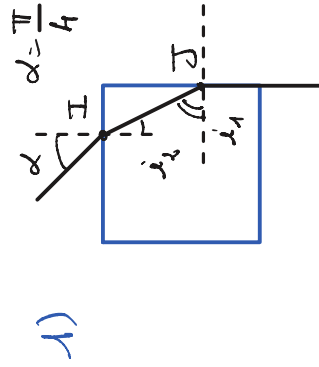


Correction TD1

QCR: 1-betd, 2-b, 3-c, 4-a, 5-b

Exercise 1



2) refraction en I: $\sin \alpha = n \sin i_2$

$$\Rightarrow m_{\text{spin}} \dot{x}_2 = \frac{\sqrt{2}}{2}$$

substitution en J: $n \sin i_n = 1$

3) $\vec{x}_2 = \frac{1}{\sigma} \vec{x}_1 \Rightarrow \sin \vec{x}_2 = \sin \vec{x}_1$
 $m \sin \vec{x}_2 = \frac{1}{\sigma} m \sin \vec{x}_1$

$$n \sin i_c = \frac{\sqrt{2}}{2}$$

$$\cos i_1^2 = 1 - \sin^2 i_1 \quad \cos i_1 = \sqrt{1 - 1/m^2}$$

$$m^2 \left(1 - \frac{1}{m^2} \right) = \frac{1}{m^2} \Rightarrow m^2 = 1 \Rightarrow m = 1$$

Exercise 2

$$1) \theta_2 = \arcsin\left(\frac{m_p \sin \theta}{m_2}\right) = 47.8^\circ$$

2) Calcul de l'angle limite : $\theta_{2, \ell} = \arcsin \left(\frac{n_{\text{air}}}{n_5} \right) = 40,2^\circ$

Diagram illustrating the refraction of light at the interface between air and water. The incident ray is in the air, and the refracted ray is in the water. The angle of incidence is θ_2 and the angle of refraction is θ_1 .

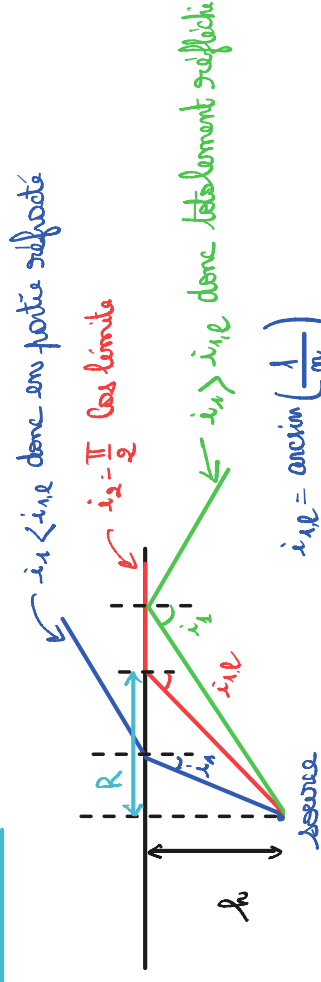
3) même scissellement sur une interface eau(liquide)/verre

$$\Theta_{2e} = \arctan\left(\frac{n_{\text{neu}}}{n_j}\right) = \underline{59,1^\circ}$$

$$\Theta_2 < \Theta_{2,0} \rightarrow \text{pas de saffronisme total en présence de pluie}$$

4.) En présence de pluie, le rayon incident est en partie réfracté et l'intensité du rayon réfléchi est donc plus faible.

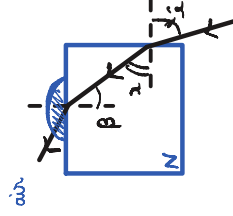
Exercise 3



$$R = \tan i_e$$

Exercice 4

1) la goutte est lumineuse si le rayon lumineux est réfléchi donc si $\beta < \beta_c$



