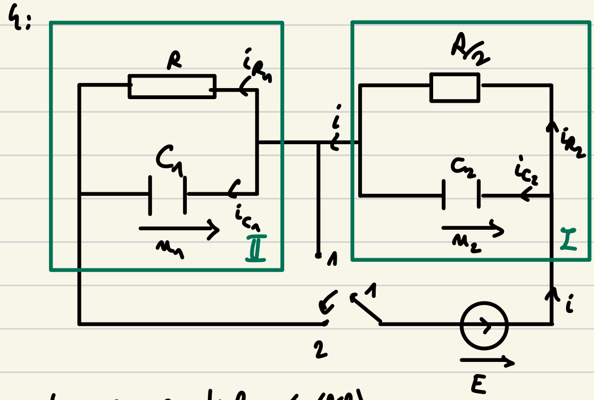
Ex 4:

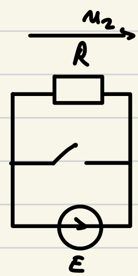


· $t=0^-$: · C_1 déchargé (fil)

· C_2 chargé (interrupteur ouvert)

Eq° du circuit 1 :

$u_E = u_R \Rightarrow E = \frac{R}{2} i$



à $t=0^+$, $u_2 = E$, $u_1 = 0$

$E = \underbrace{u_{II}}_{E} + \underbrace{u_I}_{0}$

Loi des noeuds :

· $i = i_{C_2} + i_{R_2} = C\frac{du_1}{dt} + \frac{u_1}{R}$

· $i = i_{C_1} + i_{R_1} = C\frac{du_2}{dt} + 2\frac{u_2}{R}$

$\Rightarrow \frac{RC}{2}\left[\frac{du_2}{dt} - \frac{du_1}{dt}\right] + u_2 - \frac{u_1}{2} = 0$

$\xRightarrow{E=u_2+u_1} \frac{RC}{2}\left[\frac{du_2}{dt} - \frac{dE}{dt} + \frac{du_2}{dt}\right] + u_2 - \frac{E}{2} + \frac{u_2}{2} = 0$

$\Rightarrow \frac{du_2}{dt} + \underbrace{\boxed{\frac{3}{2RC}}}_{1/\tau} u_2 = \frac{E}{2RC}$

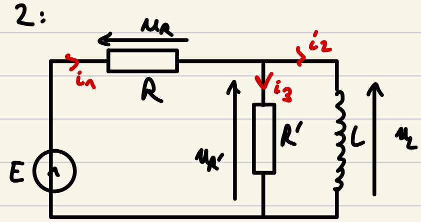
Sol. homogène: $u_{2,H} = A\exp(-t/\tau)$

Sol. part.: $u_{2,P} = \frac{E}{3}$

d'où finalement: $u_2 = Ae^{-t/\tau} + \frac{E}{3}$

or $u_2(t=0)=E \Rightarrow u_2 = \frac{E}{3}(2e^{-t/\tau}+1)$

Ex 2:



1) Loi des noeuds: $i_1 = i_2 + i_3$

Loi des mailles: $E = u_R + u_{R'}$ $\qquad u_{R'} = u_L = L\frac{di_2}{dt}$

$$\Rightarrow E = Ri_1 + L\frac{di_2}{dt}$$
$$= Ri_2 + Ri_3 + L\frac{di_2}{dt}$$
$$= Ri_2 + \frac{R}{R'}R'i_3 + L\frac{di_2}{dt}$$

$$\Rightarrow E = Ri_2 + \underbrace{\left(\frac{R}{R'}+1\right)}_{\frac{R+R'}{R'}}L\frac{di_2}{dt}$$

$$\text{donc} \quad \frac{di_2}{dt} + \underbrace{\boxed{\frac{RR'}{(R+R')L}}}_{1/\tau}i_2 = \frac{ER'}{(R+R')L}$$

2) $i_2(t) = Ae^{-t/\tau}+B$ $\qquad$ Par cont. du courant

$i_2(0)=0 \Rightarrow A+B=0$ donc $i_2(t)=B(1-e^{-t/\tau})$

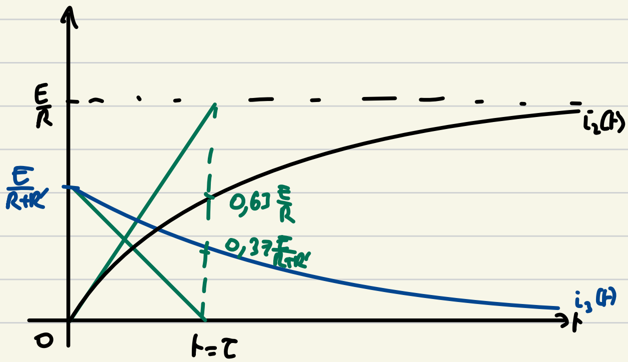
et $\frac{B}{\tau} = \frac{E}{\tau R}$ donc $B=\frac{E}{R}$

