Correction DS5

Exercice 1

Q1. $\delta = H_0M - OH = a(\sin\theta_0 - \sin\theta)$

$$\Delta\varphi = -\frac{2\pi\delta}{\lambda} \underset{n=1}{=} -\frac{2\pi\delta}{\lambda_0}$$

$$\boxed{\Delta\varphi = \frac{2\pi a}{\lambda_0}(\sin\theta - \sin\theta_0)}$$

Q2. Interférences constructives : $\Delta\varphi = 2k\pi \quad k \in \mathbb{Z}$

$$\frac{\cancel{2\pi} a}{\lambda_0}(\sin\theta_k - \sin\theta_0) = \cancel{2}k\cancel{\pi}$$

$$\boxed{\sin\theta_k = k\frac{\lambda_0}{a} + \sin\theta_0}$$

Exercice 2

Q3. Principe de superposition : $s(M,t) = s_1(M,t) + s_2(M,t)$

$$s_1(M,t) = S_{m1}\cos\left(\omega t - \frac{S_1M}{c} + \varphi_1\right) \qquad \phi_1 = -\frac{S_1M}{c} + \varphi_1$$

$$s_2(M,t) = S_{m2}\cos\left(\omega t - \frac{S_2M}{c} + \varphi_2\right) \qquad \phi_2 = -\frac{S_2M}{c} + \varphi_2$$

- On se place en $\mathbb{C}$: $\underline{s}(M,t) = \underline{s}_1(M,t) + \underline{s}_2(M,t)$

$$\underline{s}(M,t) = \underbrace{\left(S_{m1}e^{j\phi_1} + S_{m2}e^{j\phi_2}\right)}_{\underline{S}_m = S_m e^{j\varphi}} e^{j\omega t}$$

$$\boxed{s(M,t) = S_m\cos(\omega t + \varphi)}$$

- Amplitude de $s(M,t)$: $S_m = |\underline{S}_m|$

$$S_m = \left| S_{m1}\cos\phi_1 + jS_{m1}\sin\phi_2 + S_{m2}\cos\phi_2 + jS_{m2}\sin\phi_2 \right|$$

$$S_m = \left(\left(S_{m1}\cos\phi_1 + S_{m2}\cos\phi_2\right)^2 + \left(S_{m1}\sin\phi_1 + S_{m2}\sin\phi_2\right)^2\right)^{1/2}$$

$$S_m = \sqrt{S_{m1}^2 + S_{m2}^2 + 2S_{m1}S_{m2}(\cos\phi_1\cos\phi_2 + \sin\phi_1\sin\phi_2)}$$

$$\boxed{S_m = \sqrt{S_{m_1}^2 + S_{m_2}^2 + 2 S_{m_1} S_{m_2} \cos \Delta\phi}}$$ $\Delta\phi = \phi_2 - \phi_1$

Q4. Interférences constructives : S_m maximal $\boxed{\Delta\phi = 2p\pi,\ p \in \mathbb{Z}}$

$$\boxed{S_m = S_{m_1} + S_{m_2}}$$

Interférences destructives : S_m minimale $\boxed{\Delta\phi = (2p+1)\pi,\ p \in \mathbb{Z}}$

$$\boxed{S_m = S_{m_1} - S_{m_2}}$$

Q5. $\boxed{s(M,t) = S_m \cos\left(\omega\left(t - \frac{d}{c}\right) + \varphi\right)}$

d = distance parcourue jusqu'au point M

Q6. le long du bras 1 : $d_1 = L_e + 2L_1 + L_d$

$$\boxed{s_1(M,t) = S_{m_1} \cos\left(\omega\left(t - \frac{L_e + 2L_1 + L_d}{c}\right) + \varphi_1\right)}$$

Q7. le long du bras 2 : $d_2 = L_e + 2L_2 + L_e$

$$\boxed{s_2(M,t) = S_{m_2} \cos\left(\omega\left(t - \frac{L_e + 2L_2 + L_d}{c}\right) + \varphi_2\right)}$$ $k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$

$$\boxed{\Delta\phi_{2/1} = \frac{4\pi}{\lambda}(L_1 - L_2)}$$

$\varphi_2 = \varphi_1$ car les 2 ondes sont issues de la même source.