avec $\beta_k = \arcsin\left(\frac{1}{n}\right)$

$$\sin i = N \sin r = N \cos \beta \quad (r = \frac{\pi}{2} - \beta)$$

sim i function σ sur $[0, 1]^2$

1. $\frac{1}{2}$
 2. $\frac{1}{4}$
 3. $\frac{1}{8}$
 4. $\frac{1}{16}$
 5. $\frac{1}{32}$
 6. $\frac{1}{64}$
 7. $\frac{1}{128}$
 8. $\frac{1}{256}$
 9. $\frac{1}{512}$
 10. $\frac{1}{1024}$
 11. $\frac{1}{2048}$
 12. $\frac{1}{4096}$
 13. $\frac{1}{8192}$
 14. $\frac{1}{16384}$
 15. $\frac{1}{32768}$
 16. $\frac{1}{65536}$
 17. $\frac{1}{131072}$
 18. $\frac{1}{262144}$
 19. $\frac{1}{524288}$
 20. $\frac{1}{1048576}$
 21. $\frac{1}{2097152}$
 22. $\frac{1}{4194304}$
 23. $\frac{1}{8388608}$
 24. $\frac{1}{16777216}$
 25. $\frac{1}{33554432}$
 26. $\frac{1}{67108864}$
 27. $\frac{1}{134217728}$
 28. $\frac{1}{268435456}$
 29. $\frac{1}{536870912}$
 30. $\frac{1}{1073741824}$
 31. $\frac{1}{2147483648}$
 32. $\frac{1}{4294967296}$
 33. $\frac{1}{8589934592}$
 34. $\frac{1}{17179869184}$
 35. $\frac{1}{34359738368}$
 36. $\frac{1}{68719476736}$
 37. $\frac{1}{137438953472}$
 38. $\frac{1}{274877906944}$
 39. $\frac{1}{549755813888}$
 40. $\frac{1}{1099511627776}$
 41. $\frac{1}{2199023255552}$
 42. $\frac{1}{4398046511104}$
 43. $\frac{1}{8796093022208}$
 44. $\frac{1}{17592186044416}$
 45. $\frac{1}{35184372088832}$
 46. $\frac{1}{70368744177664}$
 47. $\frac{1}{140737488355328}$
 48. $\frac{1}{281474976710656}$
 49. $\frac{1}{562949953421312}$
 50. $\frac{1}{1125899906842624}$
 51. $\frac{1}{2251799813685248}$
 52. $\frac{1}{4503599627370496}$
 53. $\frac{1}{9007199254740992}$
 54. $\frac{1}{18014398509481984}$
 55. $\frac{1}{36028797018963968}$
 56. $\frac{1}{72057594037927936}$
 57. $\frac{1}{144115188075855872}$
 58. $\frac{1}{288230376151711744}$
 59. $\frac{1}{576460752303423488}$
 60. $\frac{1}{1152921504606846976}$
 61. $\frac{1}{2305843009213693952}$
 62. $\frac{1}{4611686018427387904}$
 63. $\frac{1}{9223372036854775808}$
 64. $\frac{1}{18446744073709551616}$
 65. $\frac{1}{36893488147419103232}$
 66. $\frac{1}{73786976294838206464}$
 67. $\frac{1}{147573952589676412928}$
 68. $\frac{1}{295147905179352825856}$
 69. $\frac{1}{590295810358705651712}$
 70. $\frac{1}{1180591620717411303424}$
 71. $\frac{1}{2361183241434822606848}$
 72. $\frac{1}{4722366482869645213696}$
 73. $\frac{1}{9444732965739290427392}$
 74. $\frac{1}{18889465931478580854784}$
 75. $\frac{1}{37778931862957161709568}$
 76. $\frac{1}{75557863725914323419136}$
 77. $\frac{1}{151115727451828646838272}$
 78. $\frac{1}{302231454903657293676544}$
 79. $\frac{1}{604462909807314587353088}$
 80. $\frac{1}{1208925819614629174706176}$
 81. $\frac{1}{2417851639229258349412352}$
 82. $\frac{1}{4835703278458516698824704}$
 83. $\frac{1}{9671406556917033397649408}$
 84. $\frac{1}{19342813113834066795298816}$
 85. $\frac{1}{38685626227668133590597632}$
 86. $\frac{1}{77371252455336267181195264}$
 87. $\frac{1}{154742504910672534362390528}$
 88. $\frac{1}{309485009821345068724781056}$
 89. $\frac{1}{618970019642690137449562112}$
 90. $\frac{1}{1237940039285380274899124224}$
 91. $\frac{1}{2475880078570760549798248448}$
 92. $\frac{1}{4951760157141521099596496896}$
 93. $\frac{1}{9903520314283042199192993792}$
 94. $\frac{1}{19807040628566084398385987584}$
 95. $\frac{1}{39614081257132168796771975168}$
 96. $\frac{1}{79228162514264337593543950336}$
 97. $\frac{1}{158456325028528675187087900672}$
 98. $\frac{1}{316912650057057350374175801344}$
 99. $\frac{1}{633825300114114700748351602688}$
 100. $\frac{1}{1267650600228229401496703205376}$
 101. $\frac{1}{2535301200456458802993406410752}$
 102. $\frac{1}{5070602400912917605986812821504}$
 103. $\frac{1}{10141204801825835211973625643008}$
 104. $\frac{1}{20282409603651670423947251286016}$
 105. $\frac{1}{40564819207303340847894502572032}$
 106. $\frac{1}{81129638414606681695789005144064}$
 107. $\frac{1}{162259276829213363391578010288128}$
 108. $\frac{1}{324518553658426726783156020576256}$
 109. $\frac{1}{649037107316853453566312041152512}$
 110. $\frac{1}{1298074214633706907132624082305024}$
 111. $\frac{1}{2596148429267413814265248164610048}$
 112. $\frac{1}{5192296858534827628530496329220096}$
 113. $\frac{1}{10384593717069655257060992658440192}$
 114. $\frac{1}{20769187434139310514121985316880384}$
 11

