Correction DS6

Exercice 1

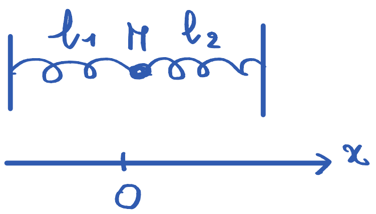
Q1. Système : {mandelette M} Réf : terrestre supposé galiléen

BdF
- poids $m\vec{g}$
- réaction du support $\vec{R} = \vec{R}_n$

} se compensent selon la verticale

- force de rappel élastique : $\vec{Fe}_1 = -K_1(l_1 - l_{0_1})\vec{u}_x$

$\vec{Fe}_2 = +K_2(l_2 - l_{0_2})\vec{u}_x$



$l_1 = l_{0_1} + x$ $l_2 = l_{0_2} - x$

$\boxed{\vec{F}_{e_1} = -K_1 x \vec{u}_x}$ $\boxed{\vec{F}_{e_2} = -K_2 x \vec{u}_x}$

PFD selon Ox : $m\ddot{x} = -(K_1+K_2)x$

$$\boxed{\ddot{x} + \frac{K_1+K_2}{m}x = 0}$$

On posera $\boxed{\omega_0 = \sqrt{\frac{K_1+K_2}{m}}}$.

Q2. $x(t) = A\cos(\omega_0 t + \varphi)$

$x(0) = x_0 = A\cos\varphi$

$\dot{x}(0) = 0 = -A\omega_0\sin\varphi$

} $\varphi = 0$ et $A = x_0$

$$\boxed{x(t) = x_0\cos(\omega_0 t)}$$

Q3. On rajoute la force de frottement fluide : $m\ddot{x} = -f\dot{x} - (K_1+K_2)x$

$$\boxed{\ddot{x} + \frac{f}{m}\dot{x} + \frac{K_1+K_2}{m}x = 0}$$

$\frac{\omega_0}{Q} = \frac{f}{m} \rightarrow \boxed{Q = \frac{\sqrt{(K_1+K_2)m}}{f}}$

Q4. • <u>Si $Q > 1/2$</u> : $\Delta < 0$ et le régime est <u>pseudopériodique</u>

$$\boxed{x(t) = \exp\left(-\frac{\omega_0 t}{2Q}\right)\left(A\cos(\Omega t) + B\sin(\Omega t)\right)} \qquad \boxed{\Omega = \omega_0\sqrt{1-\frac{1}{4Q^2}}} \qquad A \text{ et } B \in \mathbb{R}$$

pseudo-pulsation

• <u>Si $Q < 1/2$</u> : $\Delta > 0$ et le régime est <u>apériodique</u>

$$\boxed{x(t) = A\exp(r_1 t) + B\exp(r_2 t)} \qquad \boxed{r_{1,2} = -\frac{\omega_0}{2Q} \pm \omega_0\sqrt{\frac{1}{4Q^2}-1}} \qquad A \text{ et } B \in \mathbb{R}$$

• <u>Si $Q = 1/2$</u> : $\Delta = 0$ et le régime est <u>apériodique critique</u>

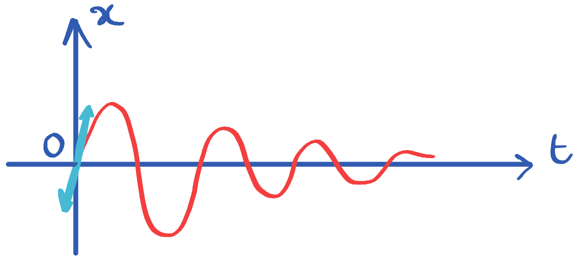
$$\boxed{x(t) = (A + Bt)\exp(-\omega_0 t)} \qquad A \text{ et } B \in \mathbb{R}$$

Q5. $x(0) = 0 \qquad A = 0$

$\dot{x}(0) = v_0 \qquad -\frac{\omega_0}{2Q}B + B\Omega = v_0 \qquad B = \frac{v_0}{\Omega - \frac{\omega_0}{2Q}}$

$$\boxed{x(t) = \frac{v_0}{\Omega - \frac{\omega_0}{2Q}}\exp\left(-\frac{\omega_0}{2Q}t\right)\sin(\Omega t)}$$

Q6.



v_0 supposée > 0

Q7. L'expression de $\vec{Fe_2}$ est modifiée : $l_2 = l_{0_2} - x + X$

$$\vec{Fe_2} = -K_2(x - X)\vec{u_x}$$

le PFD selon Ox devient : $\ddot{x} + \frac{f}{m}\dot{x} + \frac{(K_1+K_2)}{m}x = \frac{K_2}{m}X$

$$\boxed{\ddot{x} + \frac{\omega_0}{Q}\dot{x} + \omega_0^2 x = \frac{K_2}{m}a\cos(\omega t)}$$

