Pendule simple (1)

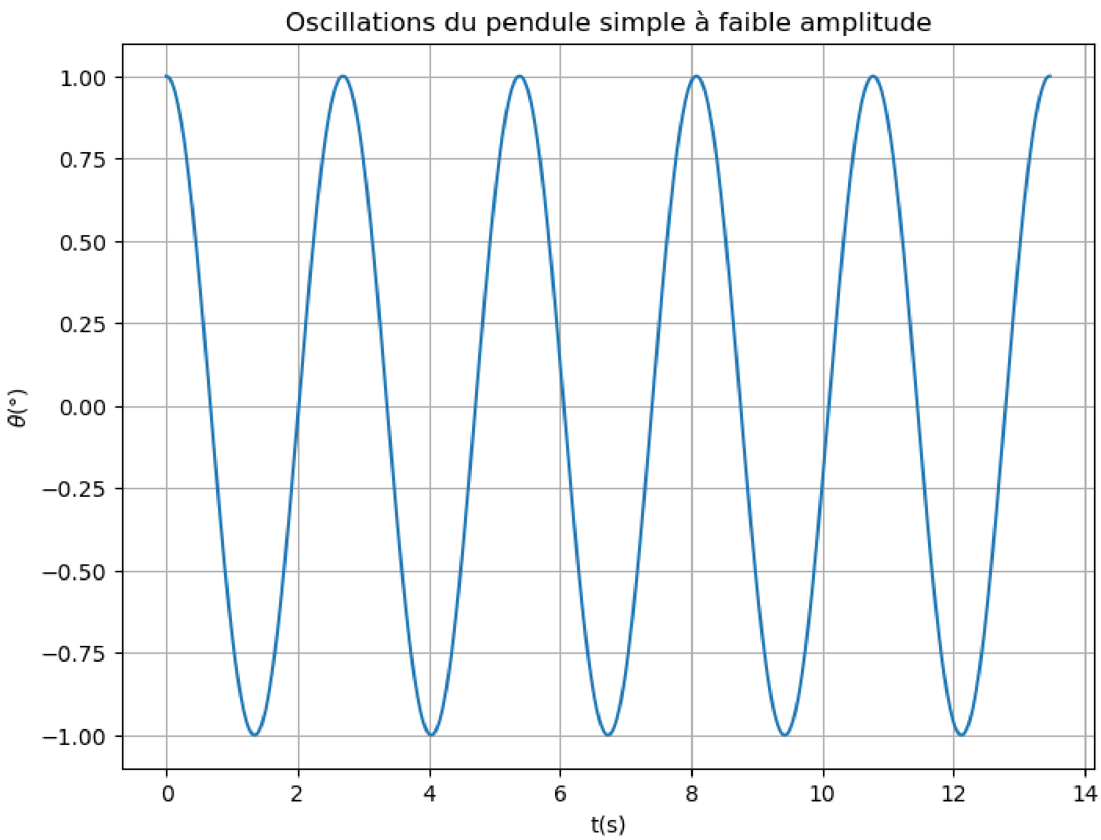
Étude cinématique: $\vec{a} = -\ell\dot{\theta}^2\vec{e_r} + \ell\ddot{\theta}\vec{e_\theta}$.

