

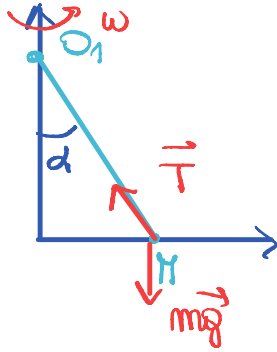
Correction TD16

Exercice 1

Système : { point matériel M }

Réf : terrestre supposé galiléen

BaF : $m\vec{g}, \vec{T}$



$$1) \vec{L}_{O_1} = \vec{O_1M} \wedge m\vec{v}$$

hyp: $\alpha = \omega t$

donc $(\vec{u}_n, \vec{u}_\theta, \vec{u}_z)$:

$$\begin{vmatrix} R & \wedge & 0 \\ 0 & & mR\omega \\ -L\cos\alpha & & 0 \end{vmatrix} = \begin{vmatrix} 0 \\ mRL\cos\alpha\omega \\ 0 \end{vmatrix}$$

$$2) \frac{d\vec{L}_{O_1}}{dt} = \sum \vec{\Pi}_{O_1} = \vec{\Pi}_{O_1}(m\vec{g})$$

$$\vec{\Pi}_{O_1}(\vec{T}) = \vec{O_1M} \wedge m\vec{g} = \vec{0}$$

$$\left. \begin{array}{l} L \begin{vmatrix} R \\ 0 \\ -L\cos\alpha \end{vmatrix} \wedge \begin{vmatrix} 0 \\ 0 \\ -mg \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ mgR \end{vmatrix} \\ \downarrow \\ mRL\cos\alpha\omega^2 \vec{u}_n \end{array} \right\} mRL\cos\alpha\omega^2 = mgR$$

$$\cos\alpha = \frac{g}{L\omega^2}$$

Exercice 2

Système : { point matériel M }

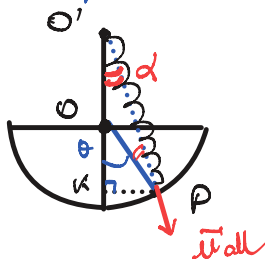
Réf : terrestre supposé galiléen

BaF : $m\vec{g}, \vec{P}_m, \vec{F}_e$

$$1) l = ((a+a\cos\theta)^2 + a^2\sin^2\theta)^{1/2} = (2a^2 + 2a^2\cos\theta)^{1/2} = \left(2a^2 \frac{1+\cos\theta}{2}\right)^{1/2} = 2a\cos\frac{\theta}{2}$$

$$l = 2a\cos\frac{\theta}{2}$$

$$2) \vec{F}_e = -k(l-b)\vec{u}_{all}$$



$\alpha = \frac{\theta}{2}$ (angle au centre = 2 x angle inscrit qui intercepte le même arc)

(autre démo : $\alpha + \frac{\pi}{2} + (\alpha + \frac{\pi}{2} - \theta) = \pi$ donc $OP \Rightarrow \alpha = \frac{\theta}{2}$)

$$\vec{\Pi}_O(\vec{F}_e) = \vec{OP} \wedge -k(l-b)\vec{u}_{all} = \begin{vmatrix} a\cos\theta & \wedge & -k(l-b)\cos\frac{\theta}{2} \\ a\sin\theta & & -k(l-b)\sin\frac{\theta}{2} \\ 0 & & 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ k(l-b)a(\cos\theta\sin\frac{\theta}{2} + \sin\theta\cos\frac{\theta}{2}) \end{vmatrix}$$

Dans $(\vec{u}_n, \vec{u}_\theta, \vec{u}_z)$

$$k(l-b)a(\cos\theta\sin\frac{\theta}{2} + \sin\theta\cos\frac{\theta}{2})$$

$$\vec{\Gamma}_0(\vec{F}_e) = \frac{k}{2} (2a \cos \frac{\theta}{2} - b) a \sin \frac{\theta}{2} \vec{e}_z$$

$$3) \frac{d\vec{L}_0}{dt} = \vec{\Gamma}_0(\vec{F}_e) + \vec{\Gamma}_0(m\vec{g}) + \vec{\Gamma}_0(\vec{R}_m)$$

$\vec{O} \text{ car } \vec{R}_m \perp \vec{OP}$

$$\vec{\Gamma}_0(m\vec{g}) = \vec{OP} \wedge m\vec{g} = -mg a \sin \theta \vec{e}_z$$

$$\vec{L}_0 = \vec{OP} \wedge m\vec{v} = a \vec{e}_n \wedge mva \dot{\theta} \vec{u}_\theta = ma^2 \dot{\theta} \vec{u}_z$$

$$\text{TRC} \Rightarrow ma^2 \ddot{\theta} = -mga \sin \theta + \frac{k}{2} (2a \cos \frac{\theta}{2} - b) a \sin \frac{\theta}{2}$$

$$\ddot{\theta} + \frac{g}{a} \sin \theta - \frac{k}{ma} (2a \cos \frac{\theta}{2} - b) \sin \frac{\theta}{2} = 0 \quad \ddot{\theta} + \left(\frac{g}{a} - \frac{k}{m} \right) \sin \theta + \frac{kb}{ma} \sin \frac{\theta}{2} = 0$$

4) A l'équilibre $\Sigma \vec{\Gamma}_0 = \vec{0}$

$$b = \frac{2mg}{a} \quad b = \frac{\sqrt{3}}{2} a$$

$$2 \left(\frac{g}{a} - \frac{k}{m} \right) \cos \frac{\theta_e}{2} \sin \frac{\theta_e}{2} + \frac{kb}{ma} \sin \frac{\theta_e}{2} = 0$$

$-\frac{k}{2mv} \quad \frac{k\sqrt{3}}{2mv}$

$$\frac{k}{mv} \sin \frac{\theta_e}{2} \left(\frac{\sqrt{3}}{2} - \cos \frac{\theta_e}{2} \right) = 0$$

↳ 3 positions d'équilibre : $\theta_{e1} = 0$ $\theta_{e2} = \frac{\pi}{3}$ $\theta_{e3} = -\frac{\pi}{3}$

$$5) \theta = \theta_e + \varepsilon \rightarrow \ddot{\varepsilon} - \frac{k}{2mv} \sin(\theta_e + \varepsilon) + \frac{k\sqrt{3}}{2m} \sin \left(\frac{\theta_e + \varepsilon}{2} \right)$$

$$\cdot \sin(\theta_e + \varepsilon) \approx \sin \theta_e \cos \varepsilon + \cos \theta_e \sin \varepsilon \approx \sin \theta_e + \varepsilon \cos \theta_e$$

$$\cdot \sin \frac{\theta_e + \varepsilon}{2} \approx \sin \frac{\theta_e}{2} + \frac{\varepsilon}{2} \cos \frac{\theta_e}{2}$$

$$\ddot{\varepsilon} - \frac{k}{2mv} \cos \theta_e \varepsilon + \frac{k\sqrt{3}}{2m} \frac{\varepsilon}{2} \cos \frac{\theta_e}{2} - \frac{k}{2mv} \sin \theta_e + \frac{k\sqrt{3}}{2m} \sin \frac{\theta_e}{2} = 0$$

0 (équilibre, voir 4))

$$\ddot{\varepsilon} + \frac{k}{2m} \left(\frac{\sqrt{3}}{2} \cos \frac{\theta_e}{2} - \cos \theta_e \right) \varepsilon = 0$$

Si $\frac{\sqrt{3}}{2} \cos \frac{\theta_e}{2} - \cos \theta_e > 0 \rightarrow \text{OHNA}$ donc stable sinon instable.