Q8. On pose en complexe : $\underline{x} = X_m e^{j(\omega t + \varphi)} \qquad \underline{X} = a e^{j\omega t}$

$$-\omega^2\underline{x} + \frac{\omega_0}{Q}j\omega\underline{x} + \omega_0^2\underline{x} = \frac{K_2}{m}\underline{X} \qquad \underline{x} = \frac{K_2/m\ \underline{X}}{\omega_0^2 - \omega^2 + j\frac{\omega\omega_0}{Q}}$$

$X_m = |\underline{x}|$

$$\boxed{X_m = \frac{K_2 a/m}{\left((\omega_0^2-\omega^2)^2 + \left(\frac{\omega\omega_0}{Q}\right)^2\right)^{1/2}}}$$

Q9. Il y a résonance si $X_m(\omega)$ passe par un maximum pour 1 pulsation non nulle ω_r.

$$\frac{dX_m}{d\omega}=0 \iff \frac{d}{d\omega}\left((\omega_0^2-\omega^2)^2+\left(\frac{\omega_0\omega}{Q}\right)^2\right)=0$$

$$-2(\omega_0^2-\omega^2)\,2\omega + 2\omega\frac{\omega_0^2}{Q^2}=0$$

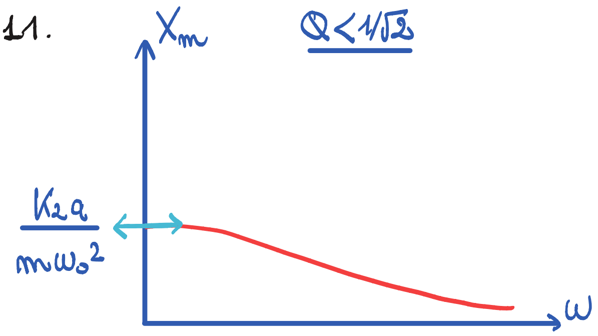
$\iff \omega=0$ ou $-2\omega_0^2+2\omega^2+\frac{\omega_0^2}{Q^2}=0 \rightarrow \omega^2=\omega_0^2\underbrace{\left(1-\frac{1}{2Q^2}\right)}_{\text{doit être }>0}$ (on admet qu'il s'agit d'1 maximum pour Xm)

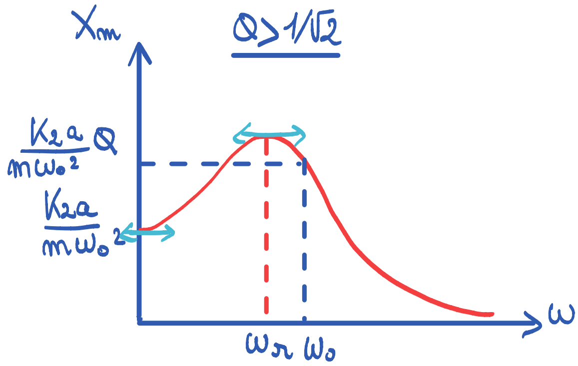
$\boxed{\omega_r \text{ existe si } Q>\frac{1}{\sqrt{2}}}$ et $\omega_r=\omega_0\sqrt{1-1/2Q^2}$

Q10 $Q \gg 1$ $\boxed{\omega_r \simeq \omega_0}$ $X_{m,max}=\frac{K_2 a}{m\omega_0^2}Q$ $\omega_0^2=\frac{K_1+K_2}{m}$

$$\boxed{X_{m,max}=\frac{K_2}{K_1+K_2}Q\,a}$$

Q11.





Q12. AN : $f_r=\frac{\omega_r}{2\pi}=\underline{25\,Hz}$

Q13. AN : $X_m(\omega_0)=\frac{K_2 a}{K_1+K_2}Q=\underline{0{,}4\,mm}$

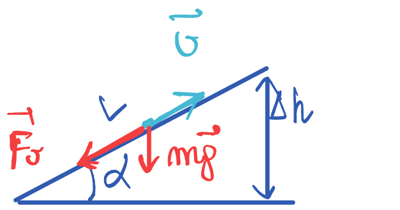
Q14. $X_m(\omega_0)>a \rightarrow$ la vibration est amplifiée

Exercice 2

Q17. $\Delta E_p = mg\,\Delta z_g$

Q18. $P = \dfrac{mg\,\Delta z_g}{\Delta t}$ $\quad \Delta t = \dfrac{a}{v}$ $\quad \rightarrow \boxed{P = \dfrac{mg\,\Delta z_g}{a}\,v}$

AN : $P = \dfrac{70 \times 9{,}8 \times 0{,}1 \times 12/3{,}6}{1} = \underline{229\,W}$



Q19. $P_{tot} = P_i + |P_{ext}|$

$$|P_{ext}| = \underbrace{\beta V^2 v}_{|P(\vec{F_v})|} + \underbrace{mg\,v \sin\alpha}_{|P(m\vec{g})|}$$

$V = v + u$

$\delta = \sin\alpha = \dfrac{\Delta h}{L}$

$$\boxed{P_{tot} = P_i + \beta\,(v+u)^2\,v + mg\,\delta\,v}$$

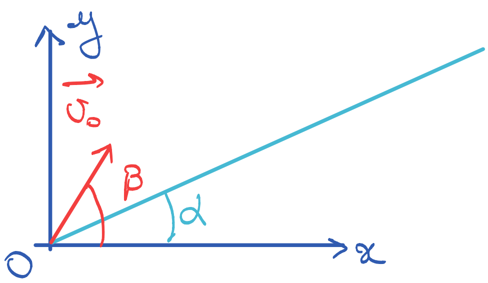
AN : $\underline{P_{tot} = 401\,W}$

Problème

Q20.

