TD 19

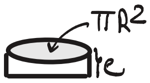
Exercice 1

Système : {Voiture}

1er principe : $\Delta(E_c^M + E_{p,ext} + U) = \underbrace{W^{nc,ext}}_{=0} + Q$

$E_{p,ext} = \text{Cte}$ (hyp : route horizontale)

$Q = 0$ car tte l'énergie dissipée est absorbée par les disques

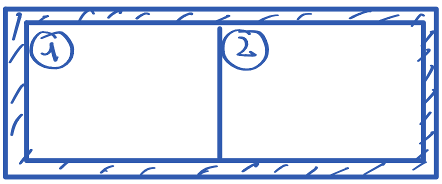


Disque

$$\Delta U = \Delta U_{freins} = 4\rho\pi r^2 e\,\Delta T = -\Delta E^M_{c\,Voiture} = \frac{1}{2}M(v_i^2 - v_f^2)$$

$$\Delta T = \frac{\frac{1}{2}M(v_i^2 - v_f^2)}{4\rho V_c}$$ $\Delta T = 40K$

Exercice 2



hyp : $V_1 = \text{Cte}$ $V_2 = \text{Cte}$

1) **Système** : {1 + 2} $\Delta U = \Delta U_1 + \Delta U_2 = m_1c_1(T_f - T_1) + m_2c_2(T_f - T_2)$

2) $\Delta U = W_p + Q = 0$ car $V = \text{Cte} \Rightarrow W_p = 0$
isolé thermiquement $Q = 0$

$$T_f = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2}$$

3) si $m_2 \gg m_1$ $T_f \simeq T_2$

Thermostat : système de grande taille pouvant participer à des échanges thermiques

sans que sa température ne varie.

4) Système : {①} $\Delta U_1 = Q_1 + \underbrace{W_{p1}}_{0 \text{ car } V_1 = Cte}$ $Q_1 = m_1 c_1 (T_f - T_1)$

Exercice 3

Système : { 0,5 mol de GP }

1) hyp : 0 → 1 isotherme réversible → $W_p = -\int_{V_0}^{V_1} P\,dV$

GP → $P = n\frac{RT}{V}$ $T = Cte$ → $W_p = -nRT_0 \ln\frac{V_1}{V_0} = nRT_0 \ln\frac{P_1}{P_0}$

$W_p = 1,97\,kJ$

2) Pour maintenir $P = Cte$, on suppose que $P_{ext} = Cte = P_1 = P_2$ $W_p = -P_{ext}(V_2 - V_1)$ avec $V_2 = \frac{nRT_2}{P_2}$, $V_1 = \frac{nRT_1}{P_1}$

$W_p = -nR(T_2 - T_1) = -208\,J$

3) 2→3 adiabatique : $W_{p\,2\to3} = \Delta U_{2\to3} = \frac{5}{2}nR(T_3 - T_2) = -1,57\,kJ$

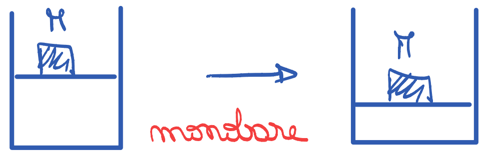
4) $V_3 = \frac{nRT_3}{P_3} = 12L = V_0$ 3→0 isochore $W_p = 0$

Exercice 4

1) Transformation rapide : hyp : adiabatique ⇒ aucune info. sur T_1.

Système : { n mol de GP }

2)



$P_1 = P_0 + \frac{Mg}{S}$

3) $\Delta U_{0\to1} = \frac{5}{2} nR(T_1 - T_0)$ avec $T_1 = \frac{P_1V_1}{nR}$ et $T_0 = \frac{P_0V_0}{nR}$

$Q_{0\to1} = 0$

$W_{p,\,0\to1} = -P_{ext}(V_1 - V_0)$ avec $P_0 + \frac{Mg}{S} = P_1$

$W_{p,\,0\to1} = -P_1(V_1 - V_0)$

$$\Delta U_{0\to1} = \frac{5}{2}(P_1V_1 - P_0V_0)$$

4) $\Delta U_{0\to1} = W_{p,\,0\to1}$

$$\frac{5}{2}P_1V_1 - \frac{5}{2}P_0V_0 = -P_1V_1 + P_1V_0 \to V_1 = \frac{2}{7}V_0\left(1 + \frac{5}{2}\frac{P_0}{P_1}\right)$$

$$T_1 = \frac{P_1V_1}{nR} = \frac{P_1V_1}{P_0V_0}T_0 \to T_1 = \frac{2}{7}T_0\left(\frac{5}{2} + \frac{P_1}{P_0}\right)$$

5) le système n'est pas isolé thermiquement : il y a des pertes par transferts thermiques au travers des parois.