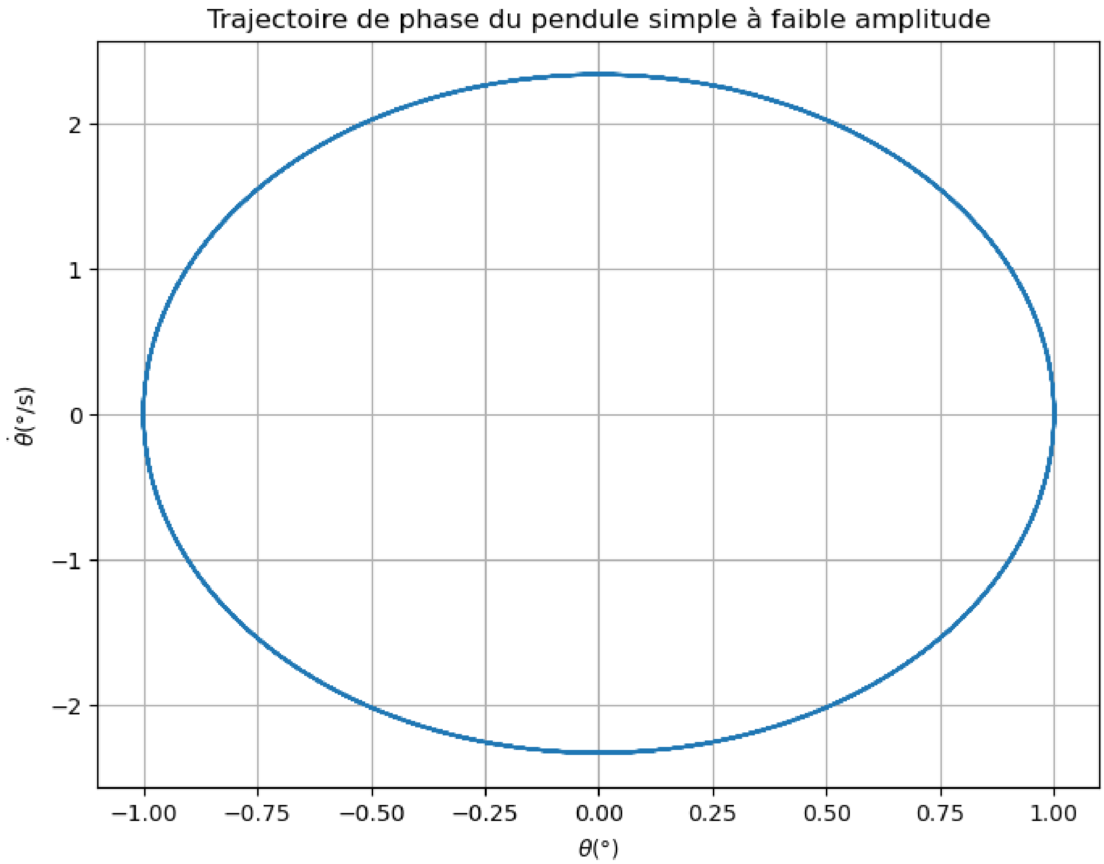
Forces: le poids $mg(\cos\theta\vec{e_r} - \sin\theta\vec{e_\theta})$ et la tension du fil $-T.\vec{e_r}$.

Le PFD projeté sur $\vec{e_\theta}$ donne $m\ell\ddot{\theta} = -mg\sin\theta$

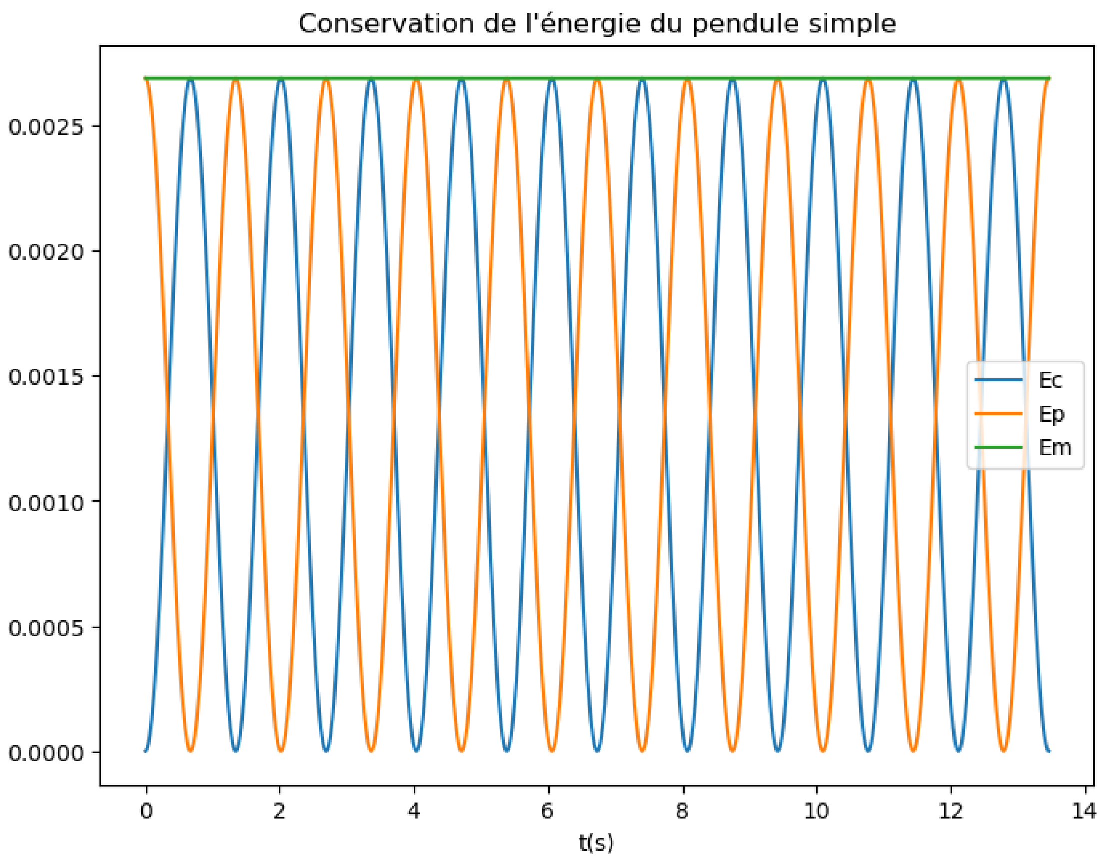
[1]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint
```

[15]:
```
L,m,g=1.8,1,9.8
w0=np.sqrt(g/L)
T0=2*np.pi/w0
def derivee(inconnues,t):
    theta=inconnues[0]
    dtheta=inconnues[1]
    ddtheta=-g*np.sin(theta)/L
    return [dtheta,ddtheta]
tab_t=np.linspace(0,5*T0,1000)
