TP16 : Pendule simple

Q1. <u>Système</u> : {masse p} <u>Réf</u> : terrestre supposé galiléen

<u>BdF</u> : $m\vec{g}$ et $\vec{T}$

PFD selon $\vec{u}_\theta$: $ml\ddot{\theta} = -mg\sin\theta$ $\rightarrow$ $\boxed{\ddot{\theta} + \frac{g}{l}\sin\theta = 0}$

non linéaire

Q2. $\sin\theta \simeq \theta$ $\rightarrow$ $\boxed{\ddot{\theta} + \frac{g}{l}\theta = 0 \quad \text{OHNA}}$

$$T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{l}{g}}$$

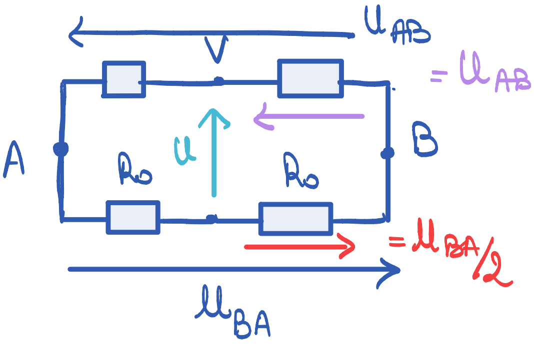
Q3. $E_c = \frac{1}{2}mv^2$ $v^2 = (l\dot{\theta})^2$ $\boxed{E_c = \frac{1}{2}ml^2\dot{\theta}^2}$

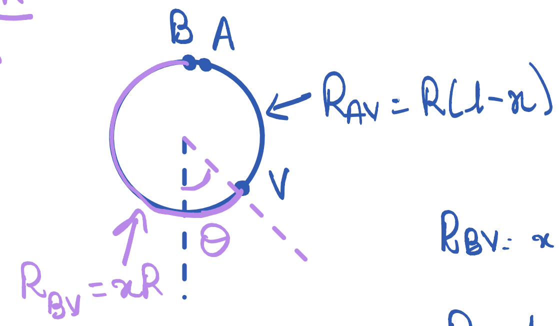
$E_p = -mgl\cos\theta + Cte$ $E_p(0) = 0 \Rightarrow Cte = mgl$

$$\boxed{E_p = mgl(1-\cos\theta)}$$

Q4. Le poids est une force conservative donc en l'absence de frottements et comme $\vec{T}$ ne travaille pas $(\vec{T} \perp \vec{v})$: $\Delta E_m = 0$ $\boxed{E_m = Cte}$

Q5. Pont diviseur de tension : $u = \frac{U_{BA}}{2} + x\,U_{AB} = U_{AB}\left(-\frac{1}{2} + x\right)$





$$\left.\begin{array}{l} R_{BV} : x \longleftrightarrow \pi+\theta \\ R : 1 \longleftrightarrow 2\pi \end{array}\right| \Rightarrow x = \frac{\pi+\theta}{2\pi}$$

$$u = U_{AB}\left(-\frac{1}{2} + \frac{1}{2} + \frac{\theta}{2\pi}\right) \rightarrow \boxed{u = \frac{U_{AB}}{2\pi}\,\theta}$$

Q11. Système conservatif $\Rightarrow$ $\frac{1}{2} m l^2 \left(\frac{d\theta}{dt}\right)^2 + mgl(1-\cos\theta) = E_m$ (constante)

$E_m = E_{P\,max} = mgl(1-\cos\theta_m)$

$$\frac{1}{2}\cancel{m}\, l^{\cancel{2}} \left(\frac{d\theta}{dt}\right)^2 = \cancel{m}g\cancel{l}(\cos\theta - \cos\theta_m) \rightarrow \left(\frac{dt}{d\theta}\right)^2 = \frac{1}{2\omega_0^2(\cos\theta - \cos\theta_m)}$$

$$dt = \pm\left(\frac{1}{2\omega_0^2(\cos\theta - \cos\theta_m)}\right)^{1/2} d\theta$$

On intègre entre $\theta = 0$ et θ_m (amplitude) ce qui correspond à $1/4$ de T.

