

Diffusion de Rutherford

1. $E_M = \text{cte} = \frac{1}{2} m v_0^2 > 0$ Hyperbole.

2. $\vec{v}_M = r \dot{\theta} \vec{e}_\theta + r \ddot{\theta} \vec{e}_\theta$

en I $r_{\min} \Rightarrow \vec{v}_I = r_{\min} \dot{\theta} \vec{e}_\theta$

3. $E_M = \text{cte} = \frac{1}{2} m v_0^2 = \frac{1}{2} m v_I^2 + \frac{2e \times Ze}{4\pi\epsilon_0 r_{\min}}$

• $\vec{L} = \text{cte} = \vec{OM} \wedge m \vec{v} = m r_{\min}^2 \dot{\theta} \vec{u}_3$
 $= m b v_0 \vec{u}_3$

4) $m b v_0 = m r_{\min}^2 \dot{\theta} \Rightarrow \dot{\theta} = \frac{b v_0}{r_{\min}^2}$

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m r_{\min}^2 \frac{b^2 v_0^2}{r_{\min}^4} + \frac{2Ze^2}{4\pi\epsilon_0 r_{\min}}$$

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m \frac{b^2 v_0^2}{r_{\min}^2} + \frac{2Ze^2}{4\pi\epsilon_0 r_{\min}}$$

$$\frac{1}{2} m v_0^2 r_{\min}^2 = \frac{1}{2} m b^2 v_0^2 + \frac{2Ze^2 r_{\min}}{4\pi\epsilon_0}$$

$$\text{donc } r_{\min} = \frac{Ze^2}{\pi\epsilon_0} + \sqrt{\frac{Z^2 e^4}{\pi^2 \epsilon_0^2} + 4m^2 v_0^4 b^2} > 0$$

$$\text{soit } r_{\min} = \frac{Ze^2}{2\pi\epsilon_0 m v_0^2} + \sqrt{\frac{Z^2 e^4}{4\pi^2 \epsilon_0^2 m^2 v_0^4} + b^2}$$

5) Si la particule fait demi-tour : $v_I = 0$

$$\frac{1}{2} m v_0^2 = \frac{2Ze^2}{4\pi\epsilon_0 r_{\min}^2}$$

$$r_{\min} = \sqrt{\frac{Ze^2}{\pi\epsilon_0 m v_0^2}} = \sqrt{\frac{Ze^2}{\pi\epsilon_0 2E}} \text{ avec } E = \frac{1}{2} m v_0^2$$