6) Equilibre thermique final : $T_2 = T_0$

Equilibre mécanique final : $P_2 = P_1 = P_0 + \frac{Mg}{S}$

GP $\Rightarrow V_2 = \frac{nRT_2}{P_2}$

$$V_2 = \frac{P_0V_0}{P_1}$$

7) $\Delta U_{1\to2} = \frac{5}{2}nR(T_2 - T_1) = \frac{5}{2}nR(T_0 - T_1)$

$W_{p,\,1\to2} = -P_1(V_2 - V_1) = -(P_0V_0 - P_1V_1)$

$Q_{1\to2} = \Delta U_{1\to2} - W_{p,\,1\to2} = \frac{5}{2}(P_0V_0 - P_1V_1) + P_0V_0 - P_1V_1$

$\frac{7}{2}nR(T_0 - T_1)$

$$Q_{1\to2} = \frac{7}{2}(P_0V_0 - P_1V_1)$$

Rq : ici on peut directement appliquer $Q_{1\to2} = \Delta H_{1\to2}$!

8) $W_{total} = W_{p,\,0\to1} + W_{p,\,1\to2} = -P_1(V_2 - V_0) = -(P_0V_0 - P_1V_0)$ $\quad W_{total} = -V_0(P_0 - P_1)$

$$W_{total} = \frac{Mg}{S}V_0$$

$$Q_{total} = Q_{1\to2} = \frac{7}{2}(P_0V_0 - P_1V_1) = \frac{7}{2}nR(T_0 - T_1)$$

Rq : $Q_{1\to2} = \Delta U_{1\to2} - W_{p,\,1\to2} = -\Delta U_{0\to1} - W_{p,\,1\to2} = -W_{p,\,0\to1} - W_{p,\,1\to2} = -W_{total}$

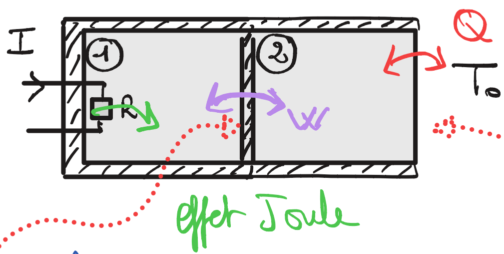
On retrouve que $W_{total} + Q_{total} = 0$ $\quad(\Delta U_{total} = 0$ car $\Delta T_{total} = 0)$

9) hyp : réversible $\to T = T_{ext}\ \forall t$ or $T_{ext} = C^{te} \to$ isotherme $T = C^{te}\ \forall t$

10) $P_f = P_1 = P_0 + \frac{Mg}{S} \to V_f = V_2 = \frac{P_0V_0}{P_1}$ (Même état final)

11) $\Delta T = 0$ donc $\Delta U = 0$ $\quad Q = -W_p = nRT_0 \ln\frac{V_2}{V_0} = nRT_0 \ln\frac{P_0}{P_1} \neq W_{p,\,0\to2}$

Exercice 5



En l'absence d'informations, on néglige la capacité thermique de la résistance.

1) Etat final compartiment 2 : $T_{2f} = T_0$ (équilibre thermique)

Equilibre mécanique : $P_{2f} = P_{1f} = 2P_0$

Conservation du volume : $V_{1f} + V_{2f} = 2V_0$

GP : $P_{2f}V_{2f} = nRT_0 = P_0V_0$ $\qquad P_{1f}V_{1f} = nRT_{1f}$

$2P_0V_{2f} = P_0V_0 \to V_{2f} = \frac{V_0}{2}$ $\qquad V_{1f} = \frac{3V_0}{2}$ $\qquad T_{1f} = 3T_0$

2) quasi-statique + contact avec 1 thermostat : hyp : isotherme réversible

3) $W_2 = -nRT_0 \ln \frac{V_{2f}}{V_0} = -nRT_0 \ln 2$

$W_1 = -W_2$ puisque le travail est échangé au travers de la paroi mobile entre les 2 compartiments.

