

$$(S) \Leftrightarrow \begin{cases} x'(t) = y(t) + 2y(t) \\ y'(t) = 2x(t) + 2y(t) \\ z'(t) = 3z(t) \end{cases} \quad (=) \quad \underbrace{\begin{pmatrix} x'(t) \\ y'(t) \\ z'(t) \end{pmatrix}}_{\frac{dX}{dt} = X'(t)} = \underbrace{\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 0 & 3 \end{bmatrix}}_A \underbrace{\begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}}_{X(t)}$$

$$L_3 \Leftrightarrow z'(t) = 3z(t)$$

$$\Leftrightarrow \exists M \in \mathbb{R}, \forall t \in \mathbb{R}, z(t) = M e^{3t}$$

$$L_3 (=) w_{t+1} = 3w_t$$

$$(=) \exists M \in \mathbb{R}, \forall t \in \mathbb{R}, w_t = M 3^n$$

$$\rightarrow X'(t) = A X(t)$$

$$\rightarrow U'(t) = A' U(t)$$

$$A \rightarrow A' = \begin{bmatrix} 1 & -1 & 3 \end{bmatrix}$$

$$X(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} \rightarrow U(t) = \begin{pmatrix} u(t) \\ v(t) \\ w(t) \end{pmatrix}$$

$$\begin{cases} u'(t) = 1 u(t) \\ v'(t) = -v(t) \\ w'(t) = 3 w(t) \end{cases} \quad (=) \quad \begin{pmatrix} u'(t) \\ v'(t) \\ w'(t) \end{pmatrix} = \begin{bmatrix} 1 & & \\ & -1 & \\ & & 3 \end{bmatrix} \begin{pmatrix} u(t) \\ v(t) \\ w(t) \end{pmatrix}$$

$$U(t) = \begin{pmatrix} u(t) \\ v(t) \\ w(t) \end{pmatrix} = \begin{pmatrix} K e^t \\ L e^{-t} \\ M e^{3t} \end{pmatrix} \Leftrightarrow X(t) = P \cdot U(t) = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 2 \\ 0 & 0 & 3 \end{bmatrix} \begin{pmatrix} K e^t \\ L e^{-t} \\ M e^{3t} \end{pmatrix}$$

$$\exists (K, L, M) \in \mathbb{R}^3, \forall t \in \mathbb{R}, \begin{cases} u(t) = K e^t \\ v(t) = L e^{-t} \\ w(t) = M e^{3t} \end{cases}$$