$\theta_{e_1} = 0$ $\frac{\sqrt{3}}{2} \cos \theta_{e_1} - \cos \theta_{e_1} = \frac{\sqrt{3}}{2} - 1 < 0$ instable

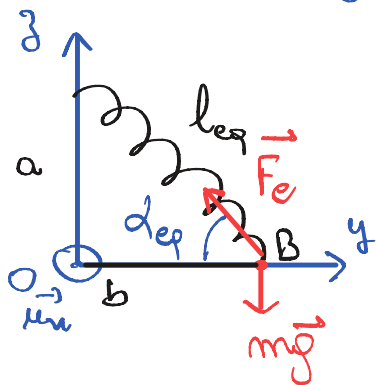
$\theta_{e_2} = +\frac{\pi}{3}$ $\frac{\sqrt{3}}{2} \cos \frac{\pi}{3} - \cos \frac{\pi}{3} = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} = 0,25 > 0$ stable

Exercice 3

1) Equilibre : $\sum \vec{F} = \vec{0}$ $\sum \vec{\Pi}_O = \vec{0}$

Système : { point B } Réf : terrestre support galiléen

BdF : poids $m\vec{g}$, force de rappel élastique \vec{F}_e , tension \vec{T} de la tige



$\vec{\Pi}_O(\vec{T}) = \vec{0}$ \vec{T} inconnue, colinéaire à \vec{OB}

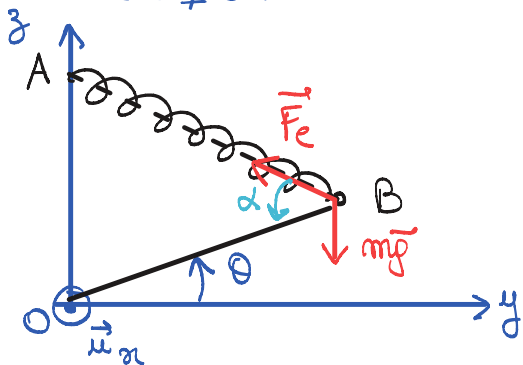
$\vec{\Pi}_O(m\vec{g}) = b\vec{u}_y \wedge -mg\vec{u}_z = -mgb\vec{u}_n$

$\vec{\Pi}_O(\vec{F}_e) = b(l_{eq} - b)b \sin \alpha \vec{u}_n$ $\sin \alpha = \frac{a}{l_{eq}}$

$mgb = \frac{b^2}{l_{eq}} a (1 - \frac{b}{l_{eq}})$ $l_{eq} = \frac{b}{1 - mgb/ka}$

$l_{eq} > 0 \Rightarrow a > \frac{mgb}{k}$

2) Pour $\theta \neq 0$:



$\vec{\Pi}_O(m\vec{g}) = \vec{OB} \wedge m\vec{g} = -mgb \cos \theta \vec{u}_n$

$\vec{\Pi}_O(\vec{F}_e) = \vec{OB} \wedge \vec{F}_e = b^2(l-b) \sin \alpha \vec{u}_n$

loi des sinus : $\frac{\sin \alpha}{a} = \frac{\sin(\pi/2 - \theta)}{l}$

$\vec{L}_O = \vec{OB} \wedge m\vec{v} = b\vec{e}_n \wedge mb\dot{\theta}\vec{u}_\theta = mb^2\dot{\theta}\vec{u}_n$

TMC : $mb^2\ddot{\theta} = -mgb \cos \theta + b^2(l-b) \frac{a \cos \theta}{l} \approx -mgb + b^2a - \frac{b^2ba}{l}$

$l^2 = a^2 + b^2 - 2ab \sin \theta \approx l_{eq}^2 - 2ab\theta$

$mb^2\ddot{\theta} = -mgb + b^2a - b^2ba \left(\frac{l_{eq}^2 - 2ab\theta}{l_{eq}^2} \right)^{-1/2} \rightarrow l_{eq}^2 \left(1 - \frac{2ab\theta}{l_{eq}^2} \right)$

$mb^2\ddot{\theta} = -mgb + b^2a - \frac{b^2ba}{l_{eq}} \left(1 - \frac{2ab\theta}{l_{eq}^2} \right)^{-1/2} \ll 1$

$\sin \alpha = a \frac{\cos \theta}{l}$

$\cos \theta \approx 1$ et $\sin \theta \approx \theta$

DL à l'ordre 1

$$mb^2 \ddot{\theta} \approx -mgb + bka - bkb a \times \frac{1}{lep} \left(1 + ab \frac{\theta}{lep^2}\right) \quad mg = k a \left(1 - \frac{b}{lep}\right)$$

$$mb^2 \ddot{\theta} = -\cancel{b a b} + \frac{k a b b}{lep} + \cancel{b k a} - \cancel{b k b a} - \cancel{b k b a^2} \frac{\theta}{lep^3}$$

$$\ddot{\theta} + \frac{k b a^2}{m lep^3} \theta = 0$$

OHNA $\omega_0 = \left(\frac{k b a^2}{m lep^3}\right)^{1/2}$ pulsation propre des petites oscillations

$$T_0 = \frac{2\pi}{\omega_0} \Rightarrow T_0 = 2\pi \sqrt{\frac{m lep^3}{k b a^2}}$$

Exercice 4

1) Système: $\{e^{-}\}$ Ref: terrestre support galiléen PDF: \vec{F}_{elec} (on néglige le poids)

$$\vec{F}_{elec} = \frac{-e^2}{4\pi\epsilon_0 r^2} \vec{u}_r \quad \begin{matrix} +e & \dots & e \\ & \leftarrow r & \rightarrow \vec{u}_r \\ & \vec{F}_{elec} & \end{matrix}$$

