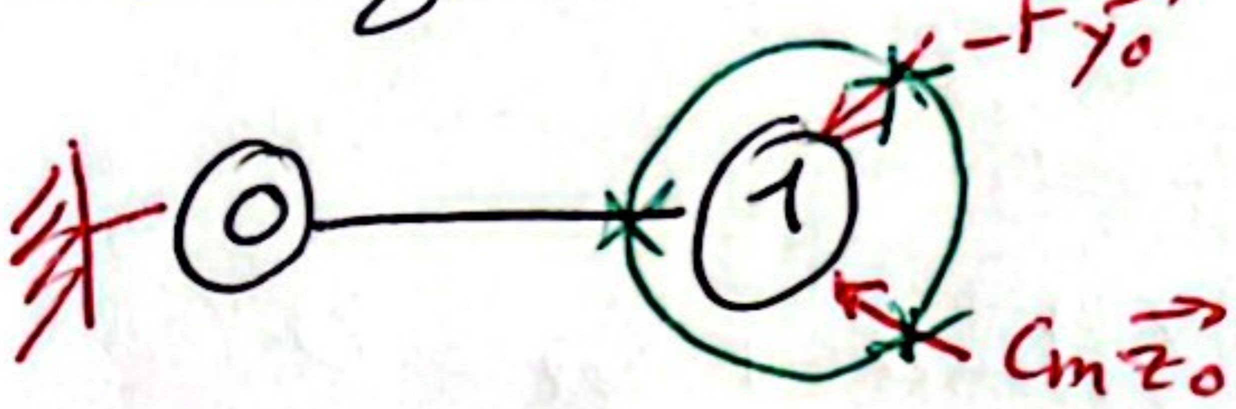
Exercices de statique corrigés

Levier

1. 

2. $\{T_{0\to1}\} = \begin{Bmatrix} X_{01} & - \\ Y_{01} & - \\ - & 0 \end{Bmatrix}_A$ pb. plan de $\perp \vec{z_0}$

3. $\{T_{0\to1}\} + \{T_{\vec F\to1}\} + \{T_{\vec{C_m}\to1}\} = \{0\}$

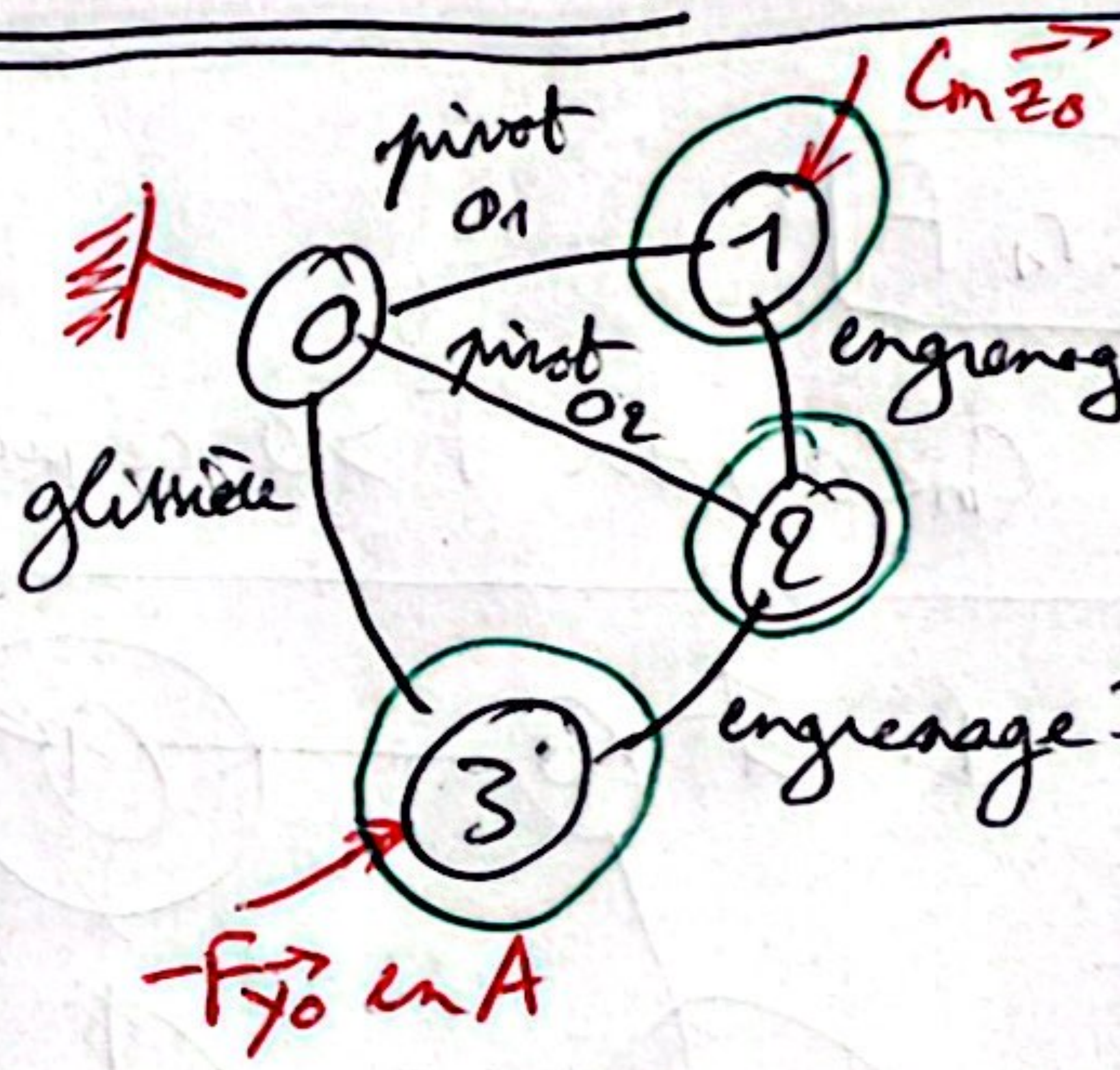
$$\begin{cases} X_{01} + 0 + 0 = 0 \\ Y_{01} - F + 0 = 0 \\ 0 - FR\cos\theta + C_m = 0 \leftarrow \text{éq}^{n} \text{ utile} \end{cases}$$