Q8. $\Delta\phi_{2/1} = \frac{4\pi}{\lambda}\left(L_{0_1} - L_{0_2} + \delta x\right) = \phi_0 + \frac{4\pi}{\lambda}\delta x$

$$I = S_m^2 = S_{m_1}^2 + S_{m_2}^2 + 2 S_{m_1} S_{m_2} \cos \Delta\phi_{2/1}$$

$$I = 2 S_{m_0}^2 \left(1 + \cos \Delta\phi_{2/1}\right)$$

$$\boxed{I = \frac{I_0}{2}\left(1+\cos\left(\Phi_0 + 2k\,\delta x\right)\right)}$$

Q9.

$$I = \frac{I_0}{2}\left(1+\cos\Phi_0\,\cos(2k\delta x) - \sin\Phi_0\,\sin(2k\delta x)\right)$$

$$I \simeq \frac{I_0}{2}\left(1+\cos\Phi_0 - \sin\Phi_0 \times 2k\,\delta x\right)$$

$$I(\delta x = 0) \simeq \frac{I_0}{2}\left(1+\cos\Phi_0\right)$$

$$\boxed{\delta I = -I_0\,k\,\delta x\,\sin\Phi_0} \qquad \boxed{A = -I_0\,k\,\delta x}$$

Q10. $|\delta I|$ maximale si $\boxed{\Phi_0 = \frac{\pi}{2} + p\pi,\ p\in\mathbb{Z}}$ $\boxed{L_{0_1} - L_{0_2} = \frac{\lambda}{8} + p\frac{\lambda}{4}}$

Q11. $\delta x = 10^{-23} \times L_{0_1} = \underline{3 \times 10^{-20}\ m}$

$$\left|\frac{\delta I}{I_0}\right| = k\,\delta x = \frac{2\pi}{1064} \times 10^{9} \times 3 \times 10^{-20} = \underline{1{,}8 \times 10^{-13}}$$

<u>extrêmement faible</u>

Q12. Il faut un filtre <u>passe-bande</u>.

Q13. Il faut choisir $\omega_0 = \Omega$ puisqu'il s'agit de la pulsation où $|\underline{H}|$ est maximal.

Q14. D'après le <u>principe de superposition</u>, 1 <u>filtre linéaire</u> restitue les mêmes composantes, il modifie juste leur amplitude et leur phase. 3 composantes en entrée $\Rightarrow$ 3 composantes de même fréquence en sortie.

$|\underline{H}|(\omega = 0) \to 0$ on élimine la composante continue $I_0 m^2$.

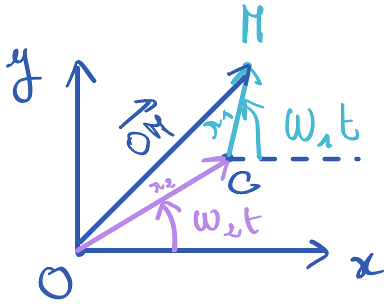
- composante Ω : $|\underline{H}| = |H_0|$
- composante 2Ω : $|\underline{H}| = \dfrac{|H_0|}{\sqrt{1+Q^2(2-1/2)^2}} = \dfrac{|H_0|}{\sqrt{1+\frac{9}{4}Q^2}}$

$$\boxed{A_2 = \gamma\, I_0\, m^2 \frac{|H_0|}{\sqrt{1+\frac{9}{4}Q^2}}}$$

$$\boxed{A_1 = \gamma |H_0| K\,\delta x\, I_0}$$

Exercice 3

Q15.



$$\vec{OM} = \vec{OC} + \vec{CM}$$

$$= \begin{vmatrix} r_2 \cos(\omega_2 t) \\ r_2 \sin(\omega_2 t) \end{vmatrix} + \begin{vmatrix} r_1 \cos(\omega_1 t) \\ r_1 \sin(\omega_1 t) \end{vmatrix}$$

$$\boxed{\vec{OM}(t) = (r_2 \cos(\omega_2 t) + r_1 \cos(\omega_1 t))\vec{e_x} + (r_2 \sin(\omega_2 t) + r_1 \sin(\omega_1 t))\vec{e_y}}$$