4) $\Delta U_2 = 0$ $\quad Q_2 = -W_2 = nRT_0 \ln 2$

5) $\Delta U_1 = \frac{nR}{\gamma - 1} \underbrace{(T_{1f} - T_0)}_{2T_0} = \underbrace{Q_{Joule}}_{RI^2\tau\ (=W_{élec})} + W_1$ $\quad$ ou $\Delta U_{1+2} = Q_2 + Q_{Joule}$

$$\tau = \frac{2P_0V_0/\gamma-1 + P_0V_0 \ln 2}{RI^2} = 569\ s$$

6) Compartiment 2 : adiabatique réversible GP → on peut appliquer les lois de Laplace.

$\underset{2P_0}{P_{2f}} V_{2f}^{\gamma} = P_0 V_0^{\gamma}$

(équilibre mécanique $P_{1f} = P_{2f} = 2P_0$)

$V_{2f} = V_0 \left(\frac{P_0}{P_{2f}}\right)^{1/\gamma}$ $\quad V_{2f} = \frac{V_0}{2^{1/\gamma}}$

$T_{2f} = \frac{P_{2f} V_{2f}}{nR} = \frac{T_0}{2^{\frac{1}{\gamma}-1}}$

$Q_2 = 0$ $\quad \Delta U_2 = W_2 = \frac{nR}{\gamma - 1}(T_{2f} - T_0)$

$W_2 = \frac{P_{2f}V_{2f} - P_0V_0}{\gamma - 1} = \frac{P_0V_0}{\gamma-1}\left(2^{1-1/\gamma} - 1\right)$ → $W_1 = -W_2$

$V_{1f} = 2V_0 - V_{2f}$ $\quad V_{1f} = V_0\left(2 - 1/2^{1/\gamma}\right)$

$T_{1f} = \frac{P_{1f}V_{1f}}{nR} = T_0\, 2\left(2 - 1/2^{1/\gamma}\right)$

$\Delta U_1 = \frac{nR}{\gamma-1}(T_{1f} - T_0) = \frac{P_0V_0\left(2(2 - 1/2^{1/\gamma}) - 1\right)}{\gamma - 1}$

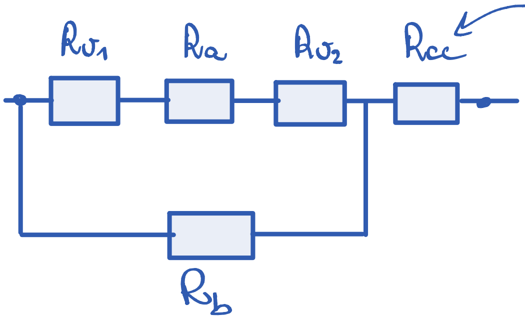
$Q_1 = \Delta U_1 - W_1$

$Q_1 = \frac{2P_0V_0}{\gamma - 1} = Q_{Joule} = RI^2\tau$

$\tau = 500\ s$

Exercice 6

1) Schéma électrique équivalent :



phénomènes conducto-convectifs

$\downarrow$

$$R_{cc} = \frac{1}{h(LH+S_b)}$$

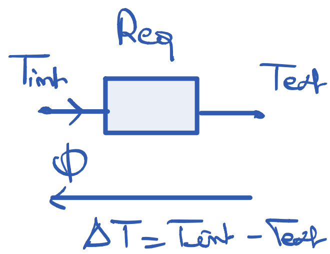
$$R_{v_1} = R_{v_2} = \frac{e_v}{\lambda_v LH}$$

$$R_a = \frac{e_a}{\lambda_a LH} \qquad R_b = \frac{e_b}{\lambda_b S_b}$$

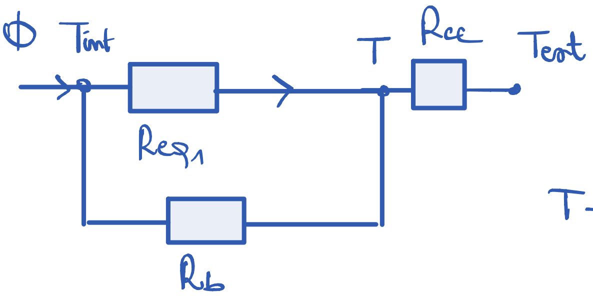
2) Associat° série (3 couches) : $R_{eq_1} = 2R_{v_1} + R_a = 0{,}75\ K.W^{-1}$

