Correction DS3

Exercice 1

Q1. loi des mailles : $E = r_E i + u_c$ $\quad i = C_0 \dfrac{du_c}{dt}$

$$\boxed{E = r_E C_0 \frac{du_c}{dt} + u_c}$$

Q2. On pose $\boxed{\tau = r_E C_0}$ $\quad \underline{\tau = 3200\,s}$

Q3. $u_c(t) = A \exp\left(-\dfrac{t}{\tau}\right) + E$

solution particulière

CI : $u_c(0^+) = u_c(0^-) = 0 \rightarrow A = -E$

$$\boxed{u_c(t) = E\left(1 - \exp\left(-\frac{t}{\tau}\right)\right)}$$

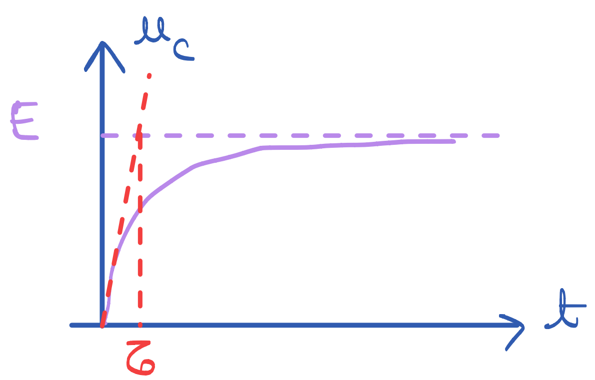
Q4. $u_c(t_1) > U_s$ $\quad 1 - \exp(-t_1/\tau) > \dfrac{U_s}{E}$ $\quad \boxed{t_1 > -\tau \ln\left(1 - \dfrac{U_s}{E}\right)}$

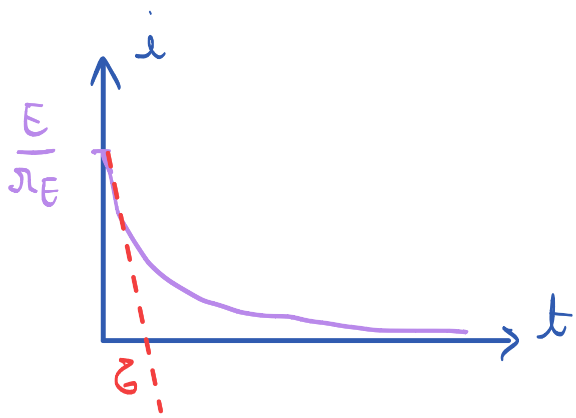
$t_1 = 6{,}65 \times 10^3\,s$

$\underline{t_1 \simeq 111\,\text{min}}$

Q5. $i(t) = C \dfrac{du_c}{dt}$ $\quad \boxed{i(t) = \dfrac{E}{r_E} \exp\left(-\dfrac{t}{\tau}\right)}$

Q6.





τ donne une indication sur la durée du régime transitoire :

durée $\simeq 5\tau$ $\quad$ ($u_c(5\tau) = 99\%\, E$, $i(5\tau) \simeq 1\% \frac{E}{r_E}$)

Q7. $E_m = \frac{1}{2} C U_m^2 \rightarrow$ $\boxed{C = \frac{2E}{U_m^2}}$ $\quad$ $\underline{C = 800F}$

Q8. Durée de charge $\simeq 5\tau = 5 r_E C = \underline{4000\,s}$

Q9. $P_G(t) = E\, i(t)$

$\boxed{P_G(t) = \frac{E^2}{r_E} \exp(-t/\tau)}$ $\quad$ $\boxed{P_{G,max} = \frac{E^2}{r_E}} = \underline{250\text{ kW}}$

Q10. $E_G = \int_0^{\infty} E\, i\, dt = \int_0^E E C\, du_c = C E^2$ $\quad$ $\boxed{E_G = CE^2} = \underline{200MJ}$

