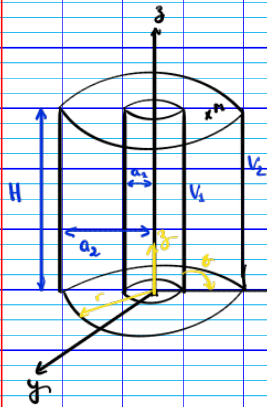


DM1

LABVTHIE
dynamique

Exercice 1

1.



1) Systeme de coordonnées

coordonnées cylindriques

$$\vec{E}(M) = E_r(r, \theta, z) \vec{e}_r + E_\theta(r, \theta, z) \vec{e}_\theta + E_z(r, \theta, z) \vec{e}_z$$

2) Symetrie

$$\pi_1 = (M, \vec{e}_r, \vec{e}_z)$$

$$\pi_2 = (M, \vec{e}_r, \vec{e}_\theta) \text{ sont des } \pi^*$$

le champ $\vec{E}$ appartient à ces deux plans

$$\vec{E}(r, \theta, z) = E_r(r, \theta, z) \vec{e}_r$$

3) Invariances

Invariance par translation selon z

Invariance par rotation selon θ

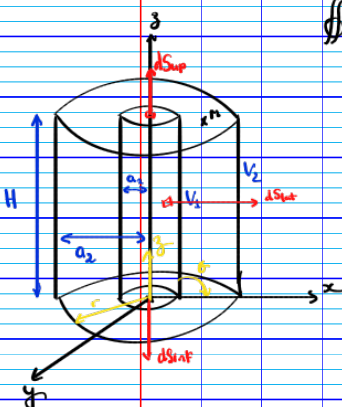
d'où

$$\vec{E}(M) = E_r(r) \vec{e}_r$$

2.

Surface de gauss on choisit un cylindre de rayon r et de hauteur h .

Calcul de flux



$$\oint \vec{E} \cdot d\vec{S} = \iint_{\text{sup}} \vec{E} \cdot d\vec{S}_{\text{sup}} + \iint_{\text{inf}} \vec{E} \cdot d\vec{S}_{\text{inf}} + \iint_{\text{lat}} \vec{E} \cdot d\vec{S}_{\text{lat}}$$

$$\bullet d\vec{S}_{\text{sup}} = dS_{\text{sup}} \vec{e}_z \perp \vec{E} = \iint_{\text{sup}} \vec{E} \cdot d\vec{S}_{\text{sup}} = 0$$

$$\bullet \text{idem pour } d\vec{S}_{\text{inf}}$$

$$\begin{aligned} \oint \vec{E} \cdot d\vec{S} &= \iint_{\text{lat}} \vec{E}(M) \cdot d\vec{S}_{\text{lat}} \\ &= \iint_{\text{lat}} E_r(r) \vec{e}_r \cdot d\vec{S}_{\text{lat}} \vec{e}_r \\ &= \iint_{\text{lat}} E_r(r) dS_{\text{lat}} \\ &= E(r) 2\pi r h \end{aligned}$$

$$\oint \vec{E} \cdot d\vec{S} = E(r) 2\pi r h$$

Calcul de Q_{int}

$$Q_{int} = Q$$

Th de Gauss

$$\oint \vec{E} \cdot d\vec{S} = \frac{Q_{int}}{\epsilon_0}$$

$$\Leftrightarrow E(r) 2\pi r H = \frac{Q}{\epsilon_0}$$

$$\Leftrightarrow E(r) = \frac{Q}{2\pi \epsilon_0 H r}$$

$$\vec{E}(M) = \frac{Q}{2\pi \epsilon_0 H r} \vec{e}_r$$

3.

$$Q = C U$$

$$\Leftrightarrow C = \frac{Q}{U} \quad U = V_1 - V_2$$

donc

$$C_0 = \frac{Q}{V_1 - V_2}$$

4.

D'après la Q3

$$C_0 = \frac{Q}{V_1 - V_2}$$

$$\text{et } \vec{E} = -\vec{\text{grad}}(V)$$

$$\vec{E}(M) = -\frac{dV}{dr} \vec{e}_r$$

$$dV = -E(r) dr$$

$$V_2 - V_1 = \int_{a_1}^{a_2} -\vec{E}(M) \cdot d\vec{r}$$

$$\begin{aligned} V_2 - V_1 &= \int_{a_1}^{a_2} \frac{Q}{2\pi \epsilon_0 H r} dr \\ &= \frac{Q}{2\pi \epsilon_0 H} \left[\ln(r) \right]_{a_1}^{a_2} \\ &= \frac{Q}{2\pi \epsilon_0 H} \ln\left(\frac{a_2}{a_1}\right) \end{aligned}$$

$$\text{donc } C_0 = \frac{Q}{V_2 - V_1} = \frac{Q}{\frac{Q}{2\pi \epsilon_0 H} \ln\left(\frac{a_2}{a_1}\right)} = \frac{2\pi \epsilon_0 H}{\ln\left(\frac{a_2}{a_1}\right)}$$

