("Circuit RL soumis à une tension sinusoïdale")
plt.legend()
plt.show()
```

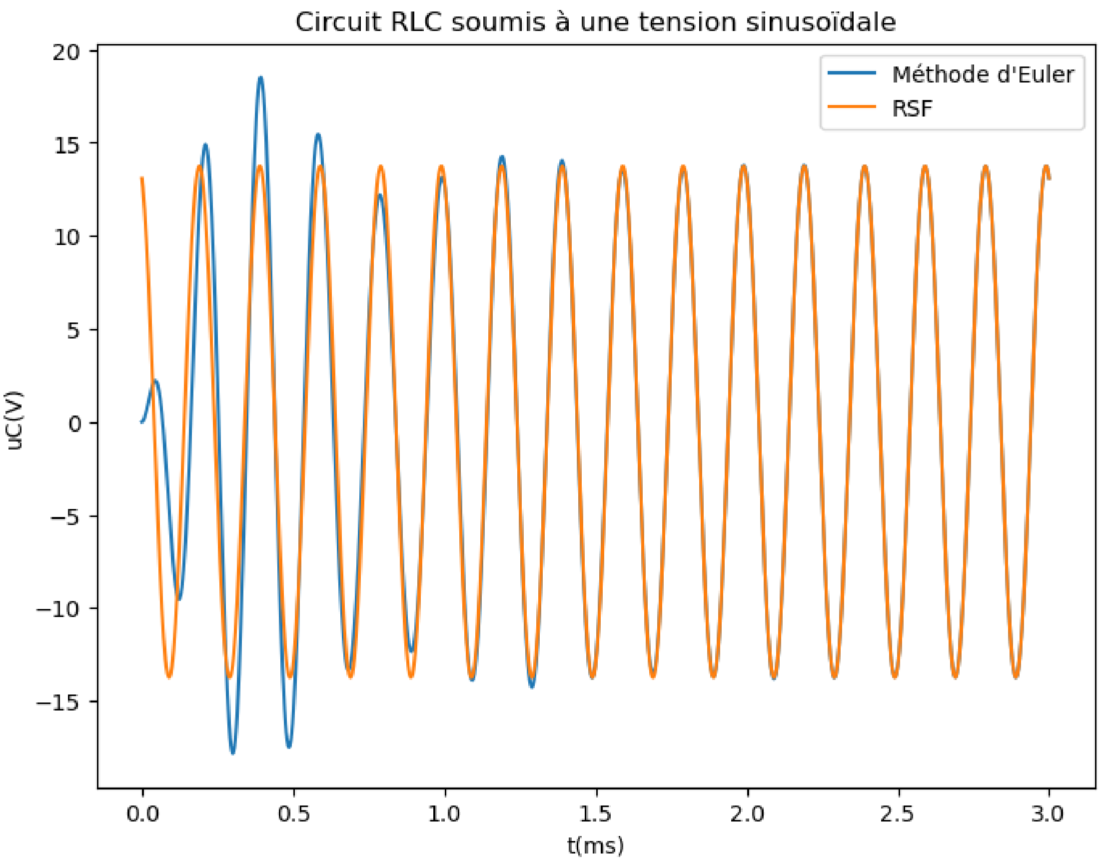


Cette fois l'ED est $LC\frac{d^2u_C}{dt^2} + RC\frac{du_C}{dt} + u_C = e$.

