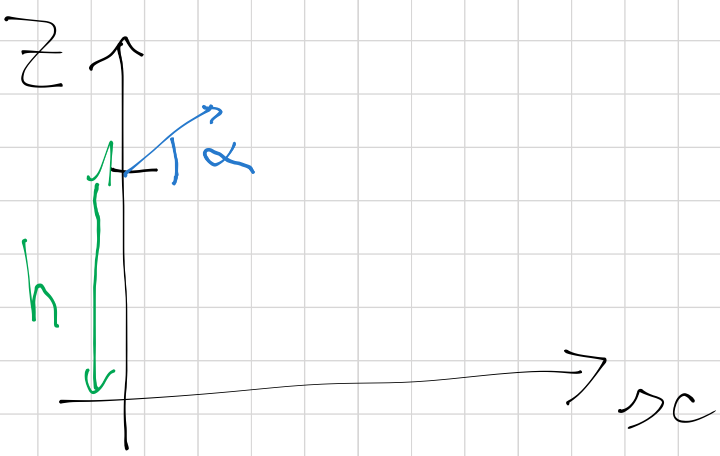
Méconique

Exercice 4 :

$\vec{g}$



Bilan des forces : $\vec{P}$

$\vec{P} = -mg\,\vec{u}_z$.

1) PFD :

$\cancel{m}\,\vec{a} = -\cancel{m}\,g\,\vec{u}_z$

↳ $\vec{v} = (-gt + v_0 \sin(\alpha))\,\vec{u}_z + v_0 \cos(\alpha)\,\vec{u}_x$

$\vec{OM} = (-\frac{1}{2}gt^2 + V_0 t \sin\alpha)\,\vec{u}_z + v_0 t \cos(\alpha)\,\vec{u}_x$

2) Position la plus haute

$\alpha < 0$: h ?

$\alpha > 0$: $t = \frac{v_0 \sin\alpha}{h}$

$$z = \frac{1}{2g}(v_0 \sin\alpha)^2 + h$$

3) $M(x_0, z_0)$ sous la parabole ssi $\exists t \in \mathbb{R}^+$

$$\begin{cases} -\frac{1}{2}gt^2 + v_0 t \sin\alpha + h - z_0 = 0 \\ V_0 t \cos\alpha = x_0 \end{cases}$$

$$\Leftrightarrow \begin{cases} -\frac{1}{2}g\left(\frac{x_0}{V_0\cos(\alpha)}\right)^2 + V_0 \sin\alpha \,\frac{x_0}{V_0\cos\alpha} + h - z_0 = \\ t = \frac{x_0}{V_0\cos\alpha} \end{cases}$$

$$\Leftrightarrow \left\{ -\frac{1}{2}g\left(\frac{x_0}{V_0\cos\alpha}\right)^2 + x_0 \tan\alpha + h - z_0 = 0 \right.$$

Rappel :

TEC : $\Delta E_c = \sum W(\vec{F})$

TEM : Forces conservative

3) $\Delta E_m = 0$.

$$-\frac{1}{2}\frac{g^2 x_0^2}{v_0^2(\cos\alpha)^2} + \tan\alpha\, x_0 + h - z_0 = 0$$

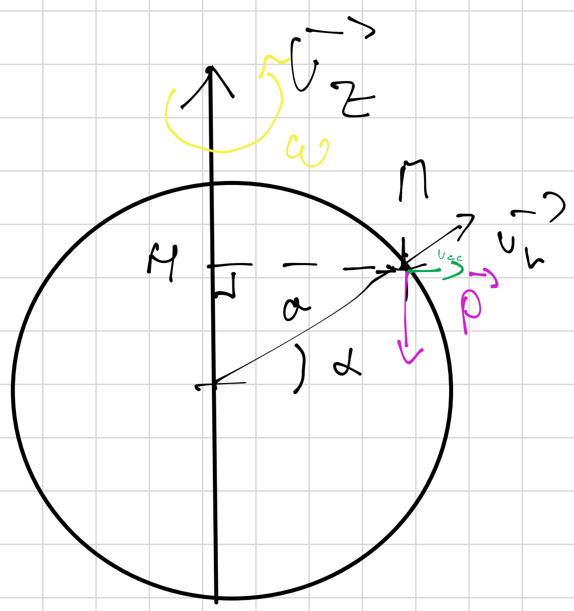
Idée : posons $X = \tan\alpha$.

