

0 - a n g

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 0 & 0 & 3 \end{bmatrix} \begin{matrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{matrix} \xrightarrow{P} \boxed{A'} = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{matrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{matrix} \quad P = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ \varepsilon_1 & \varepsilon_2 & \varepsilon_3 \end{bmatrix}$$

2)  $A^n = ?$

$$\xrightarrow{P} \boxed{A'}^n \begin{bmatrix} 1^n & -1^n & 3^n \\ 0 & 1^n & 1^n \\ 0 & 0 & 3^n \end{bmatrix} = \begin{bmatrix} 1 & (-1)^n & 3^n \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

$A^n = (P A' P^{-1})^n$

$$= \cancel{P A' P^{-1}} \dots \cancel{P A' P^{-1}} = \boxed{A'}^n \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$$

= le résultat qui est dans le poly.

3)  $\begin{cases} u_{n+1} \\ v_{n+1} \\ w_{n+1} \end{cases} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} u_n \\ v_n \\ w_n \end{bmatrix}$  complètes

$3w_n \Leftrightarrow \exists M \in \mathbb{R}, \forall n \in \mathbb{N}, w_n = M 3^n$

$\Rightarrow \begin{pmatrix} u_{n+1} \\ v_{n+1} \\ w_{n+1} \end{pmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix} \begin{pmatrix} u_n \\ v_n \\ w_n \end{pmatrix} \Leftrightarrow \underbrace{U_{n+1}}_{\substack{\text{colonne} \\ \text{de } U_{n+1}}} = \underbrace{A}_{\substack{\text{matrice} \\ \text{de } U_{n+1}}} \underbrace{U_n}_{\substack{\text{colonne} \\ \text{de } U_n}}$

$A \xrightarrow{P} A' = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

$A' = P^{-1} A P$

$X = P X'$

$U'_{n+1} = A' U'_n$

$\begin{pmatrix} u'_{n+1} \\ v'_{n+1} \\ w'_{n+1} \end{pmatrix} = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{pmatrix} u'_n \\ v'_n \\ w'_n \end{pmatrix}$

$\begin{cases} u'_{n+1} = 1 u'_n \\ v'_{n+1} = -1 v'_n + 3 w'_n \\ w'_{n+1} = 3 w'_n \end{cases}$

$\begin{cases} u_n = K + L(-1)^n + 2M3^n \\ v_n = \dots \\ w_n = \dots \end{cases}$

$\exists (K, L, M) \in \mathbb{R}^3, \forall n, \begin{cases} u'_n = K 1^n = K \\ v'_n = L (-1)^n \\ w'_n = M 3^n \end{cases}$

$\hat{=} U'_n = \begin{pmatrix} u'_n \\ v'_n \\ w'_n \end{pmatrix} = \begin{pmatrix} K \\ L(-1)^n \\ M 3^n \end{pmatrix} \Leftrightarrow U_n = P U'_n \Leftrightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{pmatrix} K \\ L(-1)^n \\ M 3^n \end{pmatrix} = \begin{pmatrix} K + L(-1)^n + 2M3^n \\ \dots \\ \dots \end{pmatrix}$