pour avoir $P < P_0$ il faut $i > i_0$ où i_0 est tel que $\sin i_0 = N \cos \theta_0$

$$2) \sin \theta_0 = \frac{n}{N} \quad \sin i_0 = N \sqrt{1 - \sin^2 \theta_0} = \sqrt{N^2 - n^2}$$

$$n^2 = N^2 - \sin^2 i_0 \Rightarrow n = \sqrt{N^2 - \sin^2 i_0}$$

$$3) -1 \leq \sin i_0 \leq 1 \quad \text{donc } 0 \leq \sin^2 i_0 \leq 1$$

$$\sqrt{N^2 - 1} \leq n \leq N$$

$$AN: 1,12 \leq n \leq 1,5$$

Exercice 5

$$1) i > i_0 \text{ tel que } \sin i_0 = \frac{n_1}{n}$$

$$2) \sin \theta = n \sin \alpha = n \cos i \quad (n = \frac{\pi}{2} - i)$$

$$i > i_0 \text{ donc } \cos i < \cos i_0 \quad (\cos i \text{ fonction décroissante sur } [0, \frac{\pi}{2}])$$

$$\sin \theta < n \cos i_0 \quad (\sin i \text{ fonction croissante sur } [0, \frac{\pi}{2}])$$

$$\theta < \theta_0 \text{ tel que } \sin \theta_0 = n \cos i_0 = n \sqrt{1 - \sin^2 i_0}$$

$$ON = \sin i_0 = \sqrt{n^2 - n_1^2}$$

$$3) ON = 0,30$$

$$4) \theta = 0 : t_{\min} = \frac{L}{v} = \frac{L n}{c}$$

$$\theta = \theta_0 : t_{\max} = \frac{L}{\sin i_0 v} = \frac{L n^2}{c n_1} \quad (\sin i_0 = \frac{n_1}{n})$$

$$\delta t = \frac{L n}{c} \left(\frac{n}{n_1} - 1 \right)$$

$$5) \delta t = \frac{L n}{c} \left((1 - 2\Delta)^{-1/2} - 1 \right) \quad \text{DL: } (1 - 2\Delta)^{-1/2} \approx 1 + \Delta$$

$$\delta t = \frac{L n \Delta}{c}$$

$$6) \Delta t_0 \quad t_0 + \delta t \quad t_1 \quad t_2 \quad t_0 + \delta t$$

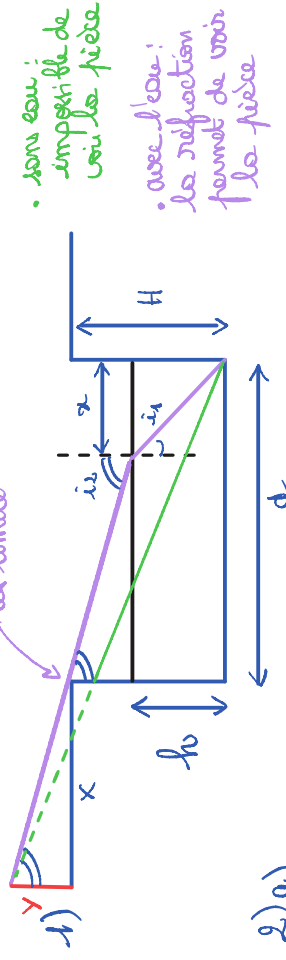
$$7) \text{non recouvrement si } T > \delta t = \frac{L n \Delta}{c} \quad (\text{en négligeant les devant})$$

$$8) B = L n f \quad L n = \frac{c T}{n \Delta} \Rightarrow B = \frac{c}{n \Delta}$$