Q16. $\vec{a} = \dfrac{d^2}{dt^2}(\vec{OC} + \vec{CM})$ $\boxed{\vec{a} = -\omega_2^2 \vec{OC} - \omega_1^2 \vec{CM}}$

• $\|\vec{a}\|$ maximale

O C M

alignement O, C, M

$$\boxed{a_M = \omega_1^2 r_1 + \omega_2^2 r_2}$$

Q17. $\|\vec{a}\|$ minimale

O M C

alignement O, M, C

$$\boxed{a_m = |\omega_1^2 r_1 - \omega_2^2 r_2|}$$

Q18. Si $\omega_1 = -\omega_2$

$$x(t) = (r_1 + r_2)\cos(\omega_1 t)$$

$$y(t) = (r_1 - r_2)\sin(\omega_1 t)$$

$$\boxed{\left(\frac{x}{r_1 + r_2}\right)^2 + \left(\frac{y}{r_1 - r_2}\right)^2 = 1}$$

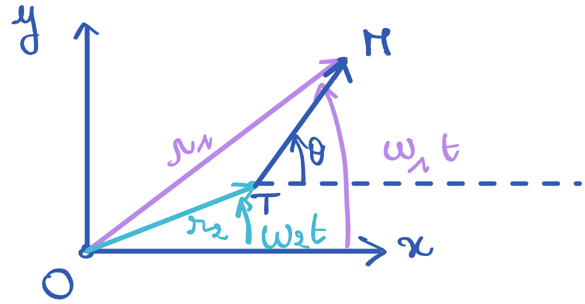
équation d'1 ellipse

Q19. $r^2 = \left((r_1 \cos(\omega_1 t) + r_2 \cos(\omega_2 t))^2 + (r_1 \sin(\omega_1 t) + r_2 \sin(\omega_2 t))^2\right)$

$$\boxed{r^2 = r_1^2 + r_2^2 + 2 r_1 r_2 \cos((\omega_2 - \omega_1)t)}$$

période de changement d'éclat : $\boxed{T = \dfrac{2\pi}{|\omega_2 - \omega_1|}}$

Q20. $\vec{TM} = \vec{OM} - \vec{OT}$



(Soleil)

$$\boxed{\vec{TM} = (r_1\cos(\omega_1 t) - r_2\cos(\omega_2 t))\vec{e}_x + (r_1\sin(\omega_1 t) - r_2\sin(\omega_2 t))\,\vec{e}_y}$$

Q21.

$$\boxed{\tan\theta = \frac{r_1\sin(\omega_1 t) - r_2\sin(\omega_2 t)}{r_1\cos(\omega_1 t) - r_2\cos(\omega_2 t)}}$$

Q22. Mars rétrograde si $\dot{\theta}$ change de signe $\Leftrightarrow \dot{\theta} = 0 \Leftrightarrow \frac{d}{dt}(\tan\theta) = 0$

$$\Leftrightarrow (r_1\omega_1\cos(\omega_1 t) - r_2\omega_2\cos(\omega_2 t))(r_1\cos(\omega_1 t) - r_2\cos(\omega_2 t)) - (r_1\sin(\omega_1 t) - r_2\sin(\omega_2 t))(-r_1\omega_1\sin(\omega_1 t) + r_2\omega_2\sin(\omega_2 t)) = 0$$