Q11. $E_e = \frac{1}{2} CE^2 - \underbrace{\frac{1}{2} C u_c^2(0)}_{0}$ $\quad$ $\boxed{E_e = \frac{1}{2} CE^2} = \underline{100\,MJ}$

Q12. $E_J = E_G - E_e = \underline{100\,MJ}$

Q13. $\underline{\rho = 50\%}$

on reprend $\hat{c}$ à Q3 et Q5

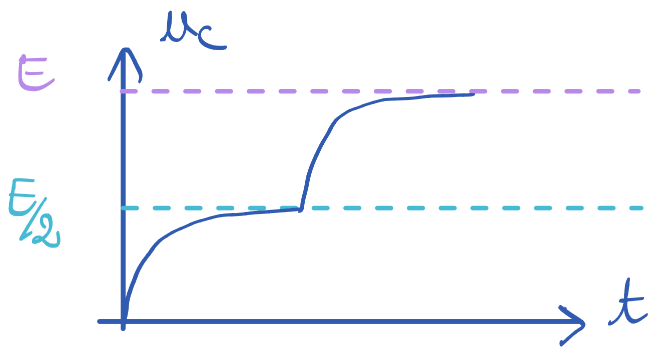
Q14. $\boxed{u_c(t) = \frac{E}{2}\left(1 - \exp(-t/\tau)\right)}$ $\quad$ $\boxed{i(t) = \frac{E}{2r_E} \exp\left(-\frac{t}{\tau}\right)}$

Q15. $u_c(t) = A \exp\left(\frac{-t}{\tau}\right) + E$ $\quad$ ⚠ $u_c(0) = \frac{E}{2}$

$\boxed{u_c(t) = E\left(1 - \frac{1}{2} \exp\left(\frac{-t}{\tau}\right)\right)}$

Q16. $\boxed{i(t) = \dfrac{E}{2r_E}\exp\left(-\dfrac{t}{\tau}\right)}$

Q17.



Q18. $\boxed{E'_e = \dfrac{1}{2} C E^2}$

Q19. $E_{G_1} = \displaystyle\int_0^{E/2} E C \, du_C = \underline{C \frac{E^2}{4}}$

$E_{G_2} = \displaystyle\int_{E/2}^{E} E C \, du_C = CE\left(E - \frac{E}{2}\right) = \underline{C \frac{E^2}{2}}$

$\boxed{E'_G = \dfrac{3}{4} C E^2}$

Q20. $\boxed{\rho = \dfrac{2}{3}}$ $\rho \simeq 67\% > 50\%$

Q21. $E'_J = E'_G - E'_e = \dfrac{3}{4} CE^2 - \dfrac{1}{2} CE^2$ $\boxed{E'_J = \dfrac{1}{4} CE^2}$ $= \underline{50\,mJ}$

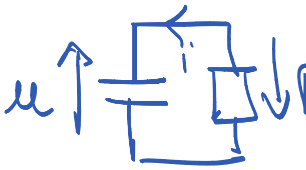
2° procédé <u>+ efficace</u> (− de pertes par effet Joule)

Exercice 2 :

Q22. Circuit A : $\boxed{E = L\frac{di}{dt}}$ $\boxed{i(t) = \frac{E}{L}t + I_0}$

$\boxed{i(\alpha T) = \frac{E\alpha T}{L} + I_0} \leftarrow I_1$

Q23. Circuit B : $\boxed{u + RC\frac{du}{dt} = 0}$ $\boxed{u(t) = U_0 \exp\left(\frac{-t}{\tau}\right)}$



$Ri = RC\frac{du}{dt}$

si $\tau = RC$

$\boxed{u(\alpha T) = U_0 \exp\left(-\frac{\alpha T}{\tau}\right)} \leftarrow U_1$

Q24. $E = L\frac{di}{dt} + u$ $i = i_R + i_C = \frac{u}{R} + C\frac{du}{dt}$

$E = LC\ddot{u} + \frac{L}{R}\dot{u} + u$

$\boxed{\ddot{u} + \frac{1}{RC}\dot{u} + \frac{1}{LC}u = \frac{E}{LC}}$

On identifie :

