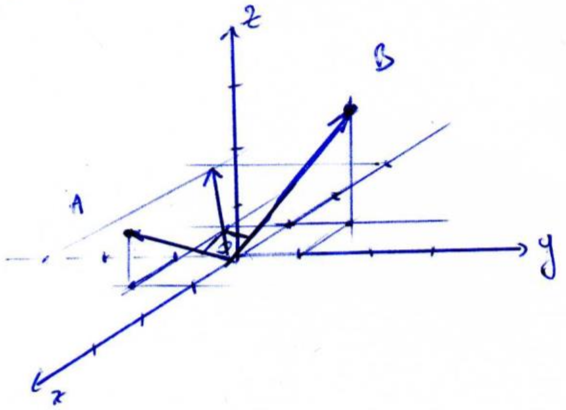
Exercice 1 :

1/ $\begin{vmatrix}1\\-1\\1\end{vmatrix} \cdot \begin{vmatrix}-1\\1\\2\end{vmatrix} = -1-1+2=0$ $\quad \vec{OA}\perp\vec{OB}$

2/



3/ $\begin{vmatrix}1\\-1\\1\end{vmatrix} \wedge \begin{vmatrix}-1\\1\\2\end{vmatrix} = \begin{vmatrix}-3\\-3\\0\end{vmatrix}$

4/ $(\vec{OA}\wedge\vec{OB})\wedge\vec{OC} = \vec{0}$ car $(\vec{OA}\wedge\vec{OB}) \parallel \vec{OC}$

5/ $\|\vec{OA}\| = \sqrt{1+1+1} = \sqrt{3}$

Exercice 2 :

$$\vec{A}\wedge(\vec{B}\wedge\vec{C}) = \begin{vmatrix}x_A\\y_A\\z_A\end{vmatrix} \wedge \left[\begin{vmatrix}x_B\\y_B\\z_B\end{vmatrix} \wedge \begin{vmatrix}x_c\\y_c\\z_c\end{vmatrix}\right] = \begin{vmatrix}x_A\\y_A\\z_A\end{vmatrix} \wedge \begin{vmatrix}y_B z_c - z_B y_c\\z_B x_c - x_B z_c\\x_B y_c - y_B x_c\end{vmatrix}$$

$$= \begin{vmatrix}y_A x_B y_c - y_A y_B x_c - z_A z_B x_c + z_A x_B z_c\\z_A y_B z_c - z_A z_B y_c - x_A x_B y_c + x_A y_B x_c\\x_A x_c z_B - x_A x_B z_c - y_A y_B z_c + y_c z_B y_A\end{vmatrix}$$

$$= \begin{vmatrix}x_B(y_A y_c + z_A z_c) - x_c(y_A y_B + z_A z_B)\\-\\-\end{vmatrix}$$

$$= \begin{vmatrix}x_B(x_A x_c + y_A y_c + z_A z_c) - x_c(x_A x_B + y_A y_B + z_A z_B)\\.\\.\end{vmatrix}$$

$$= \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$

Exercice 3 :

① 1ère méthode : eq du type $y = ax + b$

$$\Rightarrow \left.\begin{array}{l} -1 = -3a+b \\ 1 = 4a+b \end{array}\right\} \Rightarrow a = \frac{2}{7} \quad \text{et} \quad b = -\frac{1}{7}.$$

d'où $\Delta : 7y = 2x - 1$.

2ème méthode : $M(x,y) \in \Delta \Rightarrow \vec{AM} \wedge \vec{AB} = \vec{0}$

$$\Leftrightarrow \begin{vmatrix} x+3 \\ y+1 \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 7 \\ 2 \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 2(x+3) - 7(y+1) \end{vmatrix} = \vec{0}$$

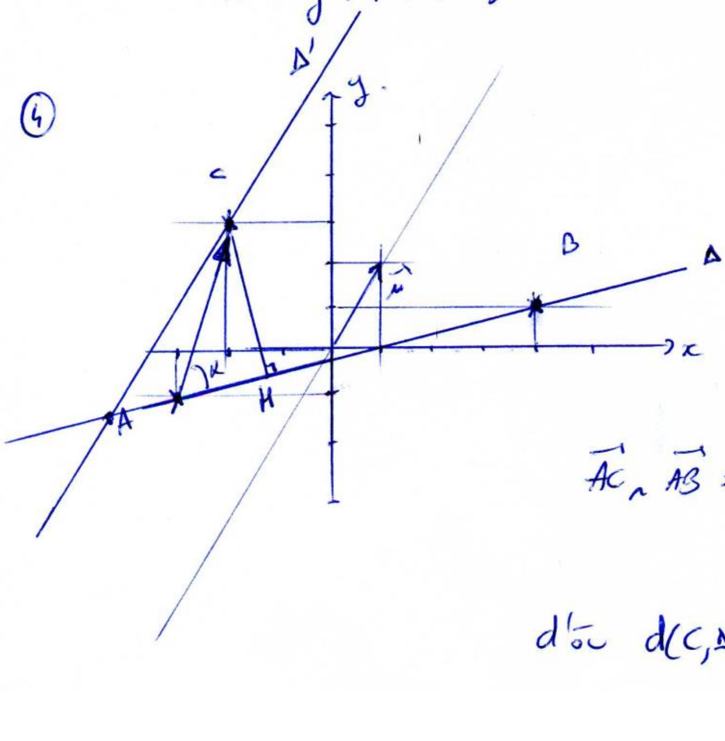
d'où $\Delta : 2x - 7y - 1 = 0$.

