Spectres – Transformée de Fourier

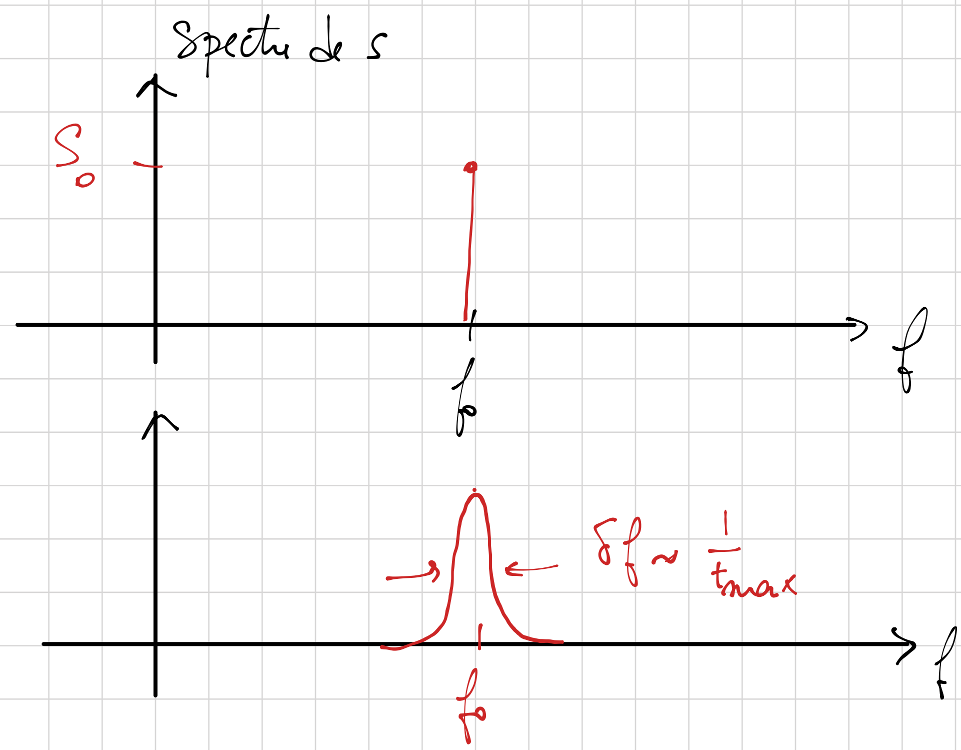
* Signal sinusoidal :

