Corrigé du TD12

Ex 2 1 et 2 : faits en classe.

3.

$$\dfrac{\dfrac{}{\Gamma \vdash p \vee \neg p}\,te \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma, p, q \vdash p}\,ax \quad \dfrac{}{\Gamma, p, q \vdash q}\,ax}{\Gamma, p, q \vdash p \wedge q}\,\wedge i \quad \dfrac{}{\Gamma, p, q \vdash \neg(p \wedge q)}\,ax}{\Gamma, p, q \vdash \bot}\,\neg e}{\Gamma, p \vdash \neg q}\,\neg i}{\Gamma, p \vdash \neg p \vee \neg q}\,\vee i.g \qquad \dfrac{\dfrac{}{\Gamma, \neg p \vdash \neg p}\,ax}{\Gamma, \neg p \vdash \neg p \vee \neg q}\,\vee i.d$$

$$\dfrac{\Gamma \vdash p \vee \neg p \qquad \Gamma, p \vdash \neg p \vee \neg q \qquad \Gamma, \neg p \vdash \neg p \vee \neg q}{\underbrace{\neg(p \wedge q)}_{=\Gamma} \vdash \neg p \vee \neg q}\,\vee e$$

4.

$$\dfrac{\dfrac{}{\Gamma \vdash \neg p \vee \neg q}\,ax \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma, \neg q \vdash p \wedge q}\,ax}{\Gamma, \neg q \vdash q}\,\wedge é \quad \dfrac{}{\Gamma, \neg q \vdash \neg q}\,ax}{\Gamma, \neg q \vdash \bot}\,\neg é \qquad \dfrac{\dfrac{\dfrac{}{\Gamma, \neg p \vdash \neg p}\,ax \quad \dfrac{}{\Gamma, \neg p \vdash p \wedge q}\,ax}{\Gamma, \neg p \vdash p}\,\wedge é}{\Gamma, \neg p \vdash \bot}\,\neg é}{\neg p \vee \neg q, p \wedge q \vdash \bot}\,\vee é}{\neg p \vee \neg q \vdash \neg(p \wedge q)}\,\neg i$$

Ex 3

1.

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash p \wedge (q \vee r)}\,ax}{\Gamma \vdash q \vee r}\,\wedge é,g \qquad \dfrac{A_1}{\Gamma, q \vdash (p \wedge q) \vee (p \wedge r)} \qquad \dfrac{A_2}{\Gamma, r \vdash (p \wedge q) \vee (p \wedge r)}}{\Gamma = p \wedge (q \vee r) \vdash (p \wedge q) \vee (p \wedge r)}\,\vee é$$

avec A_1 :

$$\dfrac{\dfrac{}{\Gamma, q \vdash q}\,ax \qquad \dfrac{\dfrac{}{\Gamma, q \vdash p \wedge (q \vee r)}\,ax}{\Gamma, q \vdash p}\,\wedge é, d}{\Gamma, q \vdash p \wedge q}\,\wedge i}{\Gamma, q \vdash (p \wedge q) \vee (p \wedge r)}\,\vee i, d$$

et A_2 :

$$\dfrac{\dfrac{}{\Gamma, r \vdash r}\,ax \qquad \dfrac{\dfrac{}{\Gamma, r \vdash \Gamma}\,ax}{\Gamma, r \vdash p}\,\wedge é, d}{\Gamma, r \vdash p \wedge r}\,\wedge i}{\Gamma, r \vdash (p \wedge q) \vee (p \wedge r)}\,\vee i, g$$

2.

$$\dfrac{\dfrac{}{\Gamma \vdash \Gamma}\,ax \qquad \dfrac{A_1}{\Gamma, p\wedge q \vdash p\wedge(q\vee r)} \qquad \dfrac{A_2}{\Gamma, p\wedge r \vdash p\wedge(q\vee r)}}{\Gamma = (p\wedge q)\vee(p\wedge r) \vdash p\wedge(q\vee r)}\,\vee é$$

avec A_1 :

$$\dfrac{\dfrac{\dfrac{}{\Gamma, p\wedge q \vdash p\wedge q}\,ax}{\Gamma, p\wedge q \vdash p}\,\wedge é,d \qquad \dfrac{\dfrac{\dfrac{}{\Gamma, p\wedge q \vdash p\wedge q}\,ax}{\Gamma, p\wedge q \vdash q}\,\wedge é,g}{\Gamma, p\wedge q \vdash q\vee r}\,\vee i,d}{\Gamma, p\wedge q \vdash p\wedge(q\vee r)}\,\wedge i$$

et A_2 :

$$\dfrac{\dfrac{\dfrac{}{\Gamma, p\wedge r \vdash p\wedge r}\,ax}{\Gamma, p\wedge r \vdash p}\,\wedge é,d \qquad \dfrac{\dfrac{\dfrac{}{\Gamma, p\wedge r \vdash p\wedge r}\,ax}{\Gamma, p\wedge r \vdash r}\,\wedge é,g}{\Gamma, p\wedge r \vdash q\vee r}\,\vee i,g}{\Gamma, p\wedge r \vdash p\wedge(q\vee r)}\,\wedge i$$

3.

$$\dfrac{\dfrac{A_1}{\Gamma \vdash p \vee q} \qquad \dfrac{A_2}{\Gamma \vdash p \vee r}}{\Gamma = p \vee (q \wedge r) \vdash (p \vee q) \wedge (p \vee r)}\ \wedge i$$

A_1 :

$$\dfrac{\dfrac{}{\Gamma \vdash \Gamma}\ ax \qquad \dfrac{\dfrac{}{\Gamma, p \vdash p}\ ax}{\Gamma, p \vdash p \vee q}\ \vee i,d}{\Gamma \vdash p \vee q}\ \vee é \qquad \dfrac{\dfrac{\dfrac{}{\Gamma, q \wedge r \vdash q \wedge r}\ \wedge é, d}{\Gamma, q \wedge r \vdash q}\ \vee i,g}{\Gamma, q \wedge r \vdash p \vee q}$$

A_2 semblable.

4.

$$\dfrac{\dfrac{}{\Gamma \vdash p \vee \neg p}\ te \qquad \dfrac{\dfrac{\dfrac{}{\Gamma, p \vdash p}\ ax}{\Gamma, p \vdash p \vee (q \wedge r)}\ \vee i,d \qquad \dfrac{\dfrac{A_1}{\Gamma, \neg p \vdash q} \quad \dfrac{A_2}{\Gamma, \neg p \vdash r}}{\Gamma, \neg p \vdash q \wedge r}\ \wedge i}{\Gamma, \neg p \vdash p \vee (q \wedge r)}\ \vee i,g}{\Gamma = (p \vee q) \wedge (p \vee r) \vdash p \vee (q \wedge r)}\ \vee é$$

A_1 :

$$\dfrac{\dfrac{\dfrac{}{\Gamma, \neg p, \neg q \vdash p \vee q} \qquad \dfrac{\dfrac{}{\Gamma' \vdash p}\ ax \qquad \dfrac{}{\Gamma' \vdash \neg p}\ ax}{\Gamma' = \Gamma, \neg p, \neg q, p \vdash \bot}\ \neg é \qquad \dfrac{\dfrac{}{\Gamma'' \vdash q}\ ax \qquad \dfrac{}{\Gamma'' \vdash \neg q}\ ax}{\Gamma'' = \Gamma, \neg p, \neg q, q \vdash \bot}\ \neg é}{\Gamma, \neg p, \neg q \vdash \bot}\ \vee é}{\Gamma, \neg p \vdash q}\ abs$$