5. $U = V_1 - V_2$

$$Q = Q_{\text{eau}} + Q_{\text{air}}$$

On sait que $Q = CU$ donc

$\bullet Q_{\text{eau}} = C_{\text{eau}}(V_1 - V_2)$ $\bullet Q_{\text{air}} = C_{\text{air}}(V_1 - V_2)$ $\bullet Q = C(V_1 - V_2)$
--

d'après

$$C = \frac{Q}{V_1 - V_2} = \frac{Q_{\text{eau}} + Q_{\text{air}}}{V_1 - V_2} = \frac{Q_{\text{eau}}}{V_1 - V_2} + \frac{Q_{\text{air}}}{V_1 - V_2}$$

$$\Leftrightarrow C = \frac{C_{\text{eau}}(V_1 - V_2)}{(V_1 - V_2)} + \frac{C_{\text{air}}(V_1 - V_2)}{(V_1 - V_2)}$$

$C = C_{\text{eau}} + C_{\text{air}}$

6. $C_0 = \frac{2\pi \epsilon_0 H}{\ln\left(\frac{a_2}{a_1}\right)}$

$$\rightarrow C_{\text{eau}} = \frac{2\pi \epsilon h}{\ln\left(\frac{a_2}{a_1}\right)}$$

$$\rightarrow C_{\text{air}} = \frac{2\pi \epsilon_0 (H-h)}{\ln\left(\frac{a_2}{a_1}\right)}$$

$$\Rightarrow C(h) = C_{\text{eau}} + C_{\text{air}}$$

$$= \frac{2\pi \epsilon h}{\ln\left(\frac{a_2}{a_1}\right)} + \frac{2\pi \epsilon_0 (H-h)}{\ln\left(\frac{a_2}{a_1}\right)}$$

$$= \frac{2\pi \epsilon_0 H}{\ln\left(\frac{a_2}{a_1}\right)} \left(\frac{\epsilon h}{\epsilon_0 H} + \frac{(H-h)}{H} \right)$$

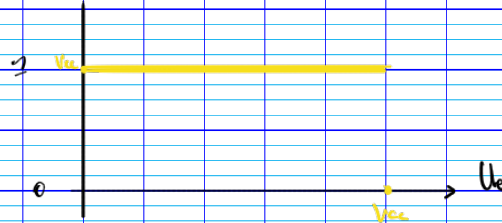
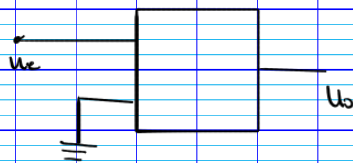
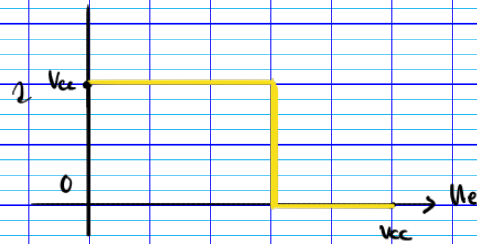
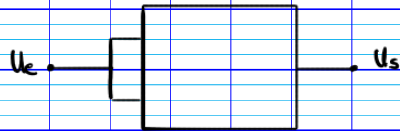
$$= C_0 \left[\frac{\epsilon h}{\epsilon_0 H} + 1 - \frac{h}{H} \right]$$

$$= C_0 \left[h \left(\frac{\epsilon - \epsilon_0}{\epsilon_0 H} \right) + 1 \right]$$

donc $A = \frac{\epsilon - \epsilon_0}{\epsilon_0 H}$ $B = 1$

EXERCICE 2

$$V_{CC} = 15V$$



7.

1^{er} experience =

si $U_e = 0$ alors $U_s = V_{CC}$

si $U_e = V_{CC}$ alors $U_s = 0$

donc $U_s = \overline{U_e}$

se comporte comme une NOT

2^{ème} experience

si $U_e = 0$ alors $U_s = V_{CC}$

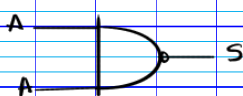
si $U_e = V_{CC}$ alors $U_s = V_{CC}$

se comporte comme une NAND

8.

NOT

$$S = \overline{A \cdot A} = \overline{A}$$



A	$\overline{A}$
0	1
1	0

AND

$$S = \overline{\overline{A \cdot B}} = A \cdot B$$



A	B	S	$\overline{A \cdot B}$
0	0	0	1
1	0	0	1
0	1	0	1
1	1	1	0

OR

$$S = \overline{A \cdot B} = \overline{A} \cdot \overline{B} = A + B$$

A	$\overline{A}$	B	$\overline{B}$	S
0	1	0	1	0
1	0	0	1	1
0	1	1	0	1
1	0	1	0	1