② $M(x,y) \in \Delta' \Rightarrow \vec{CM} \wedge \vec{u} = \vec{0} \Leftrightarrow \begin{vmatrix} x+2 \\ y-3 \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 1 \\ 2 \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 2x+4-y+3 \end{vmatrix} = \vec{0}$

d'où $\Delta' : 2x - y + 7 = 0$

③ $\left.\begin{array}{l} 2x - 7y - 1 = 0 \\ 2x - y + 7 = 0 \end{array}\right\} \Rightarrow -6y - 8 = 0 \Rightarrow y = -\frac{4}{3} \quad x = -\frac{25}{6}$

④



$$\left\| \vec{AC} \wedge \frac{\vec{AB}}{\|\vec{AB}\|} \right\| = \|\vec{AC}\| \left|\sin\left(\overset{\alpha}{\widehat{\vec{AC}, \vec{AB}}}\right)\right|$$

$CH = \|\vec{AC}\| \sin \alpha$.

d'où $d(C,\Delta) = \|\vec{AC}\| \sin\alpha$

$$= \frac{\|\vec{AC} \wedge \vec{AB}\|}{\|\vec{AB}\|}$$

$\vec{AC} \wedge \vec{AB} = \begin{vmatrix} 1 \\ 4 \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 7 \\ 2 \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ -26 \end{vmatrix}$ et $\|\vec{AB}\| = \sqrt{53}$

d'où $d(C,\Delta) = \frac{26}{\sqrt{53}} \approx 3{,}6$.

Exercice 4 :

① $\Pi \in P_1 \Rightarrow \overrightarrow{A\Pi} \perp \vec{n}$ avec $\vec{n} = \overrightarrow{AB} \wedge \overrightarrow{AC}$.

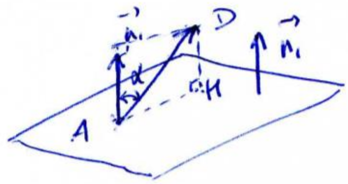
$$\Leftrightarrow \overrightarrow{A\Pi} \cdot (\overrightarrow{AB} \wedge \overrightarrow{AC}) = 0 \Leftrightarrow \begin{vmatrix} x-1 \\ y-1 \\ z \end{vmatrix} \cdot \left(\begin{vmatrix} -1 \\ 1 \\ 2 \end{vmatrix} \wedge \begin{vmatrix} 2 \\ -1 \\ 3 \end{vmatrix} \right) = 0$$

$$\Leftrightarrow \begin{vmatrix} x-1 \\ y-1 \\ z \end{vmatrix} \cdot \begin{vmatrix} 5 \\ 7 \\ -1 \end{vmatrix} = 0 \Leftrightarrow 5x + 7y - z - 12 = 0$$

② $\Delta \perp \vec{n}_1$ et $\Delta \perp \vec{n}_2$ donc vecteur directeur de Δ : $\vec{n}_1 \wedge \vec{n}_2 = \vec{u}$

$$\Rightarrow \vec{u} = \begin{vmatrix} 5 \\ 7 \\ -1 \end{vmatrix} \wedge \begin{vmatrix} 3 \\ -5 \\ 7 \end{vmatrix} = \begin{vmatrix} 44 \\ -38 \\ -46 \end{vmatrix} = \begin{vmatrix} 22 \\ -19 \\ -23 \end{vmatrix}$$

③



$$d(D, P_1) = HD$$

$$\|\overrightarrow{AD} \wedge \vec{n}_1\| = \|\overrightarrow{AD}\| \sin(\underbrace{\overrightarrow{AD}, \vec{n}_1}_{\alpha})$$ inutile.

$$|\overrightarrow{AD} \cdot \vec{n}_1| = \|\overrightarrow{AD}\| \, |\cos(\underbrace{\overrightarrow{AD}, \vec{n}_1}_{\alpha})| = HD.$$

$$\text{d'où } |HD| = \begin{vmatrix} 0 \\ -2 \\ 2 \end{vmatrix} \cdot \begin{vmatrix} 5 \\ 7 \\ -1 \end{vmatrix} = +16.$$

Exercice 5 :

1. $\dfrac{\overrightarrow{AB} \wedge \overrightarrow{AC}}{\|\overrightarrow{AB} \wedge \overrightarrow{AC}\|} = \vec{z}$

2. $\dfrac{\overrightarrow{AB}}{\|\overrightarrow{AB}\|} = \vec{x}$

3. $\vec{y} = \vec{z} \wedge \vec{x}$.