$t \in]-\infty, +\infty[$

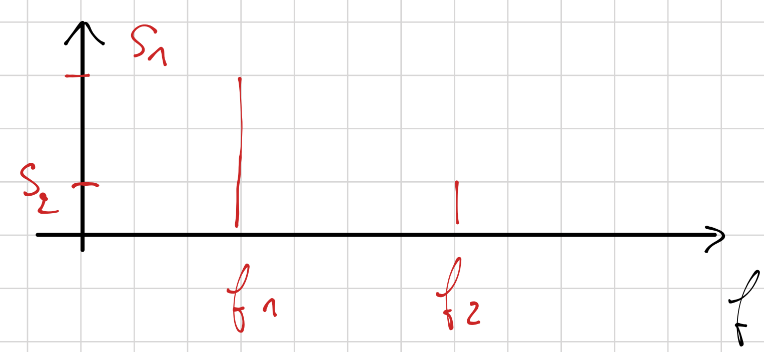
$s(t) = S_0 \cos(2\pi f_0 t)$

$t \in [0, t_{max}]$

élargissement

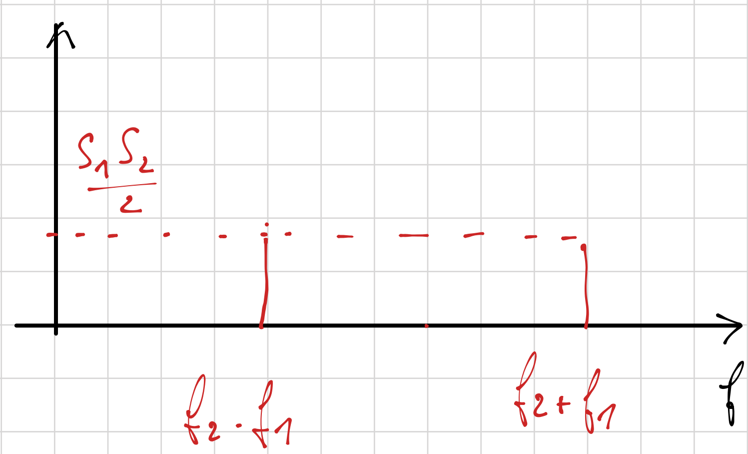


$s = S_1 \cos(2\pi f_1 t) + S_2 \cos(2\pi f_2 t)$

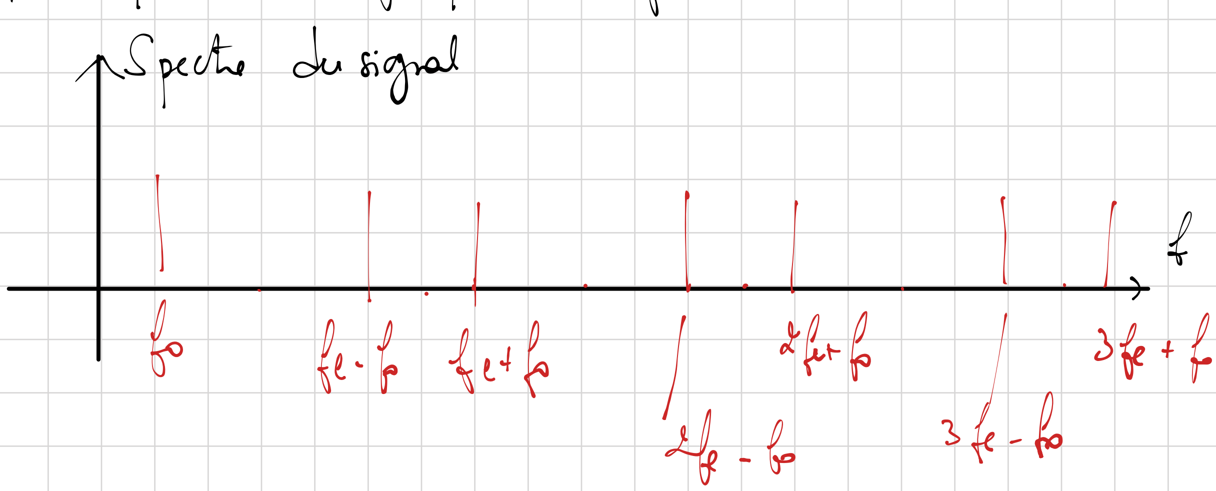


$s = S_1 \cos(2\pi f_1 t) \cdot S_2 \cos(2\pi f_2 t)$

$= \frac{S_1 S_2}{2} \left(\cos(2\pi(f_1+f_2)t) + \cos(2\pi(f_2-f_1)t) \right)$



Signal périodique de fréquence f_0 échantillonné à la fréquence $f_e > 2f_0$



$s(t)$ = signal périodique de période $T_0 = \frac{1}{f_0}$

$$\rightarrow \quad s(t) = \underbrace{S_0}_{\substack{\text{valeur} \\ \text{moyenne} \\ \text{de } s(t)}} + \underbrace{S_1 \cos(2\pi f_0 t + \varphi_1)}_{\text{fondamental}} + \underbrace{S_2 \cos(2\pi . 2f_0 t + \varphi_2) + \dots}_{\text{harmoniques}}$$

$\cos 2a = 2\cos^2 - 1$

Application si $f(x)$ = fonction L périodique, alors

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi}{L} n x\right) + b_n \sin\left(\frac{2\pi}{L} n x\right)$$

On trouve a_n, b_n en calculant :

$$\int_0^L f(x) \cos\left(\frac{2\pi}{L} n x\right) dx = a_n \int_0^L \cos^2\left(\frac{2\pi n x}{L}\right) dx$$

$$= \frac{a_n L}{2}$$

$$\int_0^L f(x) \sin\left(\frac{2\pi}{L} n x\right) dx = b_n \int_0^L \sin^2\left(\frac{2\pi n x}{L}\right) dx$$

$$= \frac{b_n L}{2}$$

$$\int_0^L f(x)\, dx = a_0 L$$