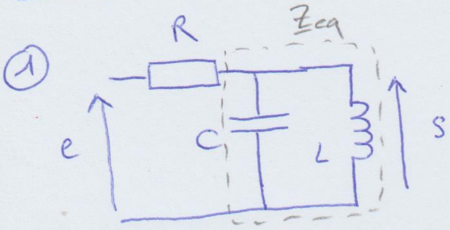
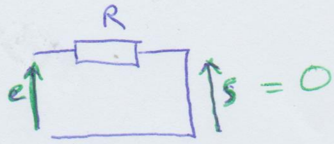
SF2 et SF3

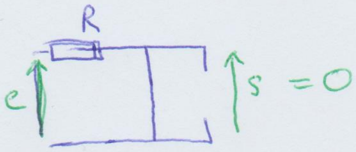
①



BF



HF

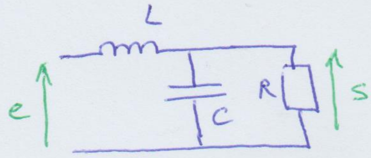


$\Rightarrow \boxed{\text{passe-bande}}$

$$\underline{H} = \frac{s}{e} = \frac{\frac{jL\omega}{1-LC\omega^2}}{R + \frac{jL\omega}{1-LC\omega^2}} = \frac{j\frac{L}{R}\omega}{1 - LC\omega^2 + j\frac{L}{R}\omega}$$

$$\left(\underline{Z}_{eq} = \frac{\frac{jL\omega}{jC\omega}}{jL\omega + \frac{1}{jC\omega}} = \frac{jL\omega}{1 - LC\omega^2} \right)$$

②



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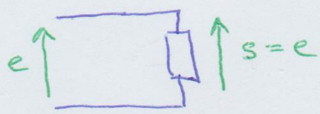
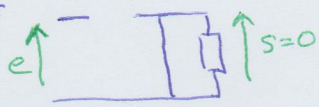


HF



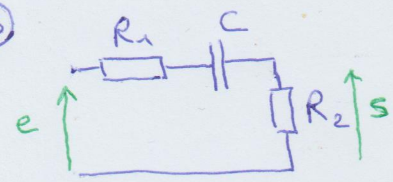
$\Rightarrow \boxed{\text{filtre passe-bas}}$

$$\underline{H} = \frac{\frac{R \cdot \frac{1}{jC\omega}}{R + \frac{1}{jC\omega}}}{jL\omega + \frac{R \cdot \frac{1}{jC\omega}}{R + \frac{1}{jC\omega}}}$$

$$\Rightarrow \underline{H} = \frac{R}{jL\omega(1 + jRC\omega) + R}$$

$$\Rightarrow \boxed{\underline{H} = \frac{1}{1 + j\frac{L}{R}\omega - LC\omega^2}}$$

③



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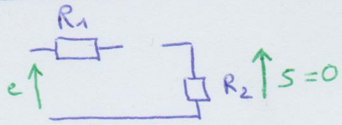
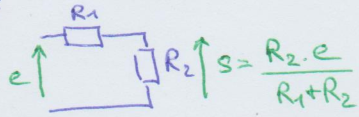


HF

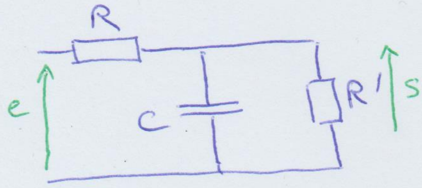


$\Rightarrow \boxed{\text{filtre passe-haut}}$

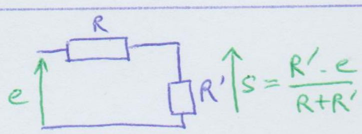
$$\underline{H} = \frac{R_2}{R_2 + R_1 + \frac{1}{jC\omega}}$$

$$\Rightarrow \boxed{\underline{H} = \frac{jR_2C\omega}{1 + j(R_2 + R_1)C\omega}}$$