On a donc

$$-\frac{g^2}{2v_0^2}(1+X^2)x_0^2 + X x_0 + h - z_0 = 0$$

Réponse : $z_0 - h = \frac{v_0^2}{2g} - \frac{g x_0^2}{2 v_0}$

Exercice 2 :



Bilan des forces :

$\vec{P} = -mg\,\vec{u_z}$

$\vec{F_c} = 2m\,\vec{\Omega}\wedge\vec{v} = \vec{0}$ ↳ Car on est à l'équilibre

$\vec{F_e} = m\,\Omega^2\,\vec{HM}$

$\vec{HM} = a\cos\alpha\,\vec{u_{cc}}$

$E_{pp} = mg\,a\sin\alpha$

Rappel : $\dfrac{dE_p}{d\alpha} = 0$

$\dfrac{d^2E_p}{d\alpha^2} > 0 \rightarrow$ Minimum $\Rightarrow$ Stable

$\dfrac{d^2E_p}{d\alpha^2} < 0 \rightarrow$ Maximum $\Rightarrow$ Instable

- $\vec{F}$ conservative $\Leftrightarrow \exists\, E_p$ tq $\vec{F} = -\overrightarrow{grad}\, E_p = -\dfrac{dE_p}{d\alpha}\, \vec{u_\alpha}$

$$\vec{F_e} = m\Omega^2 R_c\, \vec{u_{cc}} = -\frac{d}{dr}\left(-\frac{1}{2} r_c^2 m \Omega^2\right) \vec{u_{cc}}$$

donc $E_{pp} = -\dfrac{1}{2} r_c^2 m \Omega^2$

On a finalement :

$$E_p = E_{pp} + E_{pe}$$
$$= mg \sin\alpha - \frac{1}{2} a^2 \cos^2\alpha\, m\Omega^2$$

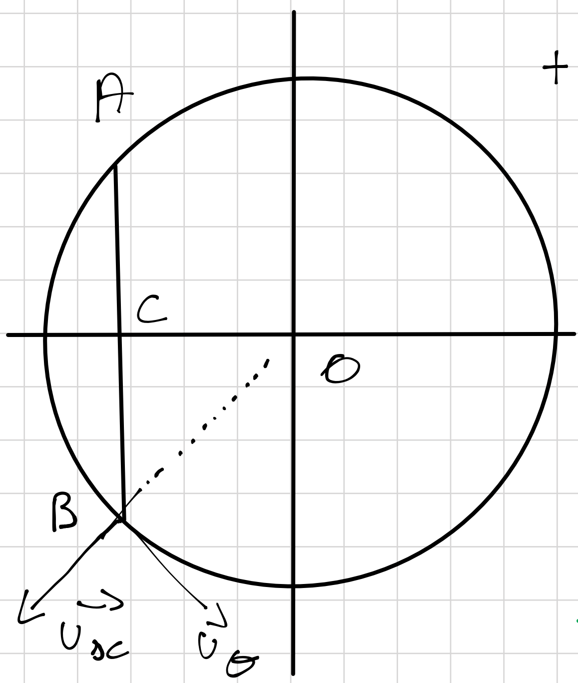
$$\frac{dE_p}{d\alpha} = mg\, a \cos\alpha + \frac{1}{2} a^2 (2 \sin\alpha \cos\alpha)\, m\Omega^2$$

Si $\dfrac{dE_p}{d\alpha} = 0$ alors $a^2 \sin\alpha \cos\alpha\, m\Omega^2 = -mg\, a \cos\alpha$

$\Leftrightarrow a \sin\alpha\, \Omega^2 = -g$

$\Leftrightarrow \sin\alpha = \dfrac{-g}{a\Omega^2}$ ou $\cos\alpha = 0$

Exercice 6 :



$+ M$

Bilan des forces.

$$\vec{F} = -\frac{G\, m\, M_T}{r^2}\, \vec{u_r}$$

Analogie Thm de Gauss.

$$\oiint_{\mathcal{S}} \vec{E} \cdot d\vec{S} = \frac{Q_{int}}{\varepsilon_0} \iff \oiint \vec{\mathcal{G}} \cdot d\vec{s} = -4\pi G M_{Int}$$

$$E \iff \vec{\mathcal{G}}$$

$$\frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \iff -\frac{G m M_T}{r^2}$$

$$\frac{1}{\varepsilon_0} \iff -4\pi G$$

$$\oiint \mathcal{G}(r) \cdot d\vec{S} = -4\pi G M_{Int}$$

$$\mathcal{G}(r) \cdot 4\pi r^2 = -4\pi G M_{int}$$

Pour $r < R_T$, ρ_T = Masse volumique de la Terre.

$$M_{int} = \rho_T \frac{4}{3}\pi r^3$$

Donc $\mathcal{G}(r) = -G\rho_T \frac{4}{3}\pi r$

Donc, finalement : $\vec{F} = -G\rho\frac{4}{3}\pi r\, \vec{u_r}$.

