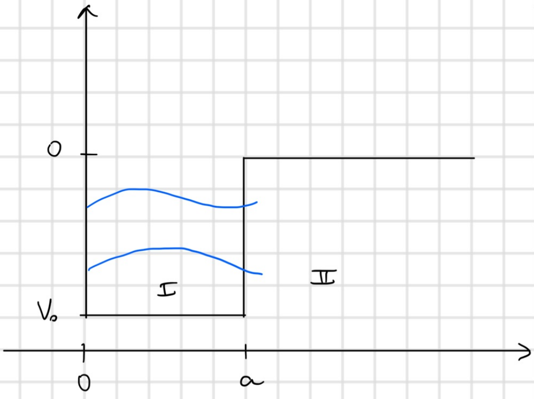
TD1 _ MECANIQUE QUANTIQUE

Exercice 2



Complément : reprise de l'exercice sans considérer un puits ∞

Zone I $\quad \varphi_I(x) = A\cos(kx) + B\sin(kx)$

Zone II $\quad \varphi_{II}(x) = \underbrace{C\,e^{qx}}_{e^{qx} \text{ diverge quand } x \to \infty} + D\,e^{-qx}$

$\Rightarrow C = 0$

$$\begin{cases} k^2 = \dfrac{2m(E-V_0)}{\hbar^2} \\ q^2 = \dfrac{2m(-E)}{\hbar^2} \end{cases}$$

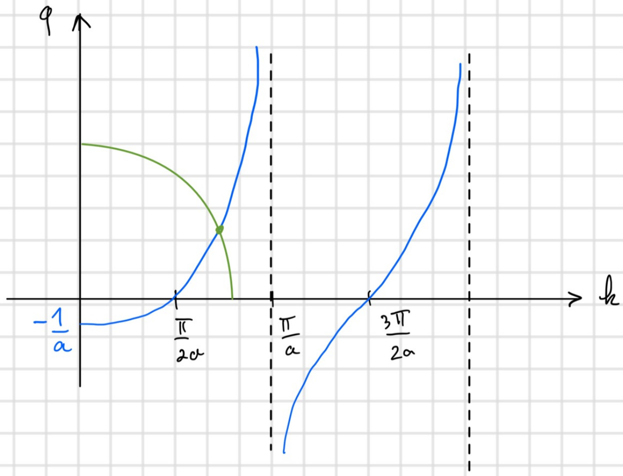
($E<0$ ici)

Conditions aux limites :

$$\begin{cases} \text{en } x=0 \text{, discontinuité } \infty \Rightarrow \varphi_I(0) = 0 \\ \text{en } x=a \text{, discontinuité finie } \begin{cases} \varphi_I(a) = \varphi_{II}(a) \\ \varphi'_I(a) = \varphi'_{II}(a) \end{cases} \end{cases}$$

On a ainsi :

$$\begin{cases} A = 0 \\ B\sin(ka) = D\,e^{-qa} \\ Bk\cos(ka) = -Dq\,e^{-qa} \end{cases} \Rightarrow \boxed{q = -k\,\mathrm{cotan}(ka)}$$



On a d'autre part :

$$\boxed{q^2 + k^2 = -\frac{2mV_0}{\hbar^2}}$$

↳ cercle de rayon $\dfrac{\sqrt{-2mV_0}}{\hbar}$

$\Rightarrow$ 1 seul point d'intersection pour $\dfrac{\pi}{2a} < \dfrac{\sqrt{-2mV_0}}{\hbar} < \dfrac{\pi}{a}$