Au total : $R_{eq} = \dfrac{R_{eq_1} R_b}{R_{eq_1} + R_b} + R_{cc} = 0{,}65\ K.W^{-1}$

3) $\Delta T = R_{eq}\,\Phi \rightarrow \Phi = \dfrac{\Delta T}{R_{eq}} = 23\ W$



4)



$$T - T_{ext} = \frac{R_{cc}}{R_{eq}} (T_{int} - T_{ext})$$

$$T = T_{ext} + \frac{R_{cc}}{R_{eq}} (T_{int} - T_{ext}) = 7{,}9\,°C$$

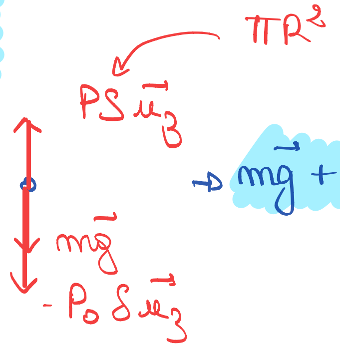
Exercice 7

1) $P_i = P_0 + \frac{mg}{S}$

2) GP subissant une transformation adiabatique réversible : $PV^\gamma = P_i V_0^\gamma$

$V = V_0 + Sx \rightarrow P = P_i \left(\frac{V_0}{V_0 + Sx}\right)^\gamma$

3) $\Sigma \vec{F}$ selon la verticale ascendante :



$\rightarrow m\vec{g} + (P - P_0) S \vec{u}_z$

4) Système : {bille} Rdf : terrestre supposé galiléen

PFD : selon $\vec{u}_x$ $\quad m\ddot{x} = -mg + (P - P_0)S = -mg + \left(\left(P_0 + \frac{mg}{S}\right)\left(\frac{V_0}{V_0 + Sx}\right)^\gamma - P_0\right)S$

$\frac{V_0}{V_0 + Sx} = \frac{1}{1 + \frac{Sx}{V_0}}$ $\quad$ DL : $\left(1 + \frac{Sx}{V_0}\right)^{-\gamma} \simeq 1 - \gamma \frac{Sx}{V_0}$

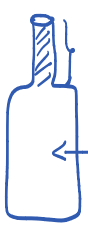
$m\ddot{x} = -mg + \left(\left(P_0 + \frac{mg}{S}\right)\left(1 - \frac{Sx}{V_0}\gamma\right) - P_0\right)S = \cancel{-mg} + \left(\cancel{P_0} + \cancel{\frac{mg}{S}} + \left(P_0 + \frac{mg}{S}\right)\left(-\frac{Sx}{V_0}\gamma\right) - \cancel{P_0}\right)S$

$$\ddot{x} + \left(P_0 + \frac{mg}{S}\right)\frac{S^2\gamma}{mV_0}\, x = 0$$

5) $f = \frac{1}{2\pi}\left(\frac{(P_0 + mg/S)S^2\gamma}{mV_0}\right)^{1/2}$

$f \simeq \frac{1}{2\pi}\left(\frac{(P_0 + mg/S)S^2\gamma}{mV_0}\right)^{1/2} = 1{,}4\,\text{Hz}$

6)

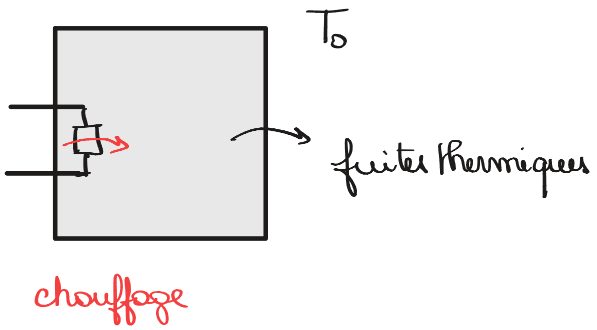


$H \simeq 10\,\text{cm} \rightarrow m = \rho_{air} S H$ $\quad$ πR^2 avec $R \simeq 1\,\text{cm}$

