Correction TD 11

Exercice 1

1) $\vec{v} = \vec{Cte}$: MRU $\quad d_R = v_0 t_R$

2) $\vec{a} = -a_0 \vec{e_x}$ $\quad \vec{v}(t) = \vec{v_0} - a_0 (t - t_R)\vec{e_x}$ ← Valable pour $t \geq t_R$

3) $x(t) = -\frac{1}{2} a_0 (t - t_R)^2 + v_0 (t - t_R) + v_0 t_R$

4) $v(t_1) = 0 \quad t_1 = \frac{v_0}{a_0} + t_R \quad d_a = x(t_1)$

$$d_a = -\frac{1}{2} a_0 \left(\frac{v_0}{a_0}\right)^2 + v_0 \frac{v_0}{a_0} + d_R \qquad d_a = \frac{v_0^2}{2a_0} + v_0 t_R$$

5) $d_a < D \quad \frac{v_0^2}{2a_0} + v_0 t_R < D \quad \frac{v_0^2}{2a_0} < D - v_0 t_R$

$$a_0 > \frac{v_0^2}{2(D - v_0 t_R)} \qquad a_0 > 12\ m.s^{-2}$$

6) Distance préconisée : $d_p = v_0 \times 2$ ← en s

$$d_a = d_p \Leftrightarrow \frac{v_0^2}{2a_0} + v_0 \underbrace{t_R}_{1s} = 2 v_0 \qquad a_0 = \frac{v_0}{2} = 18\ m.s^{-1}$$

Exercice 2

$t = 0$ départ de A $\qquad v_{max} = 144\ km.h^{-1} = 40\ m.s^{-1}$

- **Mouvement de A :**

① Tant que $v_A < v_{max}$: MRUV $\Rightarrow v_{A_1} = a_1 t$ et $x_{A_1} = \frac{a_1}{2} t^2$

↳ $0 \leq t \leq \frac{v_{max}}{a_1}$ ← $t_1 = 40s$ $\qquad x_{A_1} = \frac{t^2}{2}$

② $t \geq t_1$: MRU $\quad v_{A_2} = v_{max}$ et $x_{A_2} = v_{max}(t - t_1) + \underbrace{x_{A_1}(t_1)}_{800m}$

$$x_{A_2}(t) = 40(t - 40) + 800 = 40t - 800$$

- **Mouvement de B :** $\triangle$ pour $t \geq 10s = t_0$

① Tant que $v_B < v_{max}$: MRUV $\Rightarrow v_{B_1}(t) = a_2(t - t_0)$

$\hookrightarrow$ pour $t < t_0 + \frac{v_{max}}{a_2} = 30s$ (noté t_2)

$$x_{B_1}(t) = \frac{a_2}{2}(t-t_0)^2 \Rightarrow x_{B_1}(t) = (t-10)^2$$

② $t \geq 30s$: MRU $\quad v_{B_2} = v_{max} \quad x_{B_2} = \overbrace{x_{B_1}(t_2)}^{400\,m} + v_{max}(t - t_2)$

$$x_{B_2}(t) = 40(t-30) + 400 = 40t - 800$$

On remarque qu'à partir de $t = 40s$ $x_B(t) = x_A(t)$

$$x_A(40) = x_B(40) = 800 < 1000\,m$$

B rattrape A à $t = 40\,s$ puis les 2 voitures restent côte à côte.

Exercice 3 :

1) $L_A = \pi r_A \qquad L_B = \pi r_B + 2OO' \qquad OO' = r_A - r_B$

$$L_B = \pi r_B + 2(r_A - r_B)$$

2) Mouvement circulaire uniforme : $a = \frac{v^2}{r} \qquad a < 0,8g$

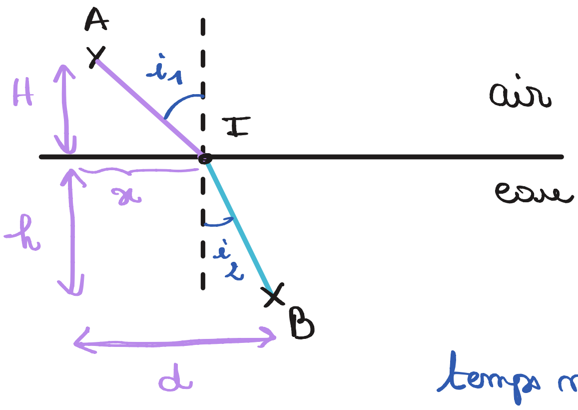
$v < \sqrt{0,8gr} \qquad r_A \Rightarrow v_A < 2,66\ m.s^{-1}$

