Correction TD 15

Exercice 1 :

1) $N_\gamma = \dfrac{\Phi S \Delta t}{hc/\lambda_m} = 2{,}5 \times 10^{21}$

2) $\Phi = \Phi_0 \, 10^{\frac{m - m_0}{-2{,}5}}$ $\qquad N_\gamma = \Phi_S \, 10^{\frac{m - m_S}{-2{,}5}} \times \pi \dfrac{d^2}{4} \times \dfrac{\Delta t}{hc} \lambda_m = 4{,}6 \times 10^3$

Exercice 2 :

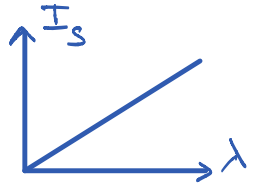
1) $W_0 = h\nu_0$

2) $E_c = h(\nu - \nu_0)$

3) $N_\gamma = \dfrac{P \Delta t}{hc/\lambda} = 1{,}79 \times 10^{18}$

4) $h\nu = \dfrac{hc}{\lambda} = 3{,}5 \text{ eV} > W_0 \rightarrow$ effet photoélectrique

5) $I_s = \dfrac{N_{e^-} \times e}{\Delta t} = P \dfrac{\lambda e}{hc} = 0{,}3 \text{ mA}$ car $N_{e^-} = N_\gamma$

6) 

pour $\lambda < \lambda_{seuil}$ $\qquad \lambda_{seuil} = \dfrac{hc}{W_0}$

7) $E_c = \dfrac{hc}{\lambda} - W_0 = 1{,}3 \text{ eV}$

8) $r = \dfrac{N_{e^-}}{N_\gamma}$ $\qquad N_\gamma = \dfrac{P \Delta t \lambda}{hc}$ $\qquad N_{e^-} = I_s \dfrac{\Delta t}{e}$ $\rightarrow r = \dfrac{hc I_s}{P \lambda e} = 6{,}7\%$

ou $\dfrac{I_s}{I_{s\max}} \leftarrow 5)$

9) $I = I_s \times 3^{10} = 1{,}18 \text{ A}$

Exercice 3 :

1) Si les détails à observer sont plus petits ou du même ordre de grandeur que λ, la qualité de l'image est altérée par un phénomène de diffraction.

2) $\lambda_{dB} = \frac{h}{p} = \frac{h}{mv}$ où $v = \sqrt{\frac{2E_c}{m}}$ $\lambda_{dB} = 0,13$ nm

$\lambda_{dB} < \lambda_{visible}$ → on peut observer des détails plus petits.

3) Avec la formule classique $v = (2E_c/m)^{1/2} = 1,9 \times 10^8 \, m.s^{-1} > \frac{c}{10}$ → relativiste

$\lambda_{dB} = \frac{h}{\gamma m v}$ $E_c = (\gamma - 1)mc^2 = \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right) mc^2$ $E_c = 100 keV \Rightarrow \gamma = 1,2$

$\frac{1}{\sqrt{1 - v^2/c^2}} = \frac{E_c}{mc^2} + 1$ $1 - \frac{v^2}{c^2} = \frac{1}{\left(1 + \frac{E_c}{mc^2}\right)^2}$ $v = c\left(1 - \frac{1}{\left(1 + E_c/mc^2\right)^2}\right)^{1/2}$

$(v = 1,66 \times 10^8 \, m.s^{-1})$

→ $\lambda_{dB} = 3,6$ pm

Exercice 4 :

1) $E_\gamma = hf$ $p_\gamma = \frac{h}{\lambda} = \frac{h}{c} f$

2) $(E_\gamma + \overleftarrow{E_{e^-}}^{0})_{avant} = (E_\gamma + E_{e^-})_{après}$

$hf = hf' + \frac{p_e^2}{2m_e}$ (1)

3) $\left(\vec{p}_\gamma + \overleftarrow{\vec{p}_{e^-}}^{\vec{0}}\right)_{avant} = \left(\vec{p}_\gamma + \vec{p}_{e^-}\right)_{après}$

$\left.\begin{array}{l} \frac{h}{c} f = \frac{h}{c} f' \cos\theta + p_e \cos\phi \quad (2) \\ 0 = \frac{h}{c} f' \sin\theta - p_e \sin\phi \quad (3) \end{array}\right\}$ projections

4) $(1) \Rightarrow \Delta f = \frac{p_e^2}{2m_e h}$

$(2) \Rightarrow p_e \cos\phi = \frac{h}{c}(f - f'\cos\theta)$ $(3) \Rightarrow p_e \sin\phi = \frac{h}{c} f' \sin\theta$