Sur 1 orbite circulaire $\vec{F}_{elec} \perp \vec{v}$, ne travaille pas donc $\Delta E_e = 0 \rightarrow v = Cté$

$$PDF \rightarrow -m_e \frac{v^2}{r} = -\frac{e^2}{4\pi\epsilon_0 r^2} \rightarrow v = \left(\frac{e^2}{m_e 4\pi\epsilon_0 r}\right)^{1/2}$$

$$2) E_e = \frac{1}{2} m_e v^2 \quad E_p = -\frac{e^2}{4\pi\epsilon_0 r} \quad E_m = \frac{1}{2} m_e v^2 - \frac{e^2}{4\pi\epsilon_0 r} = \frac{-e^2}{8\pi\epsilon_0 r}$$

$$3) \vec{L}_0 = \vec{OM}_1 \wedge m\vec{v} = m r v \vec{u}_z \quad m r v = m \hbar \quad v = \frac{m \hbar}{2\pi r m_e}$$

$$4) \frac{e^2}{m_e 4\pi\epsilon_0 r} = \frac{m^2 \hbar^2}{4\pi^2 r^2 m_e} \quad r = m^2 \frac{\hbar^2 \epsilon_0}{\pi e^2 m_e} \quad r_0 = 53 \text{ pm}$$

$$5) E_m = \frac{-e^2}{8\pi\epsilon_0 r} = \frac{-e^4 m_e}{8 \epsilon_0^2 \hbar^2} \quad E_0 = \frac{e^4 m_e}{8 \epsilon_0^2 \hbar^2} \quad E_0 = 13,6 \text{ eV}$$

$$6) v_0 = \left(\frac{e^2}{m_e 4\pi\epsilon_0 r_0}\right)^{1/2} = \left(\frac{e^2}{4\pi\epsilon_0 m_e} \times \frac{\pi e^2 m_e}{\hbar^2 \epsilon_0}\right)^{1/2} = \left(\frac{e^4}{4 \epsilon_0^2 \hbar^2}\right)^{1/2} \quad (+ \text{rapide en remonquant})$$

$$v_0 = 2,2 \times 10^6 \text{ m} \cdot \text{s}^{-1} < 0,1c \text{ (non relativiste)}$$

$$\frac{1}{2} m_e v^2 \uparrow \quad E_e = -E_m \quad \uparrow \frac{E_0}{m^2}$$

Energie d'ionisation: $\Delta E = E_{\infty} - E_1 = -E_1 = E_0 = 13,6 \text{ eV}$

$$7) \frac{h c}{\lambda_{m \rightarrow \infty}} = E_m - E_{\infty} \text{ (car } E_m > E_{\infty}) \quad \frac{1}{\lambda_{m \rightarrow \infty}} = \frac{E_0}{h c} \left(\frac{1}{m^2} - \frac{1}{\infty^2}\right)$$

$$8) E_m = -\frac{E_0}{n^2} = E_1 + \Delta E \rightarrow n = \left(\frac{-E_0}{-E_0 + \Delta E} \right)^{1/2} = 2$$

n entier : l'énergie sur l'orbite $n = 2$

9) le photon émis a l'énergie de 10,2 eV

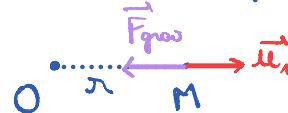
$$v = \frac{E_0}{h}$$

$$\lambda = \frac{c}{v} = 121 \text{ nm} \rightarrow \text{UV}$$

Exercices 5

1) Système : { Satellite } Ref : géocentrique supposé galiléen Bdf : $\vec{F}_{\text{grav}} = -\frac{M_T m}{r^2} \vec{u}_r$

TMC : $\frac{d\vec{L}_O}{dt} = \vec{O}M \wedge \vec{F}_{\text{grav}} = \vec{0}$
 $L \parallel OM$ (force centrale)



$\vec{L}_O = \vec{0}$ or $\vec{L}_O = \vec{O}M \wedge m\vec{v}$ $\vec{O}M$ et $\vec{v} \perp$ à \vec{L}_O par propriété du produit vectoriel

$\vec{O}M$ et $\vec{v} \in$ à 1 plan \perp à $\vec{L}_O = \vec{0}$ \rightarrow le mouvement est plan, le plan de la trajectoire contient le point O

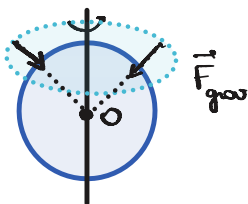
2) Orbite circulaire telle que $r = R + z$ PFD $\Rightarrow m v \frac{v^2}{r} = G \frac{M_T m}{r^2} \rightarrow v = \left(\frac{GM_T}{R+z} \right)^{1/2} = 7,6 \text{ km.s}^{-1}$

3) $E_m = \frac{1}{2} m v^2 - \frac{GM_T m}{r} = -\frac{GM_T m}{2r}$

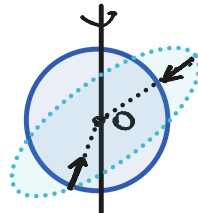
4) $T = \frac{2\pi r}{v} = 2\pi r \left(\frac{r}{GM_T} \right)^{1/2}$ $T^2 = \frac{4\pi^2}{GM_T} r^3$ 3^e loi de Kepler

5) Satellite géostationnaire : fixe pour 1 observateur terrestre
 trajectoire circulaire donc uniforme or $\vec{F}_{\text{grav}} \perp \vec{v} \Rightarrow \Delta E_c = 0$

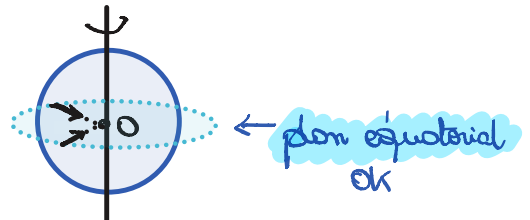
D'après 1), le plan de la trajectoire \perp à \vec{L}_O et contient O (centre de la Terre)



le plan de la trajectoire ne contient pas O



pas géostationnaire

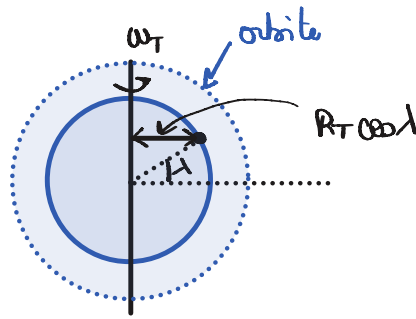


$T =$ période sidérale de la Terre = 86164 s

$T_{\text{sid}}^2 = \frac{4\pi^2}{GM_T} (R_T + z)^3 \rightarrow z \approx 35820 \text{ km}$

6) $\Delta E_m = E_m(\text{orbite}) - E_m(\text{sol}) = -\frac{GM_T m}{r} + \frac{1}{2} m v_{\text{orb}}^2 - \frac{GM_T m}{R_T}$

$$v_{\text{sol}} = \frac{1}{2} m v_{\text{T}} \cos \lambda \omega_{\text{T}}^2$$



$$\omega_{\text{T}} = \frac{2\pi}{T_{\text{sid}}}$$

$$E_{\text{m}} = -GmM_{\text{T}} \left(\frac{1}{r} - \frac{1}{R_{\text{T}}} \right) - \frac{1}{2} m R_{\text{T}} \cos \lambda \omega_{\text{T}}^2 \quad + \text{faible aux basses latitudes}$$