$\vec M(A, \vec F\to1) = \vec{AB}\wedge\vec F + \underbrace{\cancel{\vec M(B,\vec F\to1)}}_{\vec 0} = R\vec{x_1}\wedge -F\vec{y_0}$

$\vec{x_1}\wedge\vec{y_0} = -\cos\theta\,\vec{z_0}$

$\boxed{C_m = F.R\cos\theta}$

Engrenage

1. 

2. $\{T_{0\to1}\} = \begin{Bmatrix} X_{01} & - \\ Y_{01} & - \\ - & 0 \end{Bmatrix}_{O_1}$; $\{T_{0\to2}\} = \begin{Bmatrix} X_{02} & - \\ X_{02} & - \\ & 0 \end{Bmatrix}_{O_2}$; $\{T_{0\to3}\} = \begin{Bmatrix} X_3 & - \\ 0 & - \\ - & N_{03} \end{Bmatrix}_P \forall P$

$\{T_{1\to2}\} = \begin{Bmatrix} -Y_{12}\tan\alpha & \\ Y_{12} & \\ & 0 \end{Bmatrix}_I$; $\{T_{2\to3}\} = \begin{Bmatrix} Y_{23}\tan\alpha & - \\ Y_{23} & - \\ - & 0 \end{Bmatrix}_J$

3. P.F.S. à 1 en O_1 :

$$\begin{cases} X_{01} + Y_{12}\tan\alpha + 0 = 0 & (1) \\ Y_{01} + Y_{12} + 0 = 0 & (2) \\ 0 - Y_{12}.r_1 + C_m = 0 & (3) \end{cases}$$

$\vec M(O_1, 2\to1) = \underbrace{\cancel{\vec M(I, 2\to1)}}_{\vec 0} + \vec{O_1I}\wedge\vec R_{2\to1} = r_1\vec{x_0}\wedge(Y_{12}\tan\alpha\,\vec{x_0} - Y_{12}\vec{y_0}$

PFS à 2 en O_2 :

$$\begin{cases} -Y_{12}\tan\alpha & -Y_{23}\tan\alpha & +X_{02} = 0 \quad (4) \\ Y_{12} & -Y_{23} & +\cancel{Y_{02}} = 0 \quad (5) \\ -r_2 Y_{12} & -r_2 Y_{23} & +\ 0 = 0 \quad (6) \Rightarrow Y_{12} = -Y_{23} \end{cases}$$

$$\vec{M}(O_2, 1\to 2) = \vec{O_2 I} \wedge \vec{R}_{1\to 2} = -r_2 \vec{x}_0 \wedge (-Y_{12}\tan\alpha\, \vec{x}_0 + Y_{12}\vec{y}_0)$$

$$\vec{M}(O_2, 3\to 2) = \vec{O_2 J} \wedge \vec{R}_{3\to 2} = + r_2 \vec{x}_0 \wedge (-Y_{23}\tan\alpha\, \vec{x}_0 - Y_{23}\vec{y}_0)$$