$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$ $\frac{\omega_0}{Q} = \frac{1}{RC}$

$Q = \omega_0 RC$ $\boxed{Q = R\sqrt{\frac{C}{L}}}$

Q25. Equation caractéristique : $X^2 + \frac{\omega_0}{Q}X + \omega_0^2 = 0$ $\Delta = \omega_0^2\left(\frac{1}{Q^2} - 4\right)$

$\Delta < 0$ si pseudo-périodique

$X_{1/2} = -\frac{\omega_0}{2Q} \pm j\underbrace{\omega_0\sqrt{1 - \frac{1}{4Q^2}}}_{= \Omega}$

$\boxed{u_c(t) = E + \exp\left(-\frac{\omega_0 t}{2Q}\right)\left(A\cos(\Omega t) + B\sin(\Omega t)\right)}$

Q26. $u(\alpha T^-) = u(\alpha T^+) = U_1$ (par continuité de u)

$$\frac{du}{dt}(\alpha T) = \frac{i_C(\alpha T)}{C} = \frac{i(\alpha T) - i_R(\alpha T)}{C}$$

$i(\alpha T^-) = i(\alpha T^+) = I_1$ (par continuité de i)

$$i_R(\alpha T) = \frac{u(\alpha T)}{R} = \frac{U_1}{R}$$

$$\boxed{u(\alpha T) = U_1 \quad \text{et} \quad \frac{du}{dt}(\alpha T) = \frac{RI_1 - U_1}{RC}}$$

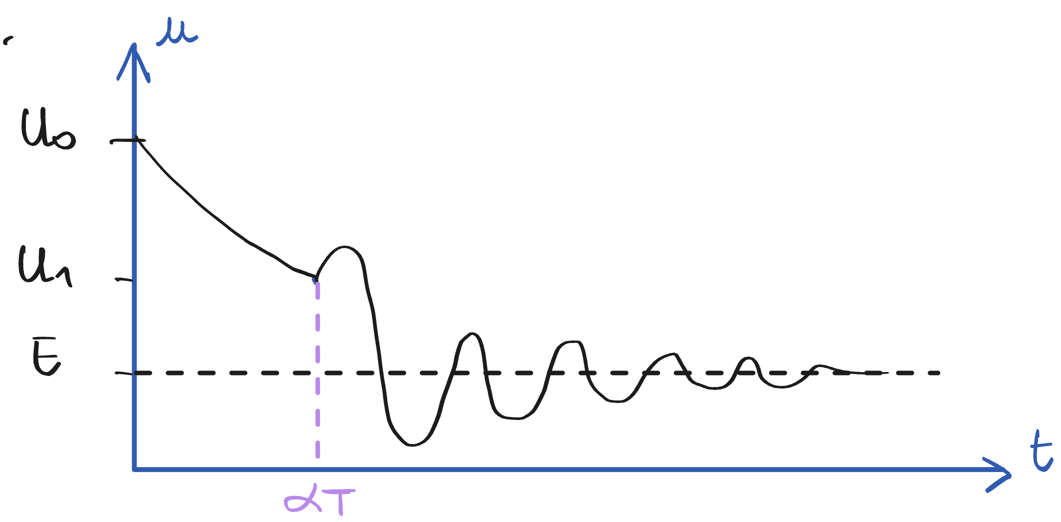
Q27. $i(\alpha T) = I_1$ (par continuité de i)

$$\frac{di}{dt} = \frac{u_L}{L} = \frac{E-u}{L} \qquad u(\alpha T) = U_1$$

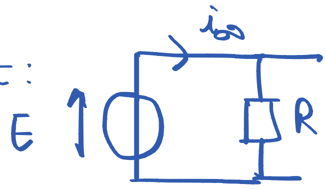
$$\boxed{i(\alpha T) = I_1 \qquad \frac{di}{dt}(\alpha T) = \frac{E - U_1}{L}}$$

Q28. Il faut $\underline{RI_1 > U_1}$ et $\underline{U_1 > E}$

Q29.