$\Rightarrow p_e^2 = \frac{h^2}{c^2}\left((f - f'\cos\theta)^2 + f'^2 \sin^2\theta\right) = \frac{h^2}{c^2}\left(f^2 - 2ff'\cos\theta + f'^2\right)$

$f' = f - \Delta f \rightarrow f^2 - 2ff'\cos\theta + f'^2 = f^2 - 2f^2\cos\theta + 2\Delta f f \cos\theta + f^2 - 2f\Delta f + \Delta f^2$

$\Delta f \ll f$

$\Rightarrow \Delta f \approx \frac{1}{2m_e h} \frac{h^2}{c^2}\left(2f^2 - 2f^2\cos\theta\right)$ $\Delta f = \frac{hf^2}{m_e c^2}(1 - \cos\theta)$

Exercice 6 :

1) $\Psi(0,t) = 0 \ \forall t$ mais il faut également que $\Psi(L,t) = 0$

$$\frac{2\pi L}{\lambda} = n\pi \qquad n \in \mathbb{N}^* \qquad \lambda = \frac{2L}{n}$$

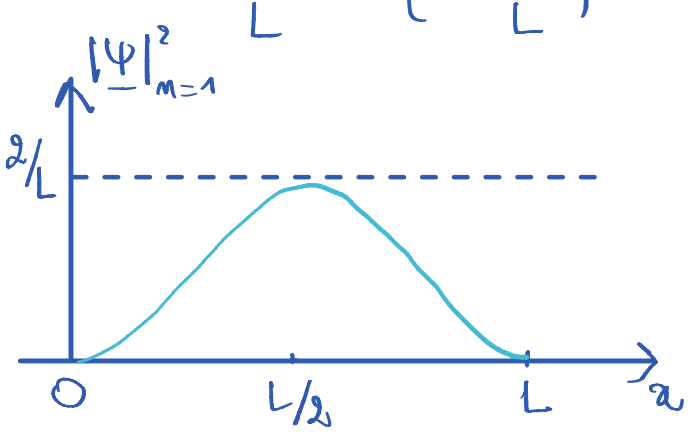
2) $$\int_0^L dP = 1 \iff \int_0^L A^2 \sin^2\left(\frac{2\pi}{\lambda} x\right) dx = 1 \qquad \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\iff \frac{A^2}{2} \int_0^L \left(1 - \cos\left(\frac{4\pi}{\lambda} x\right)\right) dx = 1$$

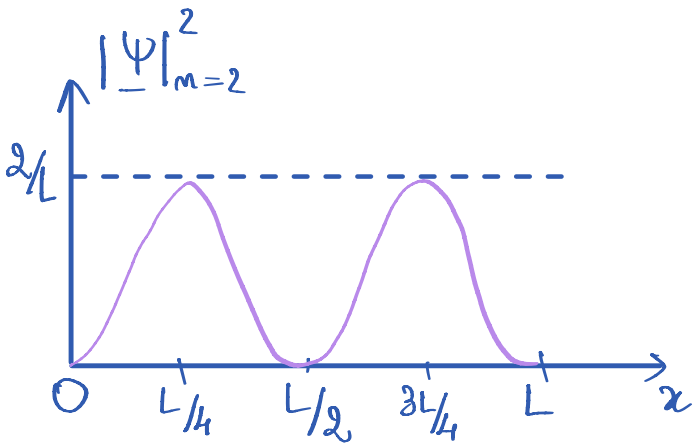
$$\iff \frac{A^2}{2}\left(L + \frac{\lambda}{4\pi} \underbrace{\sin\frac{4\pi L}{\lambda}}_{0}\right) = 1 \qquad \text{or } \lambda = \frac{2L}{n} \qquad \frac{4\pi L}{\lambda} = \frac{4\pi L}{2L} n = 2\pi n$$

$$\frac{A^2}{2} L = 1 \qquad A = \pm\sqrt{\frac{2}{L}}$$

3) $$|\Psi^2| = \frac{2}{L} \sin^2\left(\pi n \frac{x}{L}\right)$$



$n = 1$



$n = 2$

4) $$p = \frac{h}{\lambda} = \frac{h}{2L} n$$

5) $$E = \frac{p^2}{2m} = \frac{h^2 n^2}{8 L^2 m}$$

6) $E_1 = 15$ meV $\qquad E_3 = 9E_1 = 0{,}14$ eV

$E_2 = 4E_1 = 60$ meV