ci=[1*np.pi/180,0]
sol=odeint(derivee,ci,tab_t)
tab_theta=sol[:,0]
tab_dtheta=sol[:,1]
plt.figure(figsize=(8,6))
plt.plot(tab_t,tab_theta*180/np.pi)
plt.xlabel("t(s)")
plt.ylabel(r"$\theta$(°)")
plt.title("Oscillations du pendule simple à faible amplitude")
plt.grid()
plt.show()
plt.figure(figsize=(8,6))
plt.plot(tab_theta*180/np.pi,tab_dtheta*180/np.pi)
plt.xlabel(r"$\theta$(°)")
plt.ylabel(r"$\dot \theta$(°/s)")
plt.title("Trajectoire de phase du pendule simple à faible amplitude")
plt.grid()
plt.show()
```

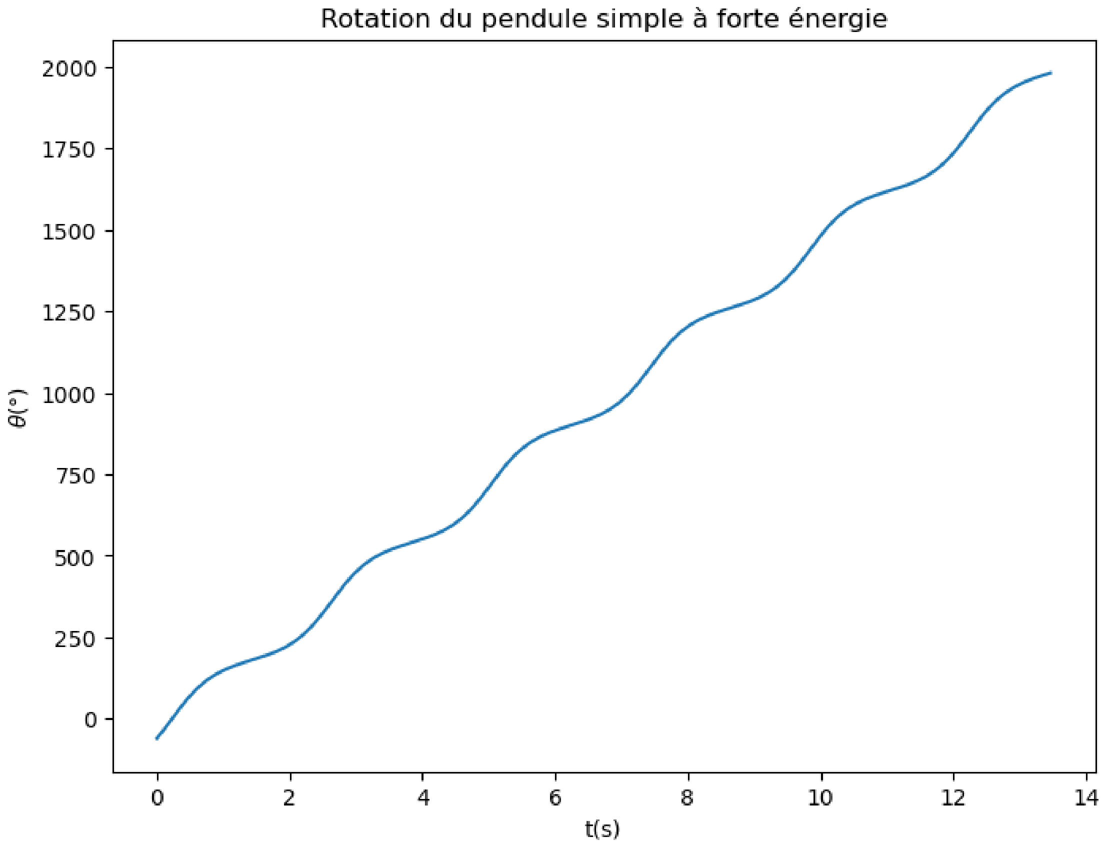