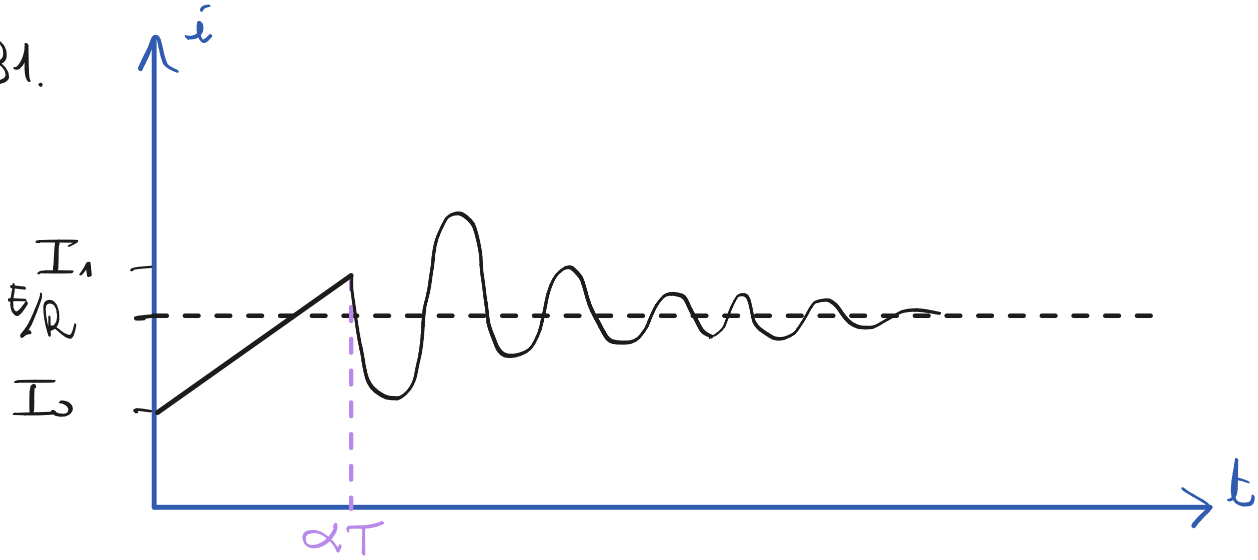


Q30. Circuit équivalent :



$$\boxed{i_\infty = \frac{E}{R}}$$

Q31.



Q32. $i(t) = i(\alpha T) + \left(\frac{di}{dt}\right)_{t=\alpha T^+}(t-\alpha T) = \boxed{I_1 + \frac{E-U_1}{L}(t-\alpha T)}$ $\quad \alpha T \leq t \leq T$

Q33. $u(t) = u(0) + \left(\frac{du}{dt}\right)_{t=0^+}(t-0) = \boxed{U_0 - \frac{U_0}{RC}t}$ $\quad 0 \leq t \leq \alpha T$

Q34. $u(t) = u(\alpha T) + \left(\frac{du}{dt}\right)_{t=\alpha T^+}(t-\alpha T) = \boxed{U_1 + \frac{RI_1 - U_1}{RC}(t-\alpha T)}$

$\alpha T \leq t \leq T$

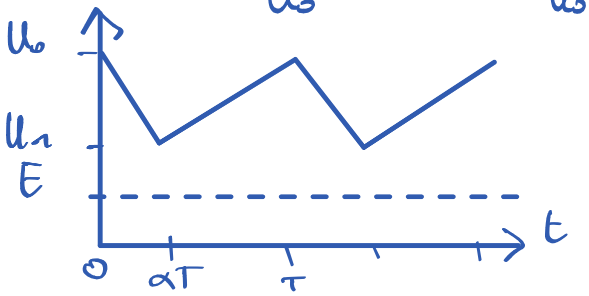
Q.35. $\boxed{u(T) = U_0 \qquad i(t) = I_0}$