\uparrow
 $R_{\text{T}} + z$

7) "libération" \rightarrow cas limite $E_{\text{m}} = 0$: satellite à l'infini donc $E_{\text{p}} = 0$ et $v_{\text{sol}} = 0$ (cas limite)
 \uparrow
 pour échapper à l'attraction terrestre

$$\frac{1}{2} m v_{\text{lib}}^2 - \frac{GmM_{\text{T}}}{R_{\text{T}}} = 0$$

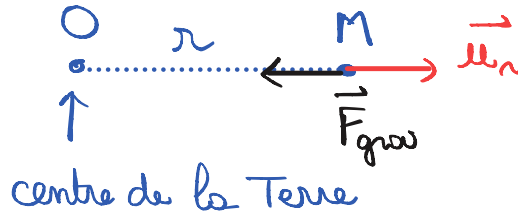
$E_{\text{m}}(\text{sol})$

$$v_{\text{lib}} = \left(\frac{2GM_{\text{T}}}{R_{\text{T}}} \right)^{1/2}$$

$$\text{AN: } 11,2 \text{ km} \cdot \text{s}^{-1}$$

Exercice 6

1) $\vec{F}_{\text{grav}} = -G \frac{mM_{\text{T}}}{r^2} \vec{u}_r$



$$\vec{F}_{\text{grav}} = -\text{grad } E_{\text{p}}$$

$$= \begin{vmatrix} -\frac{\partial E_{\text{p}}}{\partial r} \\ -\frac{1}{r} \frac{\partial E_{\text{p}}}{\partial \theta} \\ -\frac{1}{r \sin \theta} \frac{\partial E_{\text{p}}}{\partial \varphi} \end{vmatrix} = \begin{vmatrix} -G \frac{mM_{\text{T}}}{r^2} \\ 0 \\ 0 \end{vmatrix} \rightarrow E_{\text{p}} = -\frac{GmM_{\text{T}}}{r} + C_0$$

$\Rightarrow E_{\text{p}}$ ne dépend que de r

$$E_{\text{p}}(r) = 0 \rightarrow C_0 = 0$$

$$E_{\text{p}} = -\frac{GmM_{\text{T}}}{r^2}$$

2) PFD selon \vec{u}_r : $-m \frac{v^2}{r} = -G \frac{mM_{\text{T}}}{r^2} \rightarrow v = \left(\frac{GM_{\text{T}}}{r} \right)^{1/2} \quad T = \frac{2\pi r}{v}$

$$T^2 = \frac{4\pi^2}{GM_{\text{T}}} r^3 \quad \underline{\text{3}^{\circ} \text{ loi de Kepler}}$$

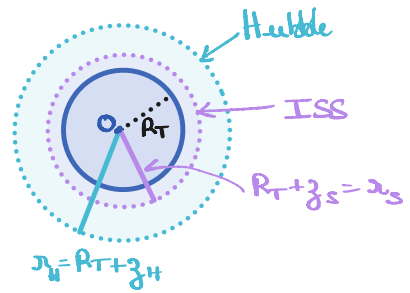
3) $E_{\text{m}} = \frac{1}{2} m v^2 + E_{\text{p}} = \frac{1}{2} \frac{GmM_{\text{T}}}{r} - \frac{GmM_{\text{T}}}{r} \Rightarrow E_{\text{m}} = -\frac{GmM_{\text{T}}}{2r}$

$$4) \frac{4\pi^2}{GM_T} = \frac{T_S^2}{r_S^3} = \frac{T_H^2}{r_H^3}$$

$$T_S = T_H \left(\frac{R_T + 3r_S}{R_T + 3r_H} \right)^{3/2}$$

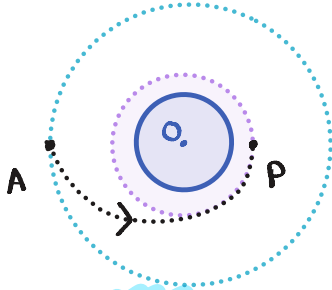
$$AN: T_S = 93 \text{ min}$$

$$v_S = \frac{2\pi r_S}{T_S} = 7,7 \cdot 10^3 \text{ m.s}^{-1}$$



$$v_H = \frac{2\pi r_H}{T_H} = 7,6 \cdot 10^3 \text{ m.s}^{-1}$$

5)



..... orbite de transfert

$$6) E_{mn} = -\frac{GmM_T}{r_S + r_H} \quad (\text{résultat admis pour 1 orbite elliptique})$$

cf cours pour la démo.

$$7) E_m(A) = \frac{1}{2} m v_A^2 - \frac{GmM_T}{r_H} = -\frac{GmM_T}{r_S + r_H} \rightarrow v_A = \sqrt{2GM_T} \left(\frac{1}{r_H} - \frac{1}{r_S + r_H} \right)^{1/2}$$

$$v_A = \left(\frac{8\pi^2}{T_H^2} r_H^3 \frac{r_S}{r_H(r_S + r_H)} \right)^{1/2}$$

$$v_A = 2\sqrt{2}\pi r_H \frac{1}{T_H} \left(\frac{r_S}{r_S + r_H} \right)^{1/2}$$

$$v_A = 7,5 \cdot 10^3 \text{ m.s}^{-1}$$

$$\text{De même : } v_P = 2\sqrt{2}\pi r_S \frac{1}{T_S} \left(\frac{r_H}{r_S + r_H} \right)^{1/2}$$

$$v_P = 7,7 \cdot 10^3 \text{ m.s}^{-1}$$

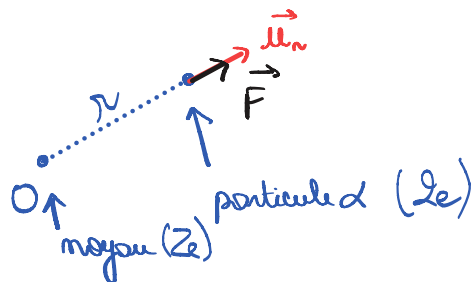
$$8) \text{Durée du voyage } \Delta t = \frac{T}{2}, \text{ orbite elliptique } T = \frac{4\pi^2}{GM_T} \left(\frac{r_H + r_S}{2} \right)^3$$

$$\Delta t = \frac{1}{2} \left(\frac{T_H^2}{r_H^3} \left(\frac{r_H + r_S}{2} \right)^3 \right)^{1/2}$$

$$\Delta t = \frac{T_H}{4\sqrt{2}} \left(1 + \left(\frac{r_S}{r_H} \right) \right)^{3/2} = 47 \text{ min}$$

