TD13

Ex 3 :

②

$$\dfrac{\dfrac{}{\Gamma \vdash \exists y, \forall x, P(y,x)}\,ax \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma, \forall x, P(y,x) \vdash \forall x, P(y,x)}\,ax}{\Gamma, \forall x, P(y,x) \vdash P(y,x)}\,\forall é}{\Gamma, \forall x, P(y,x) \vdash \exists z, P(z,x)}\,\exists i}{\Gamma; \forall x, P(y,x) \vdash \forall x, \exists z, P(z,x)}\,\forall i}{\Gamma := \exists y, \forall x, P(y,x) \vdash \forall x, \exists z, P(z,x)}\,\exists é$$

Ici on peut appliquer $\exists é$ car $y \notin FV(\Gamma) = \emptyset$ et $y \notin FV(\forall x, \exists z, P(z,x)) = \emptyset$

③

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash \exists x, \neg Q(x)}\,ax \qquad \dfrac{\dfrac{\dfrac{}{\Gamma, \neg Q(x) \vdash \forall x, Q(x)}\,ax}{\Gamma, \neg Q(x) \vdash Q(x)}\,\forall é \qquad \dfrac{}{\Gamma, \neg Q(x) \vdash \neg Q(x)}\,ax}{\Gamma, \neg Q(x) \vdash \bot}\,\neg é}{\Gamma := \exists x, \neg Q(x) ; \forall x, Q(x) \vdash \bot}\,\exists é}{\exists x, \neg Q(x) \vdash \neg(\forall x, Q(x))}\,\neg i$$

Ici on peut appliquer la règle $\exists é$ car $x \notin \underbrace{FV(\Gamma)}_{=\emptyset}$ et $x \notin FV(\bot) = \emptyset$.