<u>Système</u> : {corpuscule} $\quad$ <u>Réf</u> : terrestre supposé galiléen

<u>BdF</u> : $m\vec{g}$ $\quad$ <u>PFD</u> : $m\vec{a} = m\vec{g}$ $\quad \boxed{\vec{a} = \vec{g}}$



Dans $(O, \vec{u_x}, \vec{u_y})$:

- $\ddot{x} = 0$
- $\ddot{y} = -g$

On intègre, les conditions initiales étant $\dot{x}(0) = v_0 \cos\theta$, $\dot{y}(0) = v_0 \sin\theta$, $x(0) = y(0) = 0$.

<u>Equations horaires</u> $\quad x(t) = v_0 \cos\theta \, t$

$$y(t) = -\frac{1}{2} g t^2 + v_0 \sin\theta \, t$$

<u>Equation de la trajectoire</u> : $\quad y(x) = -g \dfrac{x^2}{v_0^2 \cos^2\theta} + \tan\theta \, x$

$$\left.\begin{array}{l} x_M = d_m \cos\alpha \\ y_M = d_m \sin\alpha \end{array}\right\} d_m \sin\alpha = -\frac{1}{2} \frac{g \cos^2\alpha}{v_0^2 \cos^2\theta} d_m^2 + \tan\theta \, d_m \cos\alpha$$

$$d_m^2 \left(\frac{g \cos^2\alpha}{2 v_0^2 \cos^2\theta} \right) + d_m \left(\sin\alpha - \tan\theta \cos\alpha \right) = 0$$

$d_m \neq 0$

$$\rightarrow \boxed{d_m = -\frac{\sin\alpha - \tan\theta \cos\alpha}{g \cos^2\alpha} 2 v_0^2 \cos^2\theta}$$

<u>Recherche d'1 extrêmum</u> $\quad \dfrac{d\, d_m}{d\theta} = 0$

$$d_m = \frac{-2 v_0^2}{g \cos^2\alpha} f(\theta) \qquad \text{avec } f(\theta) = \cos^2\theta \left(\sin\alpha - \cos\alpha \tan\theta \right)$$

$$\frac{d\, d_m}{d\theta} = 0 \iff f'(\theta) = 0$$

$$-\underbrace{2 \cos\theta \sin\theta}_{\sin 2\theta} \left(\sin\alpha - \cos\alpha \tan\theta \right) + \cancel{\cos^2\theta} \left(-\cos\alpha \times \frac{1}{\cancel{\cos^2\theta}} \right) = 0$$

$$-\sin 2\theta \sin\alpha + 2 \cos\alpha \sin^2\theta - \cos\alpha = 0 \qquad \sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$-\sin 2\theta \sin\alpha + \cancel{\cos\alpha} - \cos\alpha \cos 2\theta - \cancel{\cos\alpha} = 0$$

$$\tan 2\theta = -\frac{1}{\tan\alpha} \qquad 2\theta = \alpha \pm \frac{\pi}{2} \qquad \text{or } \theta > \alpha$$

$$\boxed{\theta_M = \frac{\alpha}{2} + \frac{\pi}{4}}$$

Expression de la portée maximale :

$$d_m = \frac{-2v_0^2}{g\cos^2\alpha}\underbrace{\left(\sin\alpha - \cos\alpha\,\tan\theta_r\right)\cos^2\theta_r}_{\sin\alpha\cos^2\theta_r - \cos\alpha\,\underbrace{\sin\theta_r\cos\theta_r}_{}}$$

$$\frac{1}{2}\sin 2\theta_r = \frac{1}{2}\sin\left(\alpha+\frac{\pi}{2}\right) = \frac{1}{2}\cos\alpha$$

$$d_m = \frac{-2v_0^2}{g\underbrace{\cos^2\alpha}_{1-\sin^2\alpha}}\left(\sin\alpha\cos^2\theta_r - \frac{1}{2}\cos^2\alpha\right)$$

$$\cos^2\theta_r = \frac{1+\cos 2\theta_r}{2} = \frac{1}{2} - \frac{\sin\alpha}{2}$$

$$d_m = -\frac{v_0^2}{g\left(1-\sin^2\alpha\right)}\underbrace{\left(\sin\alpha - \sin^2\alpha - \cos^2\alpha\right)}_{\sin\alpha - 1}$$

$$\boxed{d_m = \frac{v_0^2}{g(1+\sin\alpha)}}$$