Oscillations du pendule simple à faible amplitude
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
θ(°)
0
2
4
6
8
10
12
14
t(s)

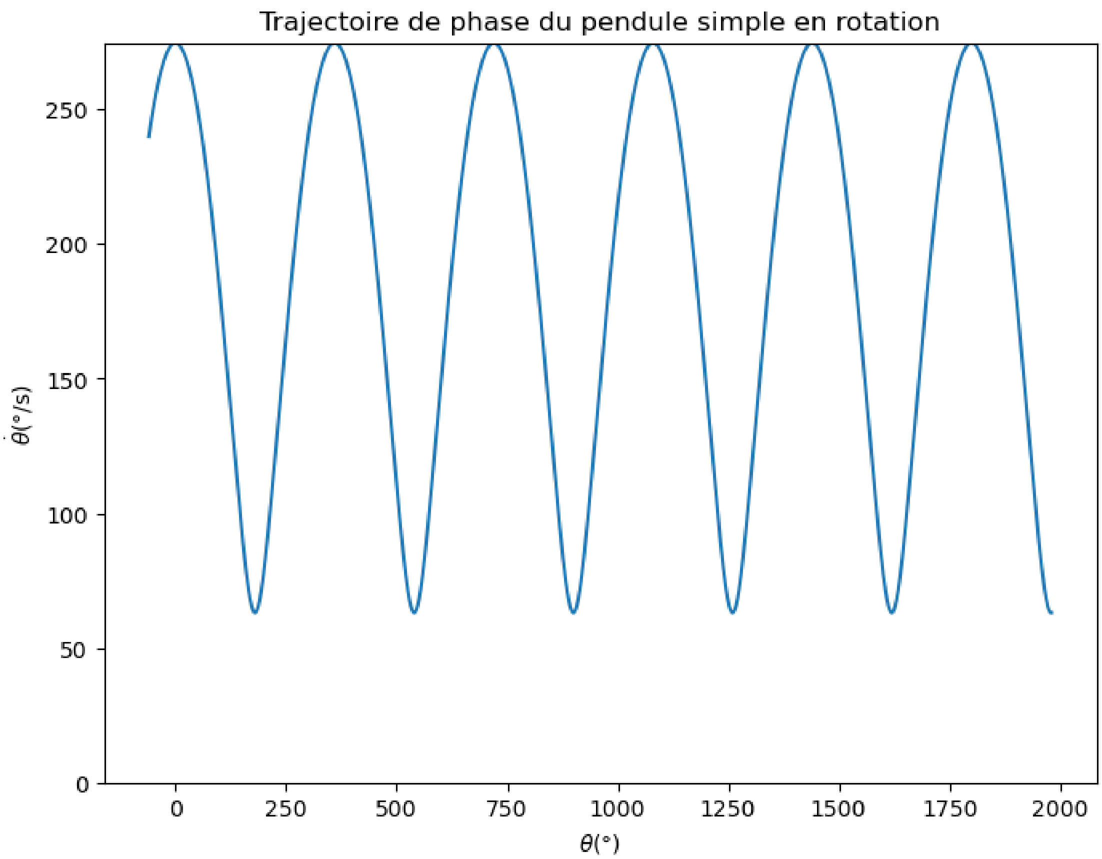


[7]:
```
tab_Ec=0.5*m*L**2*tab_dtheta**2
tab_Ep=m*g*L*(1-np.cos(tab_theta))
tab_Em=tab_Ec+tab_Ep
plt.figure(figsize=(8,6))
plt.plot(tab_t,tab_Ec,label="Ec")
plt.plot(tab_t,tab_Ep,label="Ep")
plt.plot(tab_t,tab_Em,label="Em")
plt.legend()
plt.xlabel("t(s)")
plt.title("Conservation de l'énergie du pendule simple")
plt.show()
```