$$9) \Delta = 0,0198 \quad B = 1,01 \times 10^{10} \text{ Hz} \cdot \text{m} = 1,01 \times 10^4 \text{ MHz} \cdot \text{m}$$

$$L n = 101 \text{ m} \quad \text{peu adapté aux grandes distances}$$

Exercice 6



$$2) a) \text{ --- }$$

$$b) n \sin i_1 = \sin i_2$$

$$c) \tan i_2 = \frac{d - x}{h - h} = \frac{x}{h} \quad \tan i_1 = \frac{x}{h} \rightarrow x = h \tan i_1$$

$$\frac{x}{h} (H - h) = d - h \tan i_1$$

$$h = \frac{d - \frac{x}{h} H}{\tan i_1 - \frac{x}{h}}$$

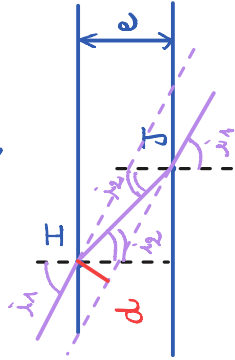
$$i_2 = \arctan \frac{x}{h} = 1,1 \text{ rad}$$

$$i_1 = \arcsin \left(\frac{\sin i_2}{n} \right) = 0,74 \text{ rad} \quad h = 0,93 \text{ m}$$

Exercice 7

1) $n_{\text{air}} < n_{\text{verre}}$ donc il y a toujours réflexion en I
 $i_1 \leq \frac{\pi}{2} \Rightarrow i_2 \leq i_{2c}$ tel que $\sin i_{2c} = \frac{n_{\text{air}}}{n_{\text{verre}}}$

en J : $i_2 \leq i_{2c} \Rightarrow$ pas de réflexion totale : le rayon emerge de la lame // au rayon incident, il existe toujours



$$2) d = IJ \sin(i_1 - i_2) \quad IJ = \frac{e}{\cos i_2}$$

$$d = \frac{e}{\cos i_2} \sin(i_1 - i_2)$$

$$3) d = e \frac{\sin i_1 \cos i_2 - \cos i_1 \sin i_2}{\cos i_2} = e \sin i_1 \left(1 - \cos i_1 \frac{n_1}{n_2 \cos i_2} \right) = \underline{6,1 \text{ cm}}$$

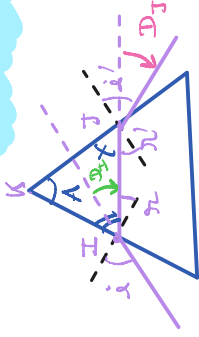
4) petits angles $\cos i_1 \approx 1 \quad \cos i_2 \approx 1 \quad \sin i_1 \approx i_1$

$$d = e \left(1 - \frac{n_1}{n_2} \right) i_1$$

Exercice 8

1) a) $\sin i = n \sin r \quad (1)$

$\sin i' = n \sin r' \quad (2)$



Dans IJK : $A + \frac{\pi}{2} - r + \frac{\pi}{2} - r' = \pi$

$$A = r + r'$$

b) Déviation en I : $D_I = i - r$ Déviation en J : $D_J = i' - r'$

Déviation totale : $D = D_I + D_J = i + i' - (r + r') = \frac{r + r'}{A} \Rightarrow D = i + i' - A$

c) - pas de réflexion totale en J $\Leftrightarrow r' < r_c = \arcsin\left(\frac{1}{n}\right)$

$i_{\text{max}} = \frac{\pi}{2} \Rightarrow r_{\text{max}} = \arcsin\left(\frac{1}{n}\right) \quad r < \arcsin\left(\frac{1}{n}\right)$

$\leftarrow$ cette condition peut se retrouver avec le principe de Fermat inverse de la lumière

$$A = r + i' < 2 \arcsin\left(\frac{1}{n}\right)$$

De plus : $r' < r_c \Rightarrow r > A - r_c \Rightarrow \sin r > \sin(A - r_c)$

$$\Leftrightarrow i > i_{\text{lim}} \text{ tel que } \sin i_{\text{lim}} = n \sin(A - \arcsin(\frac{1}{n}))$$

2) $\frac{dD}{di} = 1 + \frac{di'}{di}$

$$\frac{d}{di}(1) \Rightarrow \cos i = n \frac{dr}{di}$$

$$\frac{d}{di}(2) \Rightarrow \frac{dr}{di} = -\frac{dr'}{di}$$

$$\frac{d(2)}{di} \Rightarrow \frac{di'}{di} \cos i = n \frac{dr'}{di} \cos r' = -n \frac{dr \cos r}{di} \quad \frac{\cos i}{\cos r}$$

$$\Rightarrow \frac{di'}{di} = 1 - \frac{\cos i \cos r}{\cos i' \cos r}$$

$$3) \frac{dD}{di} = 0 \Leftrightarrow \cos i \cos r' = \cos i' \cos r \quad \text{ce qui est le cas si } i = i' \text{ et } r = r'$$