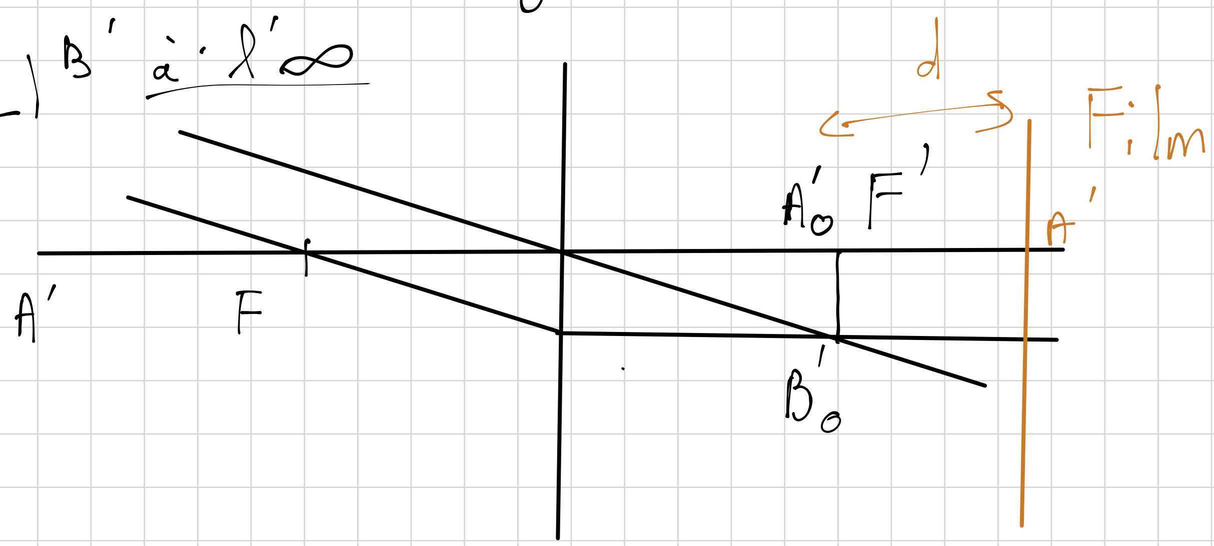
$$\vec{F} = -\overrightarrow{grad}\, E_p \Rightarrow E_p = \frac{1}{2} G\rho\frac{4}{3}\pi r^2$$

Optique

Exercice 1 :

1) On place le film dans le plan focal image.

2) B' à l'∞



3) $\overline{A_0B_0} = 60 \text{ m}$ $\qquad \overline{A_0O} = 0,3 \text{ km}$

$$\frac{1}{\overline{OA'}} - \frac{1}{\overline{OA}} = \frac{1}{f'} = \frac{1}{75\times10^{-3}}$$

$$\frac{1}{\overline{OA'}} = \frac{1}{75\times10^{-3}} + \frac{1}{\overline{OA}} = \frac{1}{75\times10^{-3}} - \frac{1}{0,3\times10^{3}} \simeq 13,33\ldots$$

$$\boxed{\overline{OA'} = 7,5\times10^{-2}}$$

$$\overline{A'B'} = \frac{\overline{OA'}\times\overline{AB}}{\overline{OA}} = \frac{7,5\times10^{-2}\times 60}{-0,3\times10^{3}} = -1,5 \text{ cm}$$

4) On prend donc une focale 2x plus grande

$\boxed{\text{Don } 15 \text{ cm}}$

5) Voir schéma.

On a $\overline{AO} = 1{,}4\ m$ et $\overline{FO} = 75 \times 10^{-3}\ m$

Donc $\overline{AO} = \overline{AF} + \overline{FO}$

$\hookrightarrow \overrightarrow{AF} = \overline{AO} - \overline{FO}$

$$\underline{= 1{,}325\ m.}$$

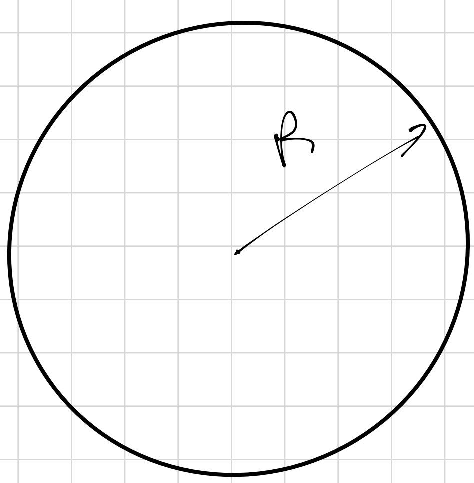
D'après les formules de conjugaison

$$\overline{FA} \times \overline{F'A'} = -f'^2 = (7{,}5 \times 10^{-2})^2$$

On a : $-\overline{AF} \times \overline{FA'} = -f'^2$

$$\boxed{\overline{F'A'} = \frac{-f'}{-AF} = 4\ mm.}$$

Balon de basquette :



$R = 10\ cm$

$f \in 20\ Hz - 20\ kHz$

$C_s = 340\ m.s^{-1}$

$2R = \frac{\lambda}{2}$

Ondes stationnaire $\Rightarrow$ mode propre.

$$\left.\lambda_0 = 4R = \frac{C_s}{f}\right\} \quad f = \frac{C_s}{4R} \qquad \underline{AN} : 850\ Hz$$