$V_0 \simeq 30\,\text{cL}$ $\quad$ $\rho_{air} \sim 1\,\text{kg/m}^3$ $\quad$ $P_0 + \frac{mg}{S} \simeq P_0$ (1 bar)

$f \simeq 200\,\text{Hz}$

Exercice 8



1) Système : { chambre + murs }

$$dU = \underset{\substack{\| \\ 0 \text{ car } V = Cte}}{\delta W} + \delta Q = (P - P_{th})\, dt$$

↑ CdT

$$\frac{dT}{dt} + \frac{1}{RC} T = \frac{1}{C}\left(P + \frac{T_0}{R}\right)$$

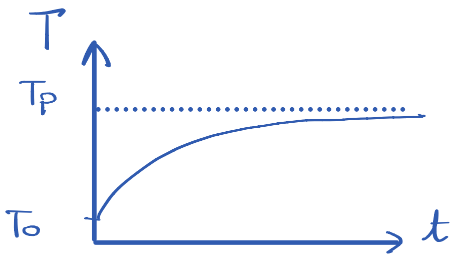
$\tau = RC \qquad T_p = T_0 + RP$

2) $T(t) = A \exp(-t/\tau) + T_0 + RP$

$T(0) = T_0$

$T_0 = A + T_0 + RP \qquad A = -RP$

$$T(t) = T_0 + RP\left(1 - \exp(-t/\tau)\right)$$



3) AN : $T_p = 320\,K$ trop chaud il faut baisser P.

Exercice 9

1) Equilibre mécanique : $\sum \vec{F} = \vec{0}$

Selon Ox :

- à gauche : $-k(\underset{\uparrow x_{G_0}}{x_{G_A}} - l_0) - P_A S = 0 \rightarrow x_{G_A} = x_{G_0} - \frac{P_A S}{k}$

- à droite : $k(\underset{\substack{\uparrow \\ x_{D_0} + l_0}}{d} - x_{D_A} - l_0) + P_A S = 0 \rightarrow x_{D_A} = x_{D_0} + \frac{P_A S}{k}$

2) $V_A = (x_{DA} - x_{GA})S = \left(L + \frac{2P_AS}{k}\right)S$ $S = \pi r_{int}^2$ $V_A = 5{,}7\,L$

3) $T_A = \frac{P_AV_A}{nR} = \frac{P_AS}{nR}\left(L + \frac{2P_AS}{k}\right) = 340\,K$

4) $V_B = \alpha V_A = 7{,}4\,L$

5) Comme au 1) : $x_{DB} = x_{D0} + \frac{P_BS}{k}$ $x_{GB} = x_{G0} - \frac{P_BS}{k}$

6) $V_B = (x_{DB} - x_{GB})S = \left(L + \frac{2P_BS}{k}\right)S$ $\rightarrow$ $P_B = \left(\frac{V_B}{S} - L\right) \times \frac{k}{2S} = 1{,}7 \times 10^4\,Pa$

7) $T_B = \frac{P_BV_B}{nR} = 760\,K$

8) $E_{pe} = \frac{1}{2}k(\ell - \ell_0)^2$ $W_{e,\,A\to B} = -\Delta E_p = E_p(A) - E_p(B)$

$$W_{e,\,A\to B} = \underbrace{\frac{1}{2}k\left[(x_{GA} - x_{G0})^2 + (x_{DA} - x_{D0})^2 - (x_{GB} - x_{G0})^2 - (x_{DB} - x_{D0})^2\right]}_{\frac{S^2(P_A^2 - P_B^2)}{k}} = -24\,J$$

Durant la transformation, le volume du gaz augmente donc le gaz cède de l'énergie.

9) 1er principe appliqué aux {n moles de GP} : $\Delta U = W_e + W_{el}$ (ou $\Delta U + \Delta E_{pe} = W_{el}$)

$n\frac{R}{\gamma - 1}(T_B - T_A)$

$W_{el} = \frac{nR}{\gamma - 1}(T_B - T_A) - W_e = 200\,J$