(car $Y_{23} = -Y_{32}$)

PFS à 3 en J :

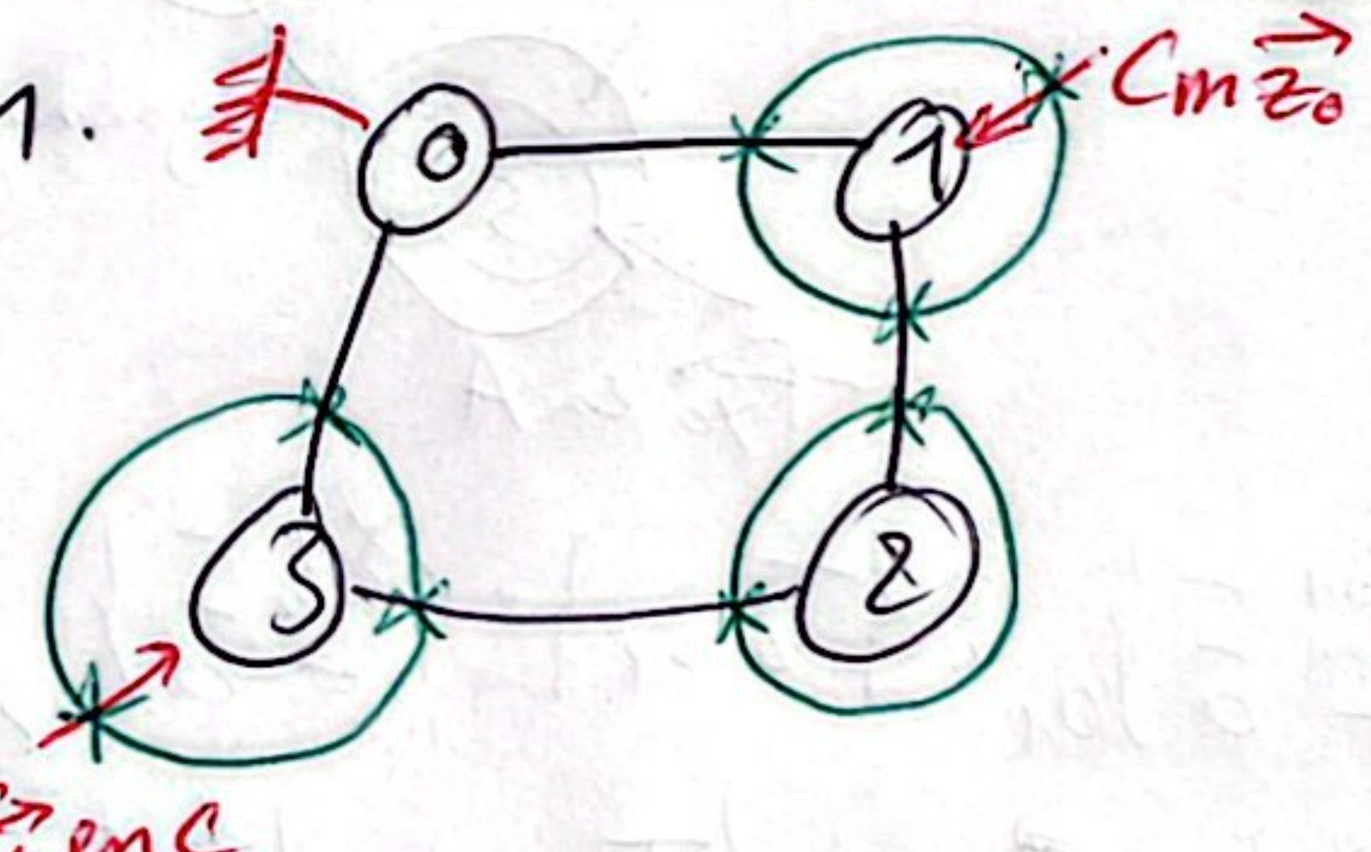
$$\begin{cases} 0 & + X_{03} & + Y_{23}\tan\alpha = 0 \quad (7) \\ -F & + 0 & + Y_{23} = 0 \quad (8) \\ 0 & + N_{03} & + 0 = 0 \quad (9) \end{cases}$$

(3), (6) et (8) $\Rightarrow \boxed{C_m = -r_1 F}$

on remarque $C_m < 0$ pour $F > 0$ ce qui est cohérent.