Q36. $I_1 + \frac{E-U_1}{L}T(1-\alpha) = I_0$ $\quad$ avec $I_1 = I_0 + \frac{E\alpha T}{L}$

$\cancel{I_0} + \frac{E\alpha \cancel{T}}{\cancel{L}} + \frac{E\cancel{T}(1-\cancel{\alpha})}{\cancel{L}} - \frac{U_1}{\cancel{L}}\cancel{T}(1-\alpha) = \cancel{I_0} \rightarrow \boxed{U_1 = \frac{E}{1-\alpha}}$

Q37. $U_1 = U_0 - \frac{U_0}{RC}\alpha T = \frac{E}{1-\alpha}$ $\boxed{U_0 = \frac{E}{(1-\alpha)(1-\frac{\alpha T}{RC})}}$

Q38. $\frac{U_0 - U_1}{U_0} = 1 - \frac{U_1}{U_0} = 1 - \left(1 - \frac{\alpha T}{RC}\right) = \boxed{\frac{\alpha T}{RC}}$



Si $T \ll RC$ $U_0 \simeq U_1 \rightarrow u(t) \simeq Cte > E$

En jouant sur α on peut faire varier cette constante $U_1 = \frac{E}{1-\alpha}$.

Q39. $\boxed{E_{m_1} = \frac{1}{2} L I_1^2 - \frac{1}{2} L I_0^2}$ $\boxed{E_{m_2} = \frac{1}{2} L I_0^2 - \frac{1}{2} L I_1^2 = -E_{m_1}}$

$E_m = E_{m_1} + E_{m_2} = 0$ sur une période.

Q40. $E_{e_1} + E_{e_2} = \frac{1}{2} C U_0^2 - \frac{1}{2} C U_0^2 = 0$ sur une période

Q.41. $\boxed{E_J = E_g}$

Q42. Si $u = Cte = U_0$ $P_J = R i_R^2 = \frac{u^2}{R} = \frac{U_0^2}{R} = Cte \rightarrow E_J = P_J \times \Delta t$

Sur 1 période $\boxed{E_J = \frac{U_0^2}{R} T}$

Q43. On modélise le système par 1 oscillateur harmonique amorti en régime pseudo-périodique.

La pseudo-période est $T = 6{,}5\,s$ $\hookrightarrow$ $Q > 1/2$ $(\Delta < 0)$

Equation type : $\ddot{x} + \frac{\omega_0}{Q}\dot{x} + \omega_0^2 x = 0$ ← oscillations autour de $x = 0$

$X^2 + \frac{\omega_0}{Q}X + \omega_0^2 = 0 \quad \Delta < 0 \Rightarrow X_{1,2} = \frac{-\frac{\omega_0}{Q} \pm j\sqrt{|\Delta|}}{2}$ Ω

Solution $x(t) = \exp\left(-\frac{\omega_0}{2Q}t\right)\left(A\cos(\Omega t) + B\sin(\Omega t)\right)$

où $\Omega = \frac{2\pi}{T} = \omega_0\sqrt{1 - 1/4Q^2}$

Sans TMD : $\frac{x(t+T)}{x(t)} = 99\% \Rightarrow Q \gg 1$ donc $\Omega \simeq \omega_0$

$$\left.\begin{array}{l}\cos(\Omega t) = \cos(\Omega(t+T)) \\ \sin(\Omega t) = \sin(\Omega(t+T))\end{array}\right\} \Rightarrow \frac{x(t+T)}{x(t)} = \exp\left(-\frac{\omega_0 T}{2Q}\right)$$

$\sim 2\pi \rightarrow$ $-\frac{\omega_0 T}{2Q} = \ln(0{,}99)$

$\boxed{Q \simeq \frac{-\pi}{\ln(0{,}99)} = 312}$ SANS

$\boxed{Q = \frac{\pi}{\ln(0{,}95)} = 61}$ AVEC