

Corrigé du DS4

①

$$\begin{array}{l} \text{---} ax \\ \text{anb} \quad \text{---} \quad \text{anb} \\ \text{---} ex \\ \text{anb} \quad \text{---} \quad a \\ \text{---} iv \\ \text{anb} \quad \text{---} \quad avb \\ \text{---} i \rightarrow \\ \vdash (anb) \rightarrow (avb) \end{array}$$

②

$$\begin{array}{l} \frac{\frac{\frac{}{P \vdash a \wedge b} \text{en}}{P \vdash \neg a \vee \neg b} \text{ax}}{\frac{P \vdash \neg a \vee \neg b; a \wedge b \vdash b; \neg a \vee \neg b; a \wedge b \vdash \neg b} \text{et}} \text{ax} \\ \frac{\frac{\frac{\frac{}{P \vdash \neg a \vee \neg b} \text{en}}{P \vdash \neg a \vee \neg b; a \wedge b \vdash b; \neg a \vee \neg b; a \wedge b \vdash \neg b} \text{et}}{\frac{P \vdash \neg a \vee \neg b; a \wedge b \vdash \perp} \text{et}} \text{ax} \\ \frac{P \vdash \neg a \vee \neg b; a \wedge b \vdash \perp}{P \vdash \neg a \vee \neg b \vdash \neg(a \wedge b)} \text{et} \\ \frac{P \vdash \neg a \vee \neg b \vdash \neg(a \wedge b)}{P \vdash (\neg a \vee \neg b) \rightarrow \neg(a \wedge b)} \text{et} \end{array}$$

③

$$\begin{array}{c}
 \frac{\frac{\frac{}{ax} \quad \frac{}{ax}}{\Gamma, p \vdash p; \Gamma, p \vdash \neg p} \quad e\gamma}{\Gamma, p \vdash q} \quad i \rightarrow \\
 \frac{\frac{ax}{\Gamma \vdash (p \rightarrow q) \rightarrow p}; \quad \Gamma \vdash p \rightarrow q}{\Gamma \vdash p} \quad e\gamma \\
 \frac{\Gamma \vdash (p \rightarrow q) \rightarrow p, \neg p \quad \vdash \perp}{\Gamma \vdash (p \rightarrow q) \rightarrow p} \quad i\gamma \\
 \frac{\Gamma \vdash (p \rightarrow q) \rightarrow p \quad \vdash \neg \neg p}{\vdash ((p \rightarrow q) \rightarrow p) \rightarrow \neg \neg p} \quad i \rightarrow
 \end{array}$$

④

Soit a une nouvelle variable non utilisée.

$$\begin{array}{c}
 \frac{\frac{}{ax} \quad \frac{}{ax}}{\Gamma \vdash \forall x (\psi \rightarrow \psi)} \quad e\forall \\
 \frac{\Gamma \vdash \psi \rightarrow \psi[a] \quad \Gamma \vdash \psi[a]}{\Gamma \vdash \psi \rightarrow \psi[a]} \quad MP \\
 \frac{\Gamma \vdash \forall x (\psi \rightarrow \psi), \forall x \psi \quad \vdash \psi[a]}{\Gamma \vdash \forall x (\psi \rightarrow \psi), \forall x \psi} \quad i\forall \\
 \frac{\Gamma \vdash \forall x (\psi \rightarrow \psi), \forall x \psi \quad \vdash \forall x \psi}{\Gamma \vdash \forall x (\psi \rightarrow \psi) \rightarrow (\forall x \psi)} \quad i \rightarrow
 \end{array}$$

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Soit t une variable non utilisée
ni dans φ ni dans ψ .

$$\begin{array}{c}
 \frac{\frac{\frac{}{ax} \Gamma \vdash \exists x (\varphi \rightarrow \psi) \quad \frac{\frac{}{ax} \Gamma, \varphi \rightarrow \psi[t] \vdash \psi[t]}{e\exists}}{\Gamma \vdash \varphi \rightarrow \psi[t]} \quad \frac{\frac{}{ax} \Gamma \vdash \forall x \varphi}{\Gamma \vdash \varphi[t]} \quad \text{MP}}{\Gamma \vdash \exists x (\varphi \rightarrow \psi), \forall x \varphi \vdash \psi[x \mapsto t]} \quad \text{e}\exists \\
 \hline
 \frac{\Gamma \vdash \exists x (\varphi \rightarrow \psi), \forall x \varphi \vdash \exists x \psi}{\Gamma \vdash \exists x (\varphi \rightarrow \psi) \vdash (\forall x \varphi) \rightarrow \exists x \psi} \quad \text{e}\exists
 \end{array}$$