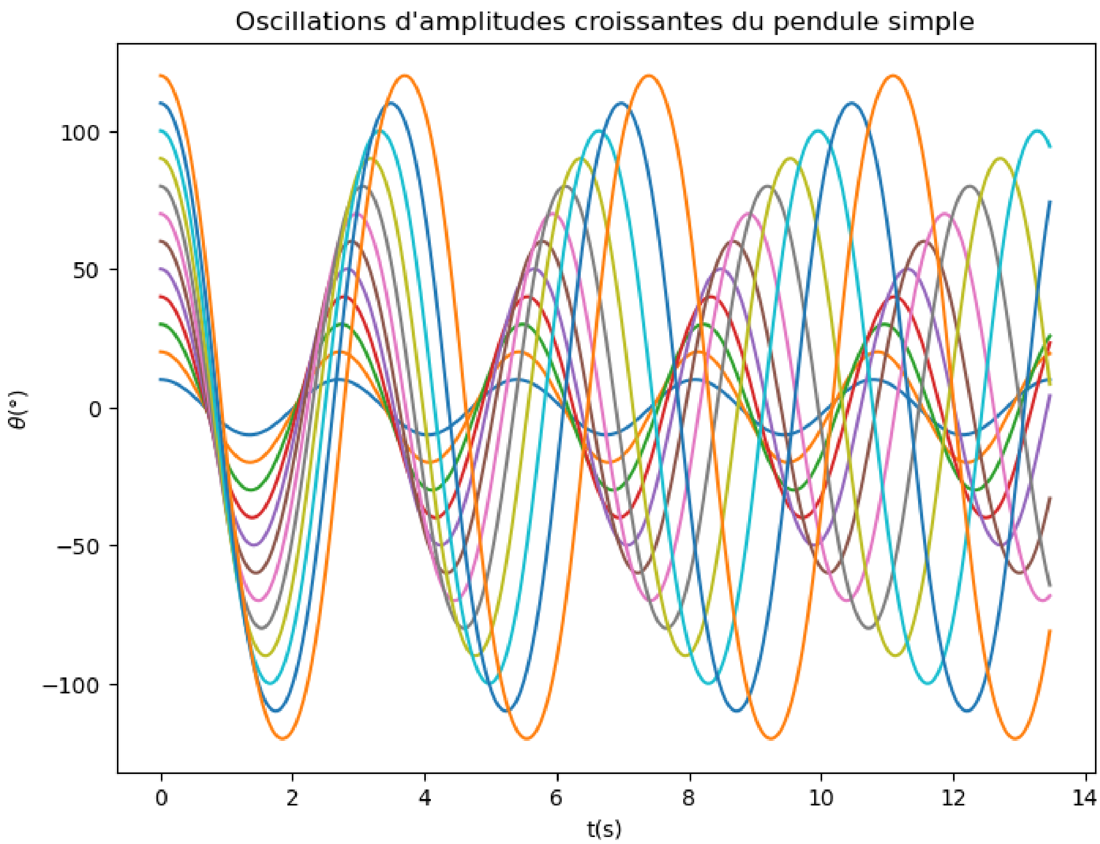


[9]:
```
tab_t=np.linspace(0,5*T0,1000)
ci=[-60*np.pi/180,240*np.pi/180]
sol=odeint(derivee,ci,tab_t)
tab_theta=sol[:,0]
tab_dtheta=sol[:,1]
plt.figure(figsize=(8,6))
plt.plot(tab_t,tab_theta*180/np.pi)
plt.xlabel("t(s)")
plt.ylabel(r"$\theta$(°)")
plt.title("Rotation du pendule simple à forte énergie")
plt.show()
plt.figure(figsize=(8,6))
plt.plot(tab_theta*180/np.pi,tab_dtheta*180/np.pi)
plt.xlabel(r"$\theta$(°)")
plt.ylabel(r"$\dot \theta$(°/s)")
plt.ylim([0,max(tab_dtheta*180/np.pi)])