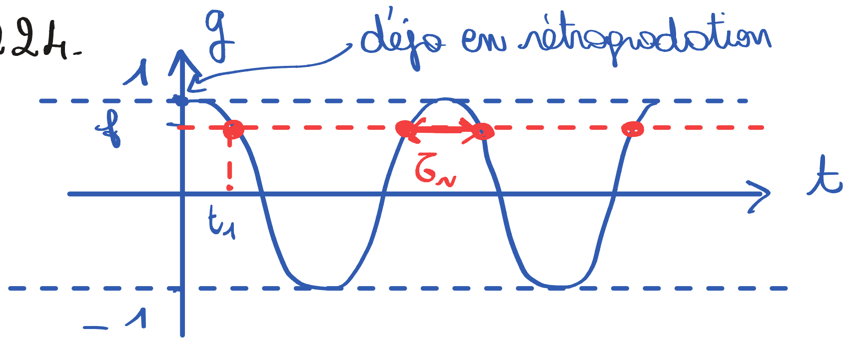
$$r_2^2\omega_2\cos^2(\omega_2 t) - r_1 r_2\omega_2\cos(\omega_2 t)\cos(\omega_1 t) - r_1 r_2\omega_1\cos(\omega_1 t)\cos(\omega_2 t) + r_1^2\omega_1\cos^2(\omega_1 t) + r_2^2\omega_2\sin^2(\omega_2 t) - r_1 r_2\omega_1\sin(\omega_2 t)\sin(\omega_1 t) - r_1 r_2\omega_2\sin(\omega_1 t)\sin(\omega_2 t) + r_1^2\omega_1\sin^2(\omega_1 t) = 0$$

$$r_2^2\omega_2 + r_1^2\omega_1 - r_1 r_2\omega_2\cos((\omega_1-\omega_2)t) - r_1 r_2\omega_1\cos((\omega_1-\omega_2)t) = 0$$

$$\boxed{\cos((\omega_1-\omega_2)t) = \frac{r_2^2\omega_2 + r_1^2\omega_1}{r_1 r_2(\omega_2+\omega_1)}}$$

Q23. $$f = \frac{2\pi/T_2 + 1{,}5^2\, 2\pi/T_1}{1{,}5\times 2\pi\left(\frac{1}{T_2} + \frac{1}{T_1}\right)} = \frac{1 + 1{,}5^2/1{,}9}{1{,}5\left(1 + \frac{1}{1{,}9}\right)} = \frac{1{,}9 + 1{,}5^2}{1{,}5\times 2{,}9} = 0{,}954$$

Q24.



$(\omega_2 > \omega_1)$

$\cos((\omega_2-\omega_1)t_1) = f(\dots)$

$$t_1 = \frac{\arccos f(\dots)}{\omega_2-\omega_1}$$

durée $= 2\,t_1$

$$\boxed{\tau_r = \frac{2\arccos(f(\dots))}{\omega_2-\omega_1}}$$

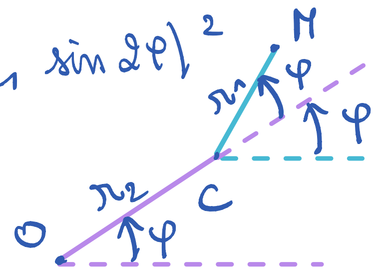
Q25. $\tau_r = \dfrac{0{,}954}{2\pi\left(\frac{1}{T_2}-\frac{1}{T_1}\right)} = \underline{75}\,j$

T_2 : 365 j T_1 : 693 j

Q26. $\vec{OM} = \vec{OC} + \vec{CM}$ $\|\vec{OM}\|^2 = r^2$

$r^2 = (r_2\cos\varphi + r_1\cos 2\varphi)^2 + (r_2\sin\varphi + r_1\sin 2\varphi)^2$

$r^2 = \underbrace{r_2^{\,2} + r_1^{\,2}}_{\sim\, r_2^{\,2}} + 2\,r_1 r_2\cos\varphi$



$$\boxed{r^2 \simeq r_2\,(r_2 + 2r_1\cos\varphi)}$$

Q27. $r \simeq r_2(1+2\varepsilon\cos\varphi)^{1/2} \simeq r_2(1+\varepsilon\cos\varphi)$

$\boxed{r \simeq r_2 + r_1\cos\varphi}$ $\boxed{h(\varphi) = \cos\varphi}$

Q28. $r(\varphi) = \dfrac{p}{1-e\cos\varphi} = p\,(1-e\cos\varphi)^{-1} \simeq p\,(1+e\cos\varphi)$ si $e \ll 1$

ainsi : $r(\varphi) \simeq p + pe\cos\varphi$

On identifie

$\boxed{p = r_2}$

$\boxed{pe = r_1}$ $e = \varepsilon = \dfrac{r_1}{r_2}$