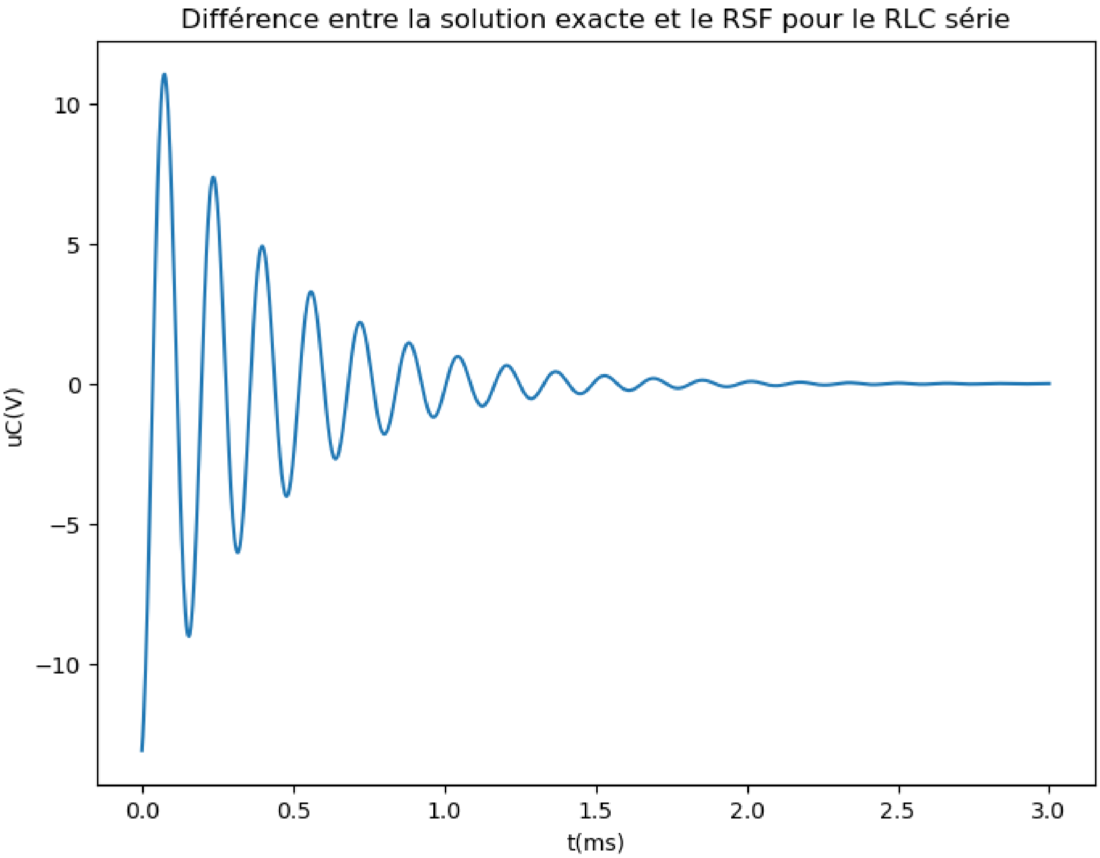
On en tire que $\underline{u_C} = \frac{5e^{j(10000\pi t+0,6}}{1+j.220.10^{-6}.10000\pi-1100.10^{-12}.(10000\pi)^2}$ puis en RSF: $u_C(t) = 13,74\cos(10000\pi t + 0,31)$

[38]:
```
from scipy.integrate import odeint
R=15
L=3e-3
C=220e-9
tmax=3e-3
N=int(tmax/pas)
def derivee(inconnues,t): # uC,duCdt
    uC,duCdt=inconnues
    dduCdtt=(e(t)-uC-R*C*duCdt)/(L*C)
    return [duCdt,dduCdtt]
tab_t=np.linspace(0,tmax,1000)
ci=[0,0]
sol=odeint(derivee,ci,tab_t)
tab_uC=sol[:,0]
tab_duCdt=sol[:,1]
```

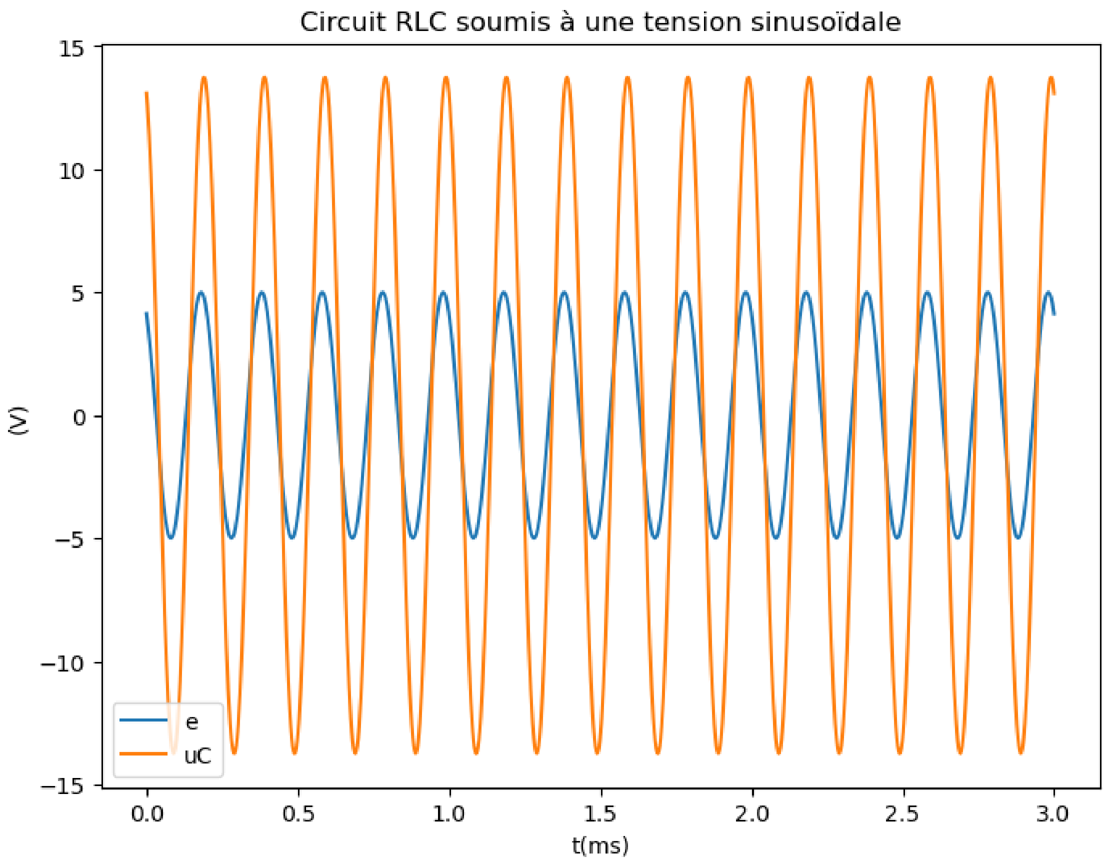
```
uc_=5*np.exp(0.6j)/(1+1j*R*C*w-L*C*w**2)
ucamp=np.abs(uc_)
ucph=np.angle(uc_)
plt.figure(figsize=(8,6))
plt.plot(tab_t*1e3,tab_uC,label="Méthode d'Euler")
plt.plot(tab_t*1e3,ucamp*np.cos(w*tab_t+ucph),label="RSF")
plt.xlabel("t(ms)")
plt.ylabel("uC(V)")
plt.title("Circuit RLC soumis à une tension sinusoïdale")
plt.legend()
plt.show()
```



[39]:
```
plt.figure(figsize=(8,6))
plt.plot(tab_t*1e3,tab_uC-ucamp*np.cos(w*tab_t+ucph))
plt.xlabel("t(ms)")
plt.ylabel("uC(V)")
plt.title("Différence entre la solution exacte et le RSF pour le RLC série")
plt.show()
```



[40]:
```
plt.figure(figsize=(8,6))
plt.plot(tab_t*1e3,5*np.cos(10000*np.pi*tab_t+0.6),label="e")
plt.plot(tab_t*1e3,13.74*np.cos(10000*np.pi*tab_t+0.31),label="uC")
plt.xlabel("t(ms)")
plt.ylabel("(V)")
plt.title("Circuit RLC soumis à une tension sinusoïdale")
plt.legend()
plt.show()
```


Circuit RLC soumis à une tension sinusoïdale
15
10
5
0
−5
−10
−15
(V)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
t(ms)
e
uC

```
[ ]:
```