plt.title("Trajectoire de phase du pendule simple en rotation")
plt.show()
```

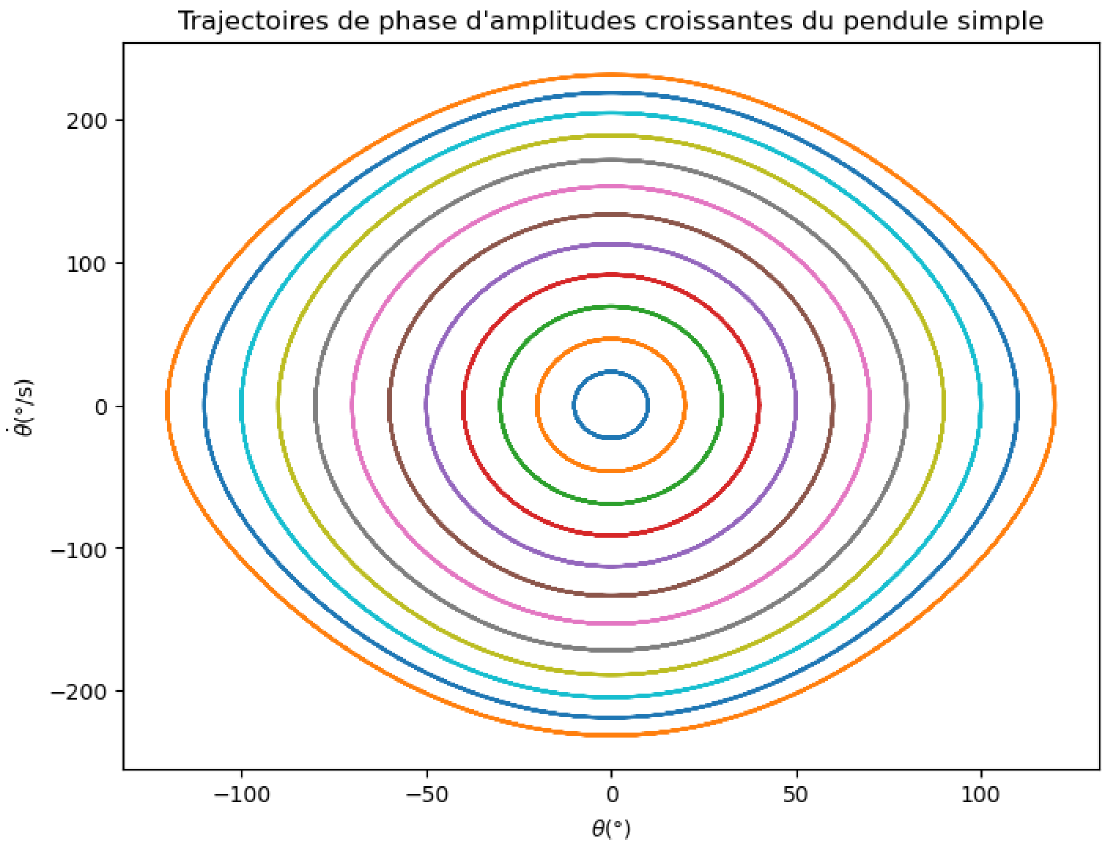

Rotation du pendule simple à forte énergie
θ(°)
t(s)
2000
1750
1500
1250
1000
750
500
250
0
0
2
4
6
8
10
12
14



[10]:
```
plt.figure(figsize=(8,6))
tab_t=np.linspace(0,5*T0,1000)
for theta0 in range(10,121,10):
    ci=[theta0*np.pi/180,0]
    sol=odeint(derivee,ci,tab_t)
    tab_theta=sol[:,0]
    plt.plot(tab_t,tab_theta*180/np.pi)
plt.xlabel("t(s)")
plt.ylabel(r"$\theta$(°)")
plt.title("Oscillations d'amplitudes croissantes du pendule simple")