Démonstration non demandée :

méthode calculatrice : $\cos i \cos r' = \cos i' \cos r \Leftrightarrow \cos^2 i \cos^2 r' = \cos^2 i' \cos^2 r$

$$(1 - \sin^2 i)(1 - \sin^2 r') = (1 - \sin^2 i')(1 - \sin^2 r)$$

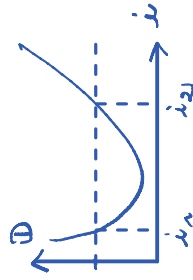
$$(1 - m^2 \sin^2 r)(1 - \sin^2 r') = (1 - m^2 \sin^2 r')(1 - \sin^2 r)$$

$$1 - \sin^2 r' - m^2 \sin^2 r \sin^2 r' = 1 - \sin^2 r - m^2 \sin^2 r' \sin^2 r \quad \text{ou } m^2 \sin^2 r \sin^2 r' = m^2 \sin^2 r' \sin^2 r$$

$$(m^2 - 1) \sin^2 r' = (m^2 - 1) \sin^2 r$$

$$\sin \left[0, \frac{\pi}{2}\right] ; r = r' \text{ et donc } i = i'$$

méthode plus astucieuse :



En dehors du minimum :

$\exists$ 2 valeurs de i, i_1 et i_2 donnant la même

déviation : on peut la comprendre du fait

du principe de retour inverse de la lumière.

incidence en I $i = i_1 \Rightarrow$ émergence en J $i' = i_2$

incidence en I $i = i_2 \Rightarrow$ émergence en J $i' = i_1$

Au minimum de déviation, $\exists$ 1 unique solution : $i_1 = i_2 \Rightarrow i = i'$

$$4) \text{ Au minimum de déviation : } A = r + r' = 2r \quad D_m = 2i - A$$

$$\sin i = m \sin r \Leftrightarrow \sin \left(\frac{D_m + A}{2} \right) = m \sin \left(\frac{A}{2} \right)$$

$$\text{On mesure } D_m \text{ et on calcule } m = \frac{\sin \left(\frac{D_m + A}{2} \right)}{\sin \left(\frac{A}{2} \right)}$$

$$5) \sin \left(\frac{D_m + A}{2} \right) = m \sin \left(\frac{A}{2} \right) \quad \frac{d}{d\lambda} \left(\frac{1}{2} \frac{dD_m}{d\lambda} \cos \left(\frac{D_m + A}{2} \right) \right) = \frac{dm}{d\lambda} \sin \left(\frac{A}{2} \right) \quad \text{ou } \frac{dm}{d\lambda} = - \frac{2b}{\lambda^3}$$

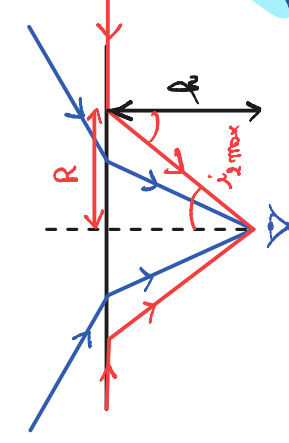
$$\rightarrow D_A = \frac{-Lb \sin \left(\frac{A}{2} \right)}{\cos \left(\frac{A + D_m}{2} \right) \lambda^3}$$

$$6) \frac{dD}{d\lambda} \approx \frac{\Delta D}{\Delta \lambda} \quad \Delta D \approx \Delta \lambda \frac{Lb \sin \left(\frac{A}{2} \right)}{\lambda^3 \cos i_m}$$

$$A = 60^\circ \quad r_m = 30^\circ \rightarrow i_m = 49.91^\circ \quad \Delta D = 2,1 \times 10^{-5} \text{ rad}$$

Cette valeur semble trop faible pour observer les 2 raies.

Exercice 9



Les rayons provenant de l'étoile sont contenus dans un cône de $\frac{1}{2}$ angle au sommet $i_{\max}$ tel que $\sin i_{\max} = \frac{1}{m}$

$$h = \frac{R}{\tan i_{\max}} \quad R = 4,2 \text{ m} \Rightarrow h = 3,7 \text{ m}$$