donc $i_2(t) = \frac{E}{R}(1-e^{-t/\tau})$

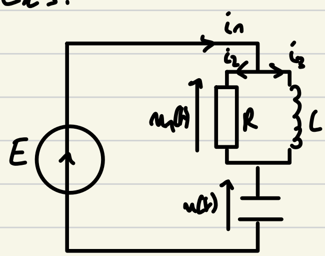
3) $u_{R'} = u_L$

donc $R' i_3(t) = L \frac{di_2}{dt} = \frac{E}{R\tau} e^{-t/\tau}$

donc $i_3(t) = \frac{E}{R+R'} e^{-t/\tau}$



Ex 3 :



Loi des noeuds :

$E = u_1(t) + u(t) \Rightarrow u_1(t) = E - u(t)$

Loi des noeuds :

$i_1 = i_2 + i_3$

$$\Rightarrow C \frac{du(t)}{dt} = \frac{u_1(t)}{R} + i_3$$

$$\Rightarrow C \frac{d^2u(t)}{dt^2} = \frac{1}{R} \frac{du_1(t)}{dt} + \frac{di_3}{dt} / \frac{U_1}{L}$$

$$\Rightarrow C \frac{d^2u(t)}{dt^2} + \frac{1}{LC} u(t) + \frac{1}{RC} \frac{du(t)}{dt} = \frac{E}{LC}$$

$$\omega_0^2 = \frac{1}{LC} \quad \text{et} \quad \frac{\omega_0}{Q} = \frac{1}{RC}$$

i.e. $\omega_0 = \frac{1}{\sqrt{LC}}$ et $Q = R\sqrt{\frac{C}{L}}$

- Sol. part : E
- Sol. générale : $Q \overset{AN}{=} 10 >> \frac{1}{2}$.

Eq carac :

$$X^2 + \frac{\omega_0}{Q} X + \omega_0^2 = 0$$

de discr. :

$$\Delta = \frac{\omega_0^2}{Q^2} - 4\omega_0^2 < 0$$
$$= -4\omega_0^2 \left(1 - \frac{1}{4Q^2}\right)$$
$$= \left(i 2 \omega_0 \sqrt{1 - \frac{1}{4Q^2}}\right)^2$$

et : $x_{1,2} = -\frac{\omega_0}{2Q} \pm i\omega_0 \sqrt{1 - \frac{1}{4Q^2}}$

on pose ici
- $\Omega = \omega_0 \sqrt{1 - \frac{1}{4Q^2}}$
- $\tau = \frac{2Q}{\omega_0}$

on a : $x_{1,2} = -\frac{1}{\tau} \pm i\Omega$.

et :

$$u(t) = e^{-t/\tau} U_0 \cos(\Omega t + \varphi) \qquad \varphi \in \mathbb{R}.$$

donc $\boxed{u_{gén}(t) = U_0 e^{-t/\tau} \cos(\Omega t + \varphi) + E_0}$.

Révision filtres d'ordre 2 :

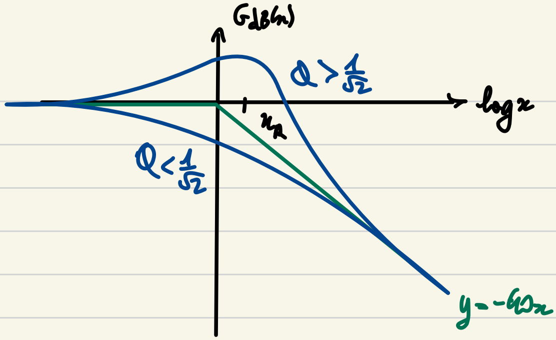
Pour $\underline{H} = \frac{1}{1 - \frac{\omega^2}{\omega_0^2} + j\frac{\omega}{Q\omega_0}}$

$x = \frac{\omega}{\omega_0}$, $\quad \underline{H} = \frac{1}{1 - x^2 + j\frac{x}{Q}}$

$\underline{H}(x) \underset{x \to 0}{\sim} 1$, $\quad \underline{H}(x) \underset{x \to +\infty}{\sim} -\frac{1}{x^2}$

$\rightarrow$ passe bas.

$G_{dB} = 20 \log(|\underline{H}|)$, $\qquad G_{dB}(x \to 0) = 0$, $G_{dB}(x \to +\infty) = -40 \log(x)$.



$$\omega_R = \omega_0\sqrt{1 - \frac{1}{2Q^2}}$$