plt.show()
```



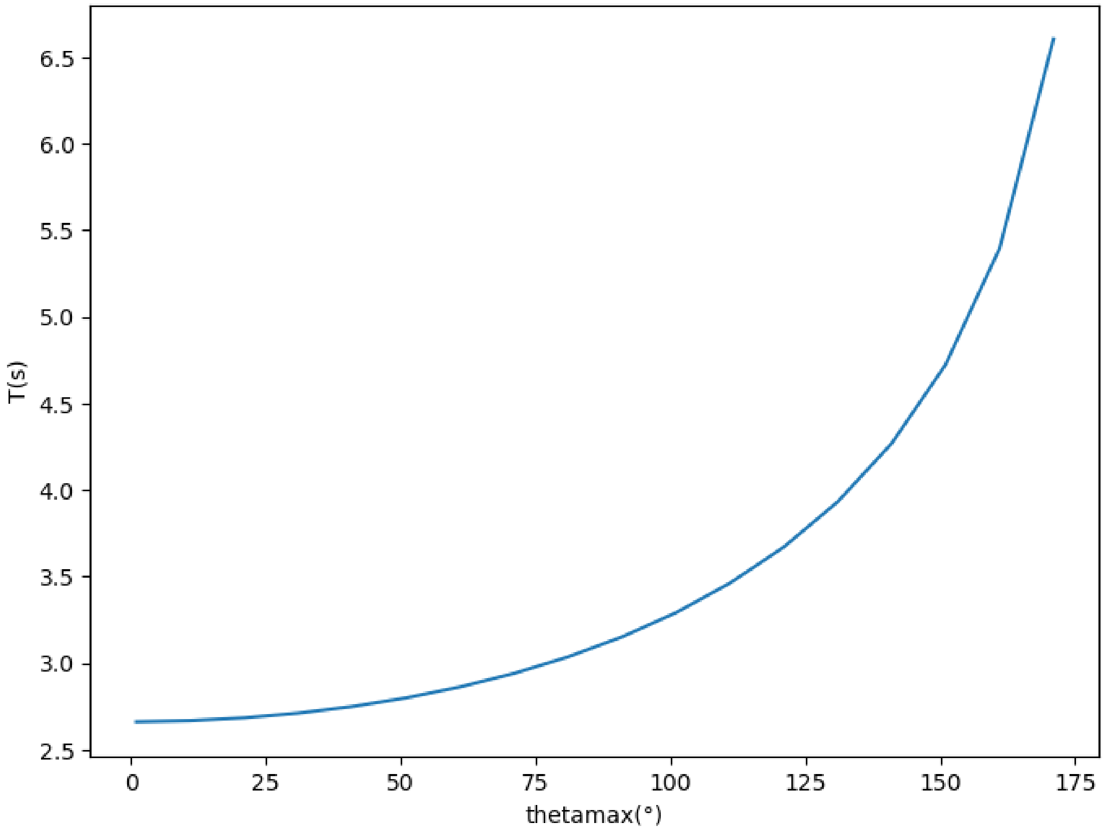
[11]:
```
plt.figure(figsize=(8,6))
tab_t=np.linspace(0,5*T0,1000)
for theta0 in range(10,121,10):
    ci=[theta0*np.pi/180,0]
    sol=odeint(derivee,ci,tab_t)
    tab_theta=sol[:,0]
    tab_dtheta=sol[:,1]
    plt.plot(tab_theta*180/np.pi,tab_dtheta*180/np.pi)
plt.xlabel(r"$\theta$(°)")
plt.ylabel(r"$\dot\theta$(°/s)")
plt.title("Trajectoires de phase d'amplitudes croissantes du pendule simple")
plt.show()
```



On constate un allongement de la période lorsque l'amplitude augmente.