Exercice 7

$$1) \vec{F} = \frac{2Ze^2}{4\pi\epsilon_0 r^2} \vec{u}_n$$



$$\vec{F} = -\vec{\text{grad}} E_p = \begin{vmatrix} -\partial E_p / \partial r \\ -\frac{1}{r} \partial E_p / \partial \theta \\ -\frac{1}{r \sin \theta} \partial E_p / \partial \phi \end{vmatrix} = \begin{vmatrix} 2Ze^2 / 4\pi\epsilon_0 r^2 \\ 0 \\ 0 \end{vmatrix} \rightarrow E_p = \frac{Ze^2}{2\pi\epsilon_0 r} + C_t$$

$$E_p(\infty) = 0 \quad C_t = 0$$

$$E_p(r) = \frac{Ze^2}{2\pi\epsilon_0 r}$$

2) \vec{F} seule force appliquée, conservative donc $E_m = Cte$

$$E_m = E_m(0) = \frac{1}{2} m v_0^2$$

3) \vec{F} force centrale $\vec{M}_O(\vec{F}) = \vec{0}$ TMC $\Rightarrow \vec{L}_O = Cte$

$$\vec{L}_O = \vec{L}_O(\omega) = \vec{OM}_0 \wedge m \vec{v}_0 = \begin{vmatrix} 0 & b \\ b & 0 \\ 0 & 0 \end{vmatrix} \wedge \begin{vmatrix} -mv_0 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ mbv_0 \end{vmatrix} \rightarrow \vec{L}_O = mbv_0 \vec{u}_z$$

(conténum)

4) $\vec{L}_O = \begin{vmatrix} r \\ 0 \\ 0 \end{vmatrix} \wedge \begin{vmatrix} m \dot{r} \\ m v \dot{\theta} \\ 0 \end{vmatrix} = m r^2 \dot{\theta} \vec{u}_z$

(cylindrique)

5) $E_m = \frac{1}{2} m (\dot{r}^2 + (r\dot{\theta})^2) + \frac{Ze}{2\pi\epsilon_0 r}$ or $m r^2 \dot{\theta} = mbv_0 \rightarrow \dot{\theta} = \frac{bv_0}{r^2}$

$$E_m = \frac{1}{2} m \dot{r}^2 + \underbrace{\frac{1}{2} m \frac{b^2 v_0^2}{r^2} + \frac{Ze^2}{2\pi\epsilon_0 r}}_{\text{énergie potentielle effective } E_{P,eff}(r)}$$

énergie potentielle effective $E_{P,eff}(r)$

6) $\dot{r}_{min} = 0$ $E_m(r_{min}) = E_{P,eff}(r_{min}) = \frac{1}{2} m v_0^2$

$$\frac{1}{2} m \frac{b^2 v_0^2}{r_{min}^2} + \frac{Ze^2}{2\pi\epsilon_0 r_{min}} = \frac{1}{2} m v_0^2$$

$$-\frac{m b^2 v_0^2}{2} - \frac{Ze^2}{2\pi\epsilon_0} r_{min} + \frac{m v_0^2}{2} r_{min}^2 = 0$$

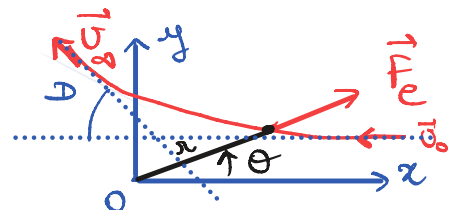
$$\Delta = \left(\frac{Ze^2}{2\pi\epsilon_0}\right)^2 + \frac{m^2 b^2 v_0^4}{4} > 0 \rightarrow 2 \text{ solutions } \in \mathbb{R}$$

$$r_{min} > 0 \quad r_{min} = \frac{\frac{Ze^2}{2\pi\epsilon_0} + \sqrt{\Delta}}{m v_0^2} = \frac{K}{m v_0^2} + \frac{1}{m v_0^2} \sqrt{K^2 + m^2 b^2 v_0^4}$$

$$r_{min} = \frac{K}{m v_0^2} \left(1 + \sqrt{1 + \left(\frac{m b v_0^2}{K}\right)^2} \right)$$

7) $m \frac{dr_n}{dt} = \frac{Ze^2}{2\pi\epsilon_0 r^2} \cos\theta$ TMC $\Rightarrow m r^2 \dot{\theta} = mbv_0$

$$m \frac{dr_n}{dt} = \frac{K}{b v_0} \cos\theta \frac{d\theta}{dt} \rightarrow \int_{-v_0}^{v_0} dr_n = \frac{K}{m b v_0} \int_{\theta(0)}^{\theta_0} \cos\theta d\theta$$



• à $t=0$: particule à l'infini $\theta=0$
 • à $t \rightarrow \infty$: " " $\theta \rightarrow \pi - D$

Conservation de E_m $\Rightarrow v_{\infty} = v_0$, selon On $v_{\infty} = -v_0 \cos D$

$$\frac{-v_0 \cos D + v_0}{v_0(1 - \cos D)} = \frac{k}{mbv_0} \frac{\sin D}{2 \cos \frac{D}{2} \sin \frac{D}{2}}$$

$$1 - \cos D = 2 \sin^2 \frac{D}{2} \Rightarrow$$

$$\tan \frac{D}{2} = \frac{k}{mbv_0^2}$$

$$8) \quad b = \frac{Ze^2}{2\pi\epsilon_0 m v_0^2 \tan \frac{D}{2}}$$

$$D_1 = 60^\circ \rightarrow b_1 = 1,06 \times 10^{-14} \text{ m} \rightarrow r_{\min, 1} = 1,62 \cdot 10^{-14} \text{ m}$$

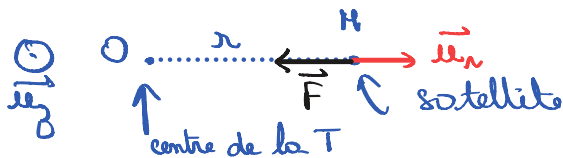
$$D_2 = 180^\circ \rightarrow b_2 = 0 \text{ m} \rightarrow r_{\min, 2} = 1,23 \cdot 10^{-14} \text{ m}$$

rayon de l'atome d'or $< r_{\min} \rightarrow$ ordre de grandeur : 10^{-14} m

Exercice 8

1) Système : { satellite } Ref : terre supposé galiléen For : force de gravitation

$$\vec{F} = -G \frac{mM_T}{r^2} \vec{u}_r \quad \text{force centrale} \rightarrow \text{TNC} \Rightarrow \vec{L}_O = Cte$$



$$\vec{L}_O = \vec{OM} \wedge m \vec{v} = \begin{vmatrix} r & 0 & 0 \\ 0 & r\omega & 0 \\ 0 & 0 & 0 \end{vmatrix} \quad \text{orbite circulaire (} r = Cte \text{)}$$

$$\vec{L}_O = m r^2 \omega \vec{u}_z = Cte$$

Donc $C = r^2 \omega = Cte$

Mouvement circulaire : $\vec{v} = R\omega \vec{u}_\theta$ $\|\vec{v}\| = \frac{C}{R} = Cte$

$$2) \text{ PFD selon } \vec{u}_r \Rightarrow -m \frac{v^2}{r} = -G \frac{mM_T}{r^2} \quad v = \left(\frac{GM_T}{r} \right)^{1/2}$$