Bielle - Manivelle

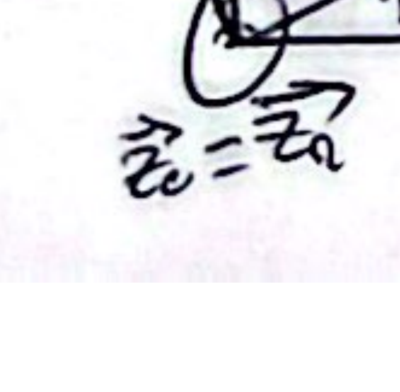
1.



2. $\{T_{1\to 2}\} = \begin{Bmatrix} X_{12} & - \\ Y_{12} & - \\ - & 0 \end{Bmatrix}_B$; $\{T_{0\to 1}\} = \begin{Bmatrix} X_{01} & - \\ Y_{01} & - \\ - & 0 \end{Bmatrix}_A$; $\{T_{2\to 3}\} = \begin{Bmatrix} X_{23} & - \\ Y_{23} & - \\ - & 0 \end{Bmatrix}_C$

$\{T_{0\to 3}\} = \begin{Bmatrix} X_{03} & - \\ 0 & - \\ - & 0 \end{Bmatrix}_{\forall P}$

3. PFS appliqué à 2 en B : $\{T_{1\to 2}\} + \{T_{3\to 2}\} = \{0\}$



$$\begin{cases} X_{12} & - X_{23} & = 0 \quad (1) \\ Y_{12} & - Y_{23} & = 0 \quad (2) \\ 0 & - L(\cos\theta_2 X_{23} + \sin\theta_2 Y_{23}) & = 0 \quad (3) \end{cases}$$

car $\vec{M}(B, 3\to 2) = \vec{M}(C, 3\to 2) + \vec{BC} \wedge \vec{R}_{3\to 2} = -L\vec{y}_2 \wedge (-X_{23}\vec{x}_0 - Y_{23}\vec{y}$

on en déduit ③ $\Rightarrow$ $\boxed{X_{23} = -Y_{23} \tan\theta_2}$ et ① $X_{12} = X_{23}$

4. P.F.S. appliqué à 3 en C :

$$\{T_{0\to3}\} + \{T_{2\to3}\} + \{T_{e\to3}\} = \{0\}$$

$$\begin{cases} X_{03} & -\tan\theta_2 . Y_{23} & + 0 & = 0 \quad ④ \\ 0 & + Y_{23} & + F & = 0 \quad ⑤ \\ N_{03} & + 0 & + 0 & = 0 \quad ⑥ \end{cases} \Rightarrow \boxed{Y_{23} = -F}$$

j'utilise le résultat de ③

PFS appliqué à 1 en A :

$$\{T_{0\to1}\} + \{T_{2\to1}\} + \{T_{e\to1}\} = \{0\}$$

$$\begin{cases} X_{01} & -\tan\theta_2 F & + 0 = 0 \quad ⑦ \\ Y_{01} & + F & + 0 = 0 \quad ⑧ \\ 0 & + eF(\cos\theta_1 + \sin\theta_1 \tan\theta_2) & + C_m = 0 \quad ⑨ \end{cases}$$

$$\vec{M}(A, 2\to1) = \vec{M}(B, 2\to1) + \vec{AB} \wedge \vec{R}_{2\to1} \quad \text{or } \vec{R}_{2\to1} = (-\tan\theta_2 F \vec{x_0} + F\vec{y_0})$$

① ③ et ⑤

$$= \vec{0} \quad + e\vec{x_1} \wedge (-\tan\theta_2 F \vec{x_0} + F\vec{y_0})$$

⑨ $\Rightarrow$ $\boxed{C_m = eF(\cos\theta_1 + \sin\theta_1 \tan\theta_2)}$

<u>remarque</u> : * $\theta_1 = 0 \Rightarrow C_m = eF$

* $\theta_1 = \frac{\pi}{2} \Rightarrow \theta_2 = 0 \Rightarrow C_m = 0$