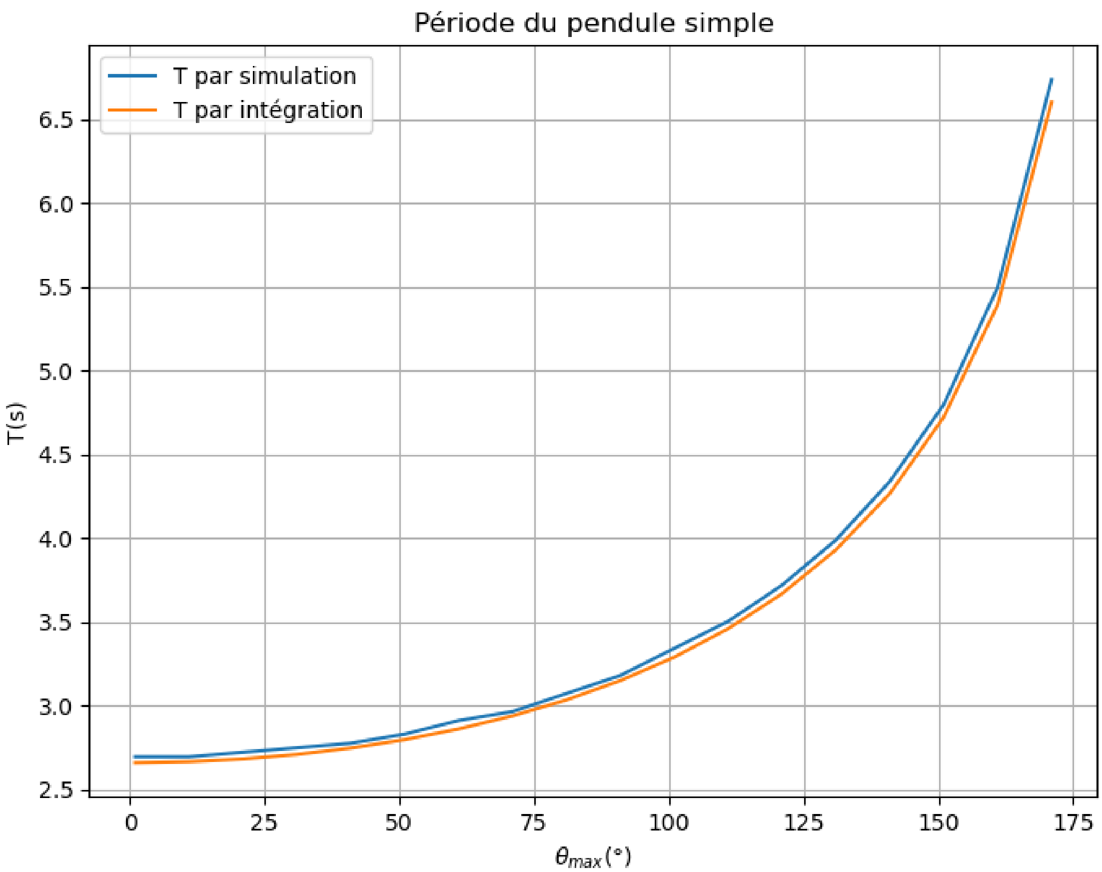
[13]:
```
def T(thetamax): # thetamax en DEGRÉS
    thetamax=thetamax*np.pi/180
    def f(theta,thetamax):
        return np.sqrt(2*L/g/(np.cos(theta)-np.cos(thetamax)))
    somme=0
    N=1000
    pas=2*thetamax/N
    for i in range(N):
        somme+=f(-thetamax+(i+0.5)*pas,thetamax)
    return somme*pas
liste_thetamax=[]
liste_T=[]
for thetamax in range(1,172,10):
    liste_thetamax.append(thetamax)
    liste_T.append(T(thetamax))
plt.figure(figsize=(8,6))
plt.plot(liste_thetamax,liste_T)
plt.xlabel("thetamax(°)")
```

```
plt.ylabel("T(s)")
plt.show()
```



[14]:
```
def T_simul(thetamax): # thetamax en DEGRÉS
    ci=[thetamax*np.pi/180,0]
    sol=odeint(derivee,ci,tab_t)
    tab_theta=sol[:,0]
    i1=0
    while tab_theta[i1]>0:
        i1+=1
    i2=i1+1
    while tab_theta[i2]<0:
        i2+=1
    return 2*(tab_t[i2]-tab_t[i1])
liste_thetamax=[]
liste_T_simul=[]
for thetamax in range(1,172,10):
    liste_thetamax.append(thetamax)
    liste_T_simul.append(T_simul(thetamax))
plt.figure(figsize=(8,6))
```

```
plt.plot(liste_thetamax,liste_T_simul,label="T par simulation")
plt.plot(liste_thetamax,liste_T,label="T par intégration")
plt.xlabel(r"$\theta_{max}$(°)")
plt.ylabel("T(s)")
plt.legend()
plt.title("Période du pendule simple")
plt.grid()
plt.show()
```

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[ ]:
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[ ]:
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