AN : $v (600 \text{ km}) = 7,4 \times 10^3 \text{ km} \cdot \text{s}^{-1}$

$$3) E_c = \frac{1}{2} m v^2 = \frac{mGM_T}{2r} \quad E_p = -G \frac{mM_T}{r} \quad E_m = -\frac{GmM_T}{2r}$$

$$4) \text{ TPM : } \frac{dE_m}{dt} = P(\vec{f})$$

$$GmM_T \frac{\dot{r}}{2r^2} = -\lambda m v^3$$

$$\vec{v} = \dot{r} \vec{u}_r + r\dot{\theta} \vec{u}_\theta \approx \sqrt{\frac{GM_T}{r}} \vec{u}_\theta \quad (\text{descente lente si } \vec{u}_r \text{ négligée})$$

$$GM_T \frac{\dot{r}}{2r^2} = -\lambda \left(\frac{GM_T}{r} \right)^{3/2}$$

$$\dot{r} = -2\lambda \sqrt{GM_T} \sqrt{r}$$

5) $\dot{r} < 0$ donc le satellite descend mais si $r \searrow$ $E_c \nearrow$, la vitesse augmente!

(ou $E_m \searrow$ car $P(\vec{f}) < 0$, $E_m = -\frac{GmM_T}{2r}$ si $E_m \searrow : r \searrow$)

$$6) \quad E_c = \frac{GmM_T}{2r(t)} \uparrow \quad E_p = -\frac{GmM_T}{r(t)} \downarrow$$

Quand r diminue, la force de gravitation travaille : son travail est positif

(E_p décroît) $\Delta E_c = \underbrace{-\Delta E_p}_{>0} + \underbrace{W(\vec{f})}_{>0}$ si $|\Delta E_p| > |W(\vec{f})|$ alors $\Delta E_c > 0$ et $v \uparrow$.

Avec total cste E_{mech} qui \downarrow .

$$7) \quad \frac{\dot{r}}{2\sqrt{r}} = -\lambda\sqrt{M_T G} \quad \text{on intègre entre } r(0) \text{ et } r(t) : \sqrt{r(t)} - \sqrt{r(0)} = -\lambda\sqrt{GM_T} t$$

$$\sqrt{r(t)} = \sqrt{r(0)} - \lambda\sqrt{GM_T} t$$

étude sur 1 jour : $r(0) = 7400 \text{ km}$ $r(1j) = (7400 - 2 \times 10^{-3}) \text{ km}$

$$\lambda = \frac{\sqrt{r(0)} - \sqrt{r(1j)}}{\sqrt{GM_T} \Delta t} = 2 \times 10^{-16} \text{ m}^{-1/2}$$

$\uparrow 24 \times 3600$

Exercice 9

1) Système : {satellite} Ref : géocentrique
suppose galiléen For : force de gravitation

$$\vec{F} = -G \frac{mM_T}{r^2} \vec{u}_r \quad \text{force centrale conservative : } E_{\text{mech}} = C_{\text{te}}$$

$$\text{TMC} \Rightarrow \vec{L}_0 = C_{\text{te}}$$

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v(r_{\text{min}})^2 - \frac{GmM_T}{r_{\text{min}}} \quad (1)$$

$$\vec{L}_0 = \vec{OH} \wedge m \vec{v} = \begin{vmatrix} b & \wedge & 0 \\ \infty & & m v_0 \\ 0 & & 0 \end{vmatrix} = m v_0 b \vec{e}_y \quad r^2 \dot{\theta} = v_0 b = \frac{L_0}{m} \quad (2)$$

$\hat{a} t = 0$ (ds $(\vec{u}_x, \vec{u}_y, \vec{u}_z)$)

$$v^2 = \dot{r}^2 + (r\dot{\theta})^2 \quad \dot{r} = 0 \text{ en } r_{\text{min}} \quad (\text{point P})$$

$$(1) \rightarrow \frac{1}{2} m v_0^2 = \frac{1}{2} m \frac{v_0^2 b^2}{r_{\text{min}}^2} - \frac{GmM_T}{r_{\text{min}}}$$

$$v_0^2 = \frac{b^2 v_0^2}{r_{\min}^2} - 2 \frac{GM_T}{r_{\min}} \quad \times r_{\min}^2 \rightarrow v_0^2 r_{\min}^2 + 2GM_T r_{\min} - b^2 v_0^2 = 0$$

$$\Delta = 4(GM_T)^2 + 4b^2 v_0^4 > 0 \quad r_{\min} > 0 \quad r_{\min} = -\frac{GM_T}{v_0^2} + \frac{1}{2v_0^2} \sqrt{\Delta}$$

$$r_{\min} = -\frac{GM_T}{v_0^2} + \sqrt{b^2 + \frac{GM_T^2}{v_0^4}}$$

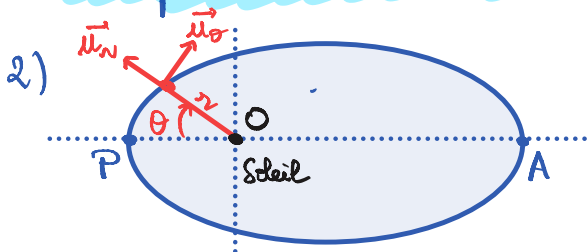
$$2) \quad r_{\min} > R_T \quad b_{\min} = \left(-\frac{GM_T}{v_0^2} + \left(R_T + \frac{GM_T}{v_0^2} \right)^2 \right)^{1/2} = \left(R_T^2 + 2R_T \frac{GM_T}{v_0^2} \right)^{1/2}$$

Exercice 10

1) La comète n'est soumise qu'à la force de gravitation, force centrale.

$$\vec{M}_O = \vec{O} \vec{n} \wedge \vec{F} = \vec{0} \quad \text{TMC} : \frac{d\vec{L}_O}{dt} = \vec{0} \quad \vec{L}_O = \vec{O} \vec{n} \wedge m \vec{v} = C \vec{e}_z$$

Propriété du produit vectoriel : $\vec{O} \vec{n}$ et $\vec{v} \perp \vec{L}_O$ donc $\vec{O} \vec{n}$ et $\vec{v} \in$ à tout instant t au plan contenant le centre de force O et \perp à \vec{L}_O .