$r_B \Rightarrow v_B < 2,43\ m.s^{-1}$

3) $t_A = \frac{L_A}{v_A} = 1,06\,s \quad < \quad t_B = \frac{L_B}{v_B} = 1,09\,s$

A gagne

Exercice 4



durée du trajet AB : $t_{AB} = t_1 + t_2$

où $t_1 = \dfrac{AI}{v_1}$ et $t_2 = \dfrac{IB}{v_2}$

$$t_{AB} = \frac{\sqrt{H^2+x^2}}{v_1} + \frac{\sqrt{h^2+(d-x)^2}}{v_2} \leftarrow \text{fonction de } x$$

temps minimal $\iff \dfrac{dt_{AB}}{dx} = 0$

$$\frac{2x}{2v_1 AI} + \frac{-2(d-x)}{2v_2 IB} = 0 \iff \frac{x}{v_1 AI} = \frac{d-x}{v_2 IB}$$

$$\iff \frac{\sin i_1}{v_1} = \frac{\sin i_2}{v_2}$$

2) Dans le cas d'une onde lumineuse : $v = \dfrac{c}{n} \rightarrow n_1 \sin i_1 = n_2 \sin i_2$

On retrouve la loi de Descartes de la réfraction.

Exercice 5

1) $\vec{OM} = \vec{OH} + \vec{HA} + \vec{AM}$ $\qquad \vec{OH} = vt\,\vec{u_x} \qquad \vec{HA} = R\,\vec{u_y}$

$\vec{AM} = -R\sin\theta\,\vec{u_x} - R\cos\theta\,\vec{u_y}$

$x_M = vt - R\sin\theta \qquad y_M = R(1-\cos\theta)$

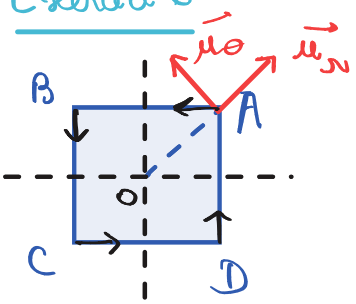
2) absence de glissement $\Rightarrow OH = \overset{\frown}{HM} \iff vt = R\theta \rightarrow \dot\theta = \dfrac{v}{R}$

$\rightarrow \ddot\theta = 0$

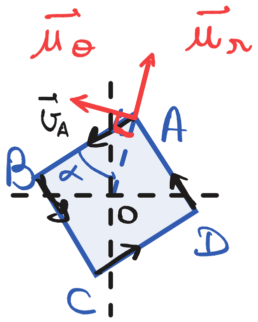
$$\vec{v} \left| \begin{array}{l} v - R\dot\theta\cos\theta = v(1-\cos\theta) \\ R\dot\theta\sin\theta = v\sin\theta \end{array} \right.$$

$$\vec{a} \left| \begin{array}{l} v\dot\theta\sin\theta = \dfrac{v^2}{R}\sin\theta \\ v\dot\theta\cos\theta = \dfrac{v^2}{R}\cos\theta \end{array} \right.$$

Exercice 6



à $t = 0$



à t

$\vec{v}_A$: vecteur vitesse de A à t

$\alpha = \frac{\pi}{4} \quad \forall t$

$\theta(0) = 0 \quad r(0) = \frac{a}{\sqrt{2}}$

$\vec{v} = v\left(-\cos\frac{\pi}{4}\vec{u}_r + \sin\frac{\pi}{4}\vec{u}_\theta\right) \Rightarrow \begin{cases} \dot{r} = -\frac{v}{\sqrt{2}} \\ r\dot{\theta} = \frac{v}{\sqrt{2}} \end{cases}$

$r(t) = -\frac{v}{\sqrt{2}}t + \frac{a}{\sqrt{2}}$

$\hookrightarrow r = \frac{a - vt}{\sqrt{2}}$

$\dot{\theta} = \frac{v}{a - vt} \rightarrow \theta(t) = -\ln(a - vt) + \ln a$

$\theta(t) = \ln\frac{a}{a - vt}$

$\hookrightarrow a - vt = a\exp(-\theta)$

$r = \frac{a}{\sqrt{2}}\exp(-\theta)$

2) $r = 0$ à $t_1 = \frac{a}{v}$

3) $\frac{ds}{dt} = v = C^{te} \quad s = vt \quad s(t_1) = vt_1 = a$