$$\boxed{T = \frac{4}{\omega_0}\int_0^{\theta_m} \frac{1}{\sqrt{2(\cos\theta - \cos\theta_m)}}\,d\theta}$$

Q12. $\cos\theta = 1 - 2\sin^2\frac{\theta}{2}$ $\qquad \cos\theta_m = 1 - 2\sin^2\frac{\theta_m}{2}$

$$\cos\theta - \cos\theta_m = 2\sin^2\frac{\theta_m}{2} - 2\sin^2\frac{\theta}{2} \rightarrow T = \frac{2}{\omega_0}\int_0^{\theta_m} \frac{1}{\sqrt{\sin^2\frac{\theta_m}{2} - \sin^2\frac{\theta}{2}}}\,d\theta$$

<u>Changement de variable :</u>

$$\sin\frac{\theta}{2} = \sin\frac{\theta_m}{2}\sin\varphi \qquad \theta = 0 \rightarrow \varphi = 0 \qquad \theta = \theta_m \rightarrow \varphi = \pi/2$$

Il faut exprimer $d\theta$.

1ère méthode : $\varphi = \arcsin\left(\frac{\sin(\theta/2)}{\sin(\theta_m/2)}\right)$ $\frac{d\varphi}{d\theta} = \frac{\cos(\theta/2)}{2\sin(\theta_m/2)} \frac{1}{\sqrt{1-\left(\frac{\sin(\theta/2)}{\sin(\theta_m/2)}\right)^2}}$

$$d\theta = \frac{2\sin\left(\frac{\theta_m}{2}\right)}{\cos\frac{\theta}{2}}\sqrt{1-\frac{\sin^2(\theta/2)}{\sin^2(\theta_m/2)}}\,d\varphi = \frac{2}{\cos\frac{\theta}{2}}\sqrt{\sin^2\left(\frac{\theta_m}{2}\right)-\sin^2(\theta/2)}\,d\varphi$$

2° méthode : en différenciant $\sin\frac{\theta}{2} = \sin\frac{\theta_m}{2}\sin\varphi$

$$\frac{1}{2}d\theta\cos\frac{\theta}{2} = \sin\frac{\theta_m}{2}\cos\varphi\, d\varphi = \sin\frac{\theta_m}{2}\left(1-\sin^2\varphi\right)^{1/2}d\varphi$$

$$d\theta = \frac{2}{\cos\theta/2}\left(\sin^2\frac{\theta_m}{2} - \sin^2\frac{\theta}{2}\right)^{1/2}d\varphi$$

On remplace $d\theta$:

$$T = \frac{2}{\omega_0}\times 2\int_0^{\pi/2}\frac{1}{\cos\theta/2}\,d\varphi = \frac{4}{\omega_0}\int_0^{\pi/2}\frac{1}{\sqrt{1-\sin^2\left(\frac{\theta}{2}\right)}}\,d\varphi$$

$$\boxed{T = \frac{4}{\omega_0}\int_0^{\pi/2}\frac{1}{\sqrt{1-\sin^2\left(\frac{\theta_m}{2}\right)\sin^2\varphi}}\,d\varphi}$$

Q13. $T = \frac{4}{\omega_0}\int_0^{\pi/2}\left(1+\frac{1}{2}\sin^2\frac{\theta_m}{2}\underbrace{\sin^2\varphi}_{\frac{1-\cos 2\varphi}{2}}\right)d\varphi = \frac{2T_0}{\pi}\left(\frac{\pi}{2}+\frac{1}{4}\sin^2\frac{\theta_m}{2}\left[\varphi - \frac{\sin 2\varphi}{2}\right]_0^{\pi/2}\right)$

$$T = T_0 + \frac{T_0}{2\pi}\underbrace{\sin^2\frac{\theta_m}{2}}_{\sim\frac{\theta_m^2}{4}}\left(\frac{\pi}{2}\right) = T_0 + T_0\frac{\theta_m^2}{16}$$

$$\boxed{T = T_0\left(1+\frac{\theta_m^2}{16}\right)}$$

<u>Régression linéaire</u> : on trace $T(\theta_m^2)$