$$OA = r_A = \frac{p}{1-e} \quad OP = r_p = \frac{p}{1+e}$$

$$p = r_A (1-e) \quad \text{ou} \quad p = r_p (1+e) \quad p = 1,16 \text{ ua}$$

$$3) \text{ 3}^\circ \text{ loi de Kepler} : \frac{T^2}{a^3} = \frac{4\pi^2}{GM_S} \quad a = \frac{r_p + r_A}{2} \rightarrow \mu_S = 2,0 \times 10^{30} \text{ kg}$$

$$4) \quad E_m = \frac{1}{2} m v^2 - \frac{GM_S m}{r} = -\frac{GM_S m}{2a} \rightarrow v = \left(GM_S \left(\frac{2}{r} - \frac{1}{a} \right) \right)^{1/2}$$

$$\text{en } r_p : v_p = 54,5 \text{ km.s}^{-1} \quad \text{idem en } r_A : v_A = 911 \text{ m.s}^{-1}$$

ou $\frac{dA}{dt} = \frac{1}{2} C$ C constante des aires $C = \frac{L_O}{m} = r^2 \dot{\theta}$ (constante car $\vec{L}_O = C \vec{e}_z$)
 \uparrow
 vitesse areolaire $\frac{dA}{dt} = \frac{S}{T}$ or en r_A et r_p $\dot{r} = 0$ donc $v_A = r_A \dot{\theta}_A$ et $v_p = r_p \dot{\theta}_p$

$$C = r_A v_A \dot{\theta}_A = r_P v_P \dot{\theta}_P \Rightarrow r_A v_A = r_P v_P = C$$

$$\frac{S}{T} = \frac{1}{2} r_A v_A \rightarrow v_A = \frac{2S}{T r_A} = \dots \quad v_P = \frac{r_A v_A}{r_P} = \dots$$

4) a) θ_0 et $-\theta_0$ correspondent à $r = r_T = 1 \text{ ua}$.

$$r_T = \frac{p}{1 + e \cos \theta_0}$$

$$\theta_0 = \arccos \left(\frac{p - r_T}{e r_T} \right)$$

$$\text{AN: } \theta_0 = 80,5^\circ$$

$$\begin{aligned} \text{b) } C &= r^2 \dot{\theta} = \frac{L_0}{m} = C t \quad \vec{L}_0 = \vec{OA} \wedge m \vec{v}_A \quad (\text{ou } \vec{OP} \wedge m \vec{v}_P) \\ &= \begin{vmatrix} r_A \wedge & 0 \\ 0 & m v_A \\ 0 & 0 \end{vmatrix} = \frac{m r_A v_A \vec{e}_z}{L_0} \end{aligned}$$

$$C = r_A v_A \quad (\text{ou } r_P v_P) \quad \text{et } p = r_A (1 - e)$$

$$\frac{r_A^2 (1 - e)^2}{(1 + e \cos \theta)^2} \frac{d\theta}{dt} = r_A v_A$$

$$\Delta t = \frac{r_A (1 - e)^2}{v_A} \int_{-\theta_0}^{\theta_0} \frac{1}{(1 + e \cos \theta)^2} d\theta$$

$$\text{AN: } e \approx 1 \Rightarrow \Delta t = 77 \text{ j}$$

Exercice 10

1) Système : {satellite} Réf : géocentrique supposé galiléen

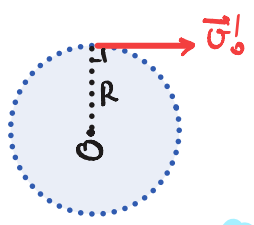
Bdf : force de gravitation

$$\text{Orbite circulaire : PFD} \Rightarrow -m \frac{v_0^2}{R} = -G \frac{m M_T}{R^2} \rightarrow v_0 = \sqrt{\frac{G M_T}{R}}$$

$$E_m = E_c + E_p = \frac{1}{2} m v_0^2 - G \frac{m M_T}{R} = -\frac{G m M_T}{2R}$$

$$3^\circ \text{ loi de Kepler : } T_0^2 = \frac{4\pi^2 R^3}{G M_T}$$

2)



O : centre la Terre

$E_m = Cte$ car système conservatif.

$$E_m = \frac{1}{2} m v_0'^2 - G \frac{m M_T}{R}$$

$$\text{ou } \frac{GM_T}{R} = v_0^2$$

$$E_m = m v \left(\frac{v'^2}{2} - v_0^2 \right)$$

3) $E_m < 0$: elliptique $\frac{v_0'}{v_0} < \sqrt{2}$

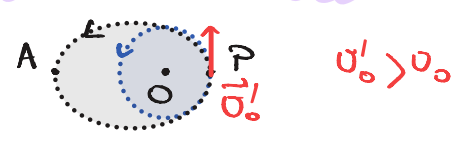
$E_m = 0$: parabolique $v_0' = \sqrt{2} v_0$

$E_m > 0$: hyperbolique $\frac{v_0'}{v_0} > \sqrt{2}$

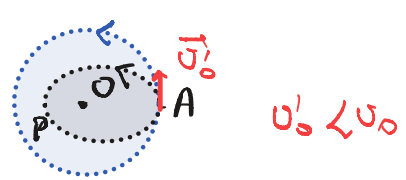
4) Erreur faible : $\frac{v_0'}{v_0} < \sqrt{2} \rightarrow$ ellipse.

\vec{v}_0' toujours orthoradial donc $\vec{r} = 0 \rightarrow r = R$ apogée ou périée.

$\Delta v > 0$: $r = R$ périée $r_{min} = R$ $r_{max} = 2a - R$



$\Delta v < 0$: $r = R$ apogée $r_{max} = R$ $r_{min} = 2a - R$



$$E_m = m v \left(\frac{v_0'^2}{2} - v_0^2 \right) = - \frac{GM_T m}{2a} \rightarrow a = \frac{-GM_T}{2v_0^2 \left(\frac{v_0'^2}{2} - 1 \right)} \quad GM_T = v_0^2 R$$

$$a = \frac{R}{2 - \left(\frac{v_0'}{v_0} \right)^2}$$

$$v_0' = v_0 + \Delta v \text{ et } \Delta v \ll v_0 \quad \left(\frac{v_0'}{v_0} \right)^2 = \left(1 + \frac{\Delta v}{v_0} \right)^2$$

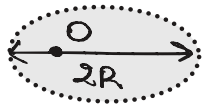
$$a \approx \frac{R}{1 - 2 \frac{\Delta v}{v_0}} \approx R \left(1 + 2 \frac{\Delta v}{v_0} \right) \quad (DL) \quad \approx 1 + 2 \frac{\Delta v}{v_0} \quad (DL)$$

$$T^2 = \frac{4\pi^2}{GM_T} a^3 = T_0^2 \times \frac{1}{\left(2 - \left(\frac{v_0'}{v_0} \right)^2 \right)^3}$$

$$T \approx T_0 \left(1 + 3 \frac{\Delta v}{v_0} \right) \quad (DL)$$

5) Cette fois-ci, comme $\|\vec{v}_0\| = \|\vec{v}'_0\|$ E_{mv} est inchangée $E_{mv} = -\frac{GmM_T}{2R} < 0 \rightarrow$ ellipse

Pour 1 ellipse : $E_{mv} = -\frac{GmM_T}{2a} \Rightarrow a = R \rightarrow r_{\min} + r_{\max} = 2R$



3^o loi de Kepler $\Rightarrow T = T_0$

Pour déterminer r_{\min} et r_{\max} on peut se servir de $\vec{L}_0 = \vec{O}\vec{C}_0$.

$$\vec{L}_0 = \vec{O}\vec{M} \wedge m\vec{v}'_0 = \begin{vmatrix} R & \wedge & m v'_0 \sin \alpha \\ 0 & & m v'_0 \cos \alpha \\ 0 & & 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ m R v'_0 \cos \alpha \end{vmatrix}$$

$$L = m v r^2 \dot{\theta} = m R v'_0 \cos \alpha \quad E_{mv} = -\frac{GmM_T}{2R} = \frac{1}{2} m (\dot{r}^2 + (r\dot{\theta})^2) - \frac{GmM_T}{r}$$

0 en A et P

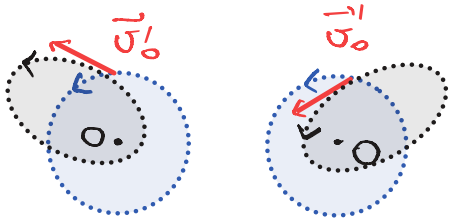
$$-\frac{GmM_T}{2R} = \frac{L^2}{2mr^2} - \frac{GmM_T}{r} \quad \text{en A et P}$$

$\nearrow r_{\max} \quad \nwarrow r_{\min}$

$$-\frac{GmM_T}{2R} = \frac{m^2 R^2 v_0^2 \cos^2 \alpha}{2m r^2} - \frac{GmM_T}{r} \rightarrow \frac{GmM_T}{2R} r^2 - GmM_T r + \frac{R^2 v_0^2 \cos^2 \alpha}{2} = 0$$

$$\Delta = (GmM_T)^2 - 4 \frac{GmM_T}{2R} \times \frac{R^2 v_0^2 \cos^2 \alpha}{2} = (GmM_T)^2 (1 - \cos^2 \alpha) = (GmM_T)^2 \sin^2 \alpha$$

$$r = \frac{GmM_T \pm \sqrt{\Delta}}{GmM_T} R = R (1 \pm |\sin \alpha|)$$



$$r_{\max} = R(1 + |\sin \alpha|) \quad r_{\min} = R(1 - |\sin \alpha|)$$

Exercice 11

1) $\vec{v} = \dot{r} \vec{u}_r + r\dot{\theta} \vec{u}_\theta + \dot{z} \vec{u}_z \quad \dot{z} = \frac{2}{a} r \dot{r}$

$$\vec{L}_0 = \begin{vmatrix} r & & \\ 0 & r^2 & \\ \frac{a}{2} & & \end{vmatrix} \sim \begin{vmatrix} m \dot{r} & & \\ m r \dot{\theta} & & \\ \frac{2m}{a} r \dot{r} & & \end{vmatrix} = \begin{vmatrix} -2m r^3 \dot{r} / a^2 & & \\ -m a^2 \dot{r} / a & & \\ m r^2 \dot{\theta} & & \end{vmatrix}$$

2) Système : { bille } Déf : terrestre supposé galiléen

Bdf : • poids : $m\vec{g}$

• $\vec{R}_m = R_{mz} \vec{u}_z + R_{m\theta} \vec{u}_\theta$

$$\vec{\Pi}_0 = \vec{OM} \wedge (m\vec{g} + \vec{R}_m) = \begin{vmatrix} r \\ 0 \\ z \end{vmatrix} \wedge \begin{vmatrix} R_{m\theta} \\ 0 \\ R_{mz} - mg \end{vmatrix} = \begin{vmatrix} 0 \\ R_{mz} - r(R_{m\theta} - mg) \\ 0 \end{vmatrix}$$

TMC : $\frac{dL_{0z}}{dt} = 0 \rightarrow L_{0z} = Cte$ $m r^2 \dot{\theta} = Cte$

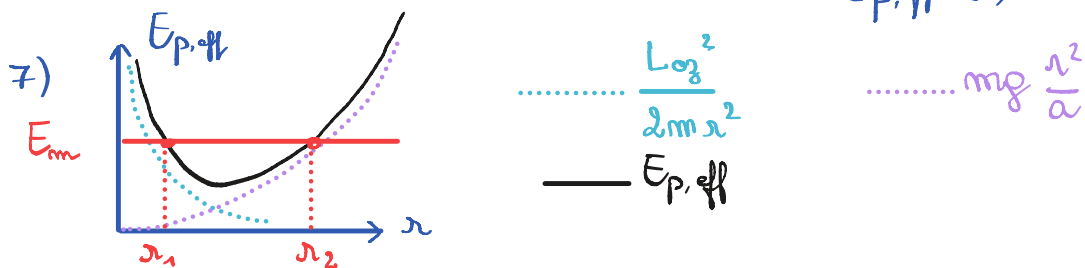
3) $E_c = \frac{1}{2} m v^2 = \frac{1}{2} m (\dot{r}^2 + (r\dot{\theta})^2 + \dot{z}^2) = \frac{1}{2} m \dot{r}^2 + \frac{L_{0z}^2}{2mr^2} + \left(\frac{2rz\dot{r}}{a}\right)^2$

4) $m\vec{g}$ est une force conservative $E_p = mgz + \frac{Cte}{\theta}$ $E_p = mg \frac{r^2}{a}$
 avec $E_p(0) = 0$

5) TEM : $\Delta E_m = \dot{W}^{nc} = 0$ $E_m = Cte$

6) $E_m = Cte \Leftrightarrow \frac{1}{2} m \dot{r}^2 + \frac{L_{0z}^2}{2mr^2} + \frac{1}{2} m \dot{r}^2 \left(\frac{4r^2}{a^2}\right) + mg \frac{r^2}{a} = Cte$

$$\underbrace{\frac{1}{2} m \dot{r}^2 \left(1 + \frac{4r^2}{a^2}\right)}_{G(r)} + \underbrace{\frac{L_{0z}^2}{2mr^2} + mg \frac{r^2}{a}}_{E_{p,eff}(r)} = Cte$$



8) Domaines accessibles à la trajectoire : $E_m \geq E_{p,eff}$

La particule évolue entre 2 disques de rayon r_1 et r_2 tels que

$E_{p,eff}(r_1) = E_{p,eff}(r_2) = E_m \leftarrow \text{constante (ce)}$