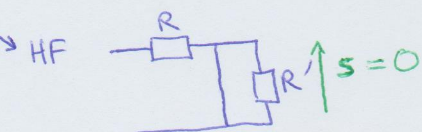
④



BF



HF



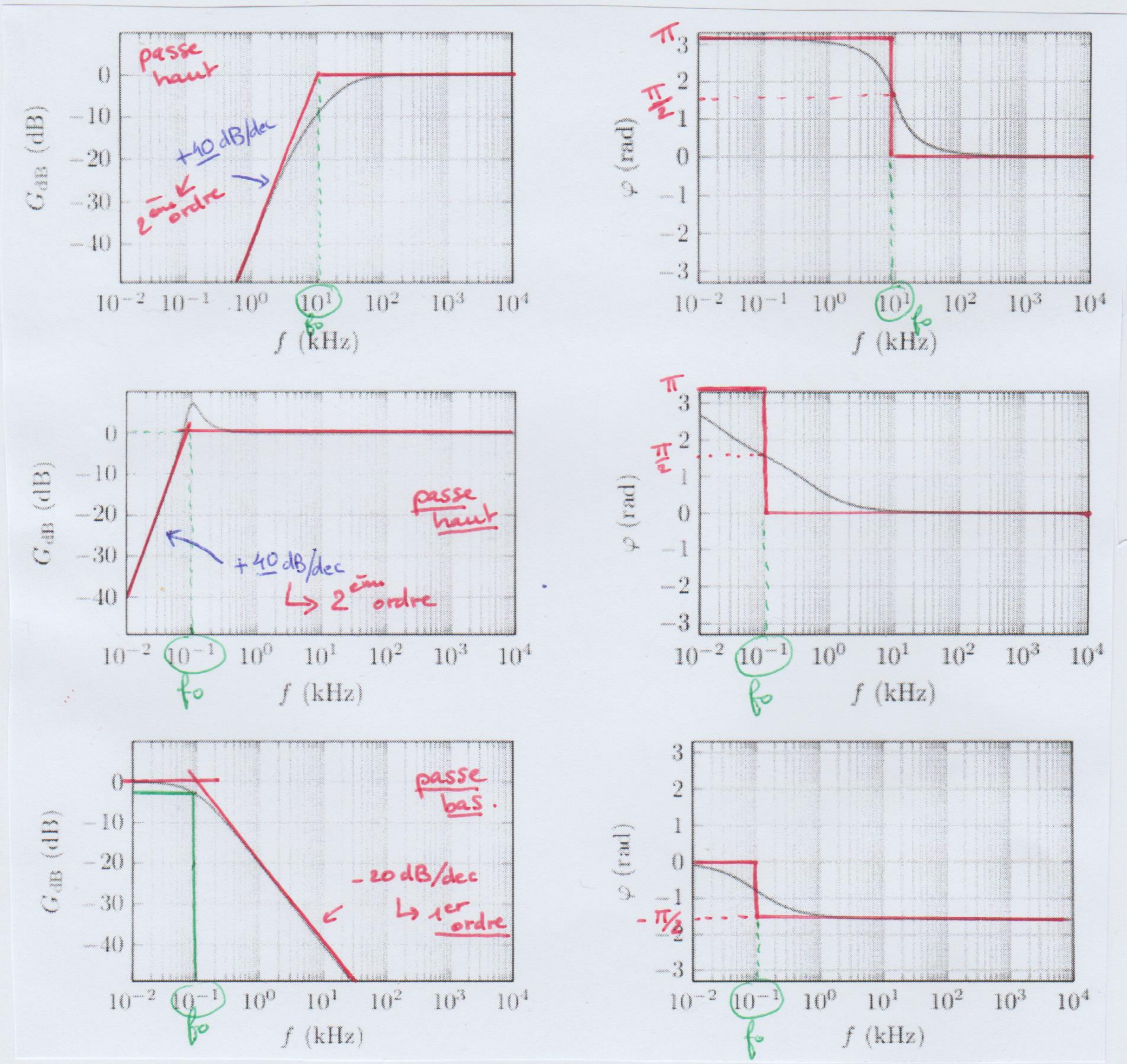
$\Rightarrow \boxed{\text{filtre passe-bas}}$

$$\underline{H} = \frac{\frac{R' \cdot \frac{1}{jC\omega}}{R' + 1/jC\omega}}{R + \frac{R' \cdot 1/jC\omega}{R' + 1/jC\omega}}$$

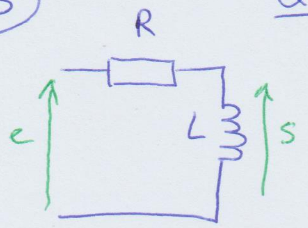
$$\underline{H} = \frac{R'}{R \cdot (1 + jR'C\omega) + R'}$$

$$\boxed{\underline{H} = \frac{\frac{R'}{R+R'}}{1 + j\left(\frac{R'R}{R+R'}\right)C\omega}}$$

SF4


passe haut
+40 dB/dec
2ème ordre
G_{dB} (dB)
f (kHz)
fo
π
π/2
φ (rad)
passe haut
+40 dB/dec
↳ 2ème ordre
passe bas.
-20 dB/dec
↳ 1er ordre
-π/2

SF5



$\underline{H}(j\omega) = \frac{\underline{s}}{\underline{e}} = \frac{jL\omega}{jL\omega + R}$

$\hookrightarrow \boxed{\underline{H}(j\omega) = \frac{j\frac{L}{R}\omega}{1 + j\frac{L}{R}\omega}}$

$\hookrightarrow \begin{cases} H_0 = 1 \\ \omega_c = \frac{R}{L} = 1{,}0 \times 10^5 \ \text{rad.s}^{-1} \end{cases}$

Q1 :

$$\underline{H}(j\omega) = H_0 \cdot \frac{j\frac{\omega}{\omega_c}}{1 + j\frac{\omega}{\omega_c}}$$

En BF : $\omega \to 0$ donc $1 \gg \frac{\omega}{\omega_c}$

$\underline{H}_{BF} = H_0 \cdot j \cdot \frac{\omega}{\omega_c} \longrightarrow G_{BF} = H_0 \cdot \frac{\omega}{\omega_c}$

imaginaire pur positif.

$\boxed{\varphi_{BF} = +\frac{\pi}{2}}$

$\longrightarrow G_{dB_{BF}} = 20 \cdot \log\left(H_0 \cdot \frac{\omega}{\omega_c}\right)$

donc $\boxed{G_{dB_{BF}} = \underbrace{20 \cdot \log(H_0)}_{\text{constante}} + \underbrace{20 \cdot \log\left(\frac{\omega}{\omega_c}\right)}_{\text{pente à } +20\,\text{dB/dec.}}}$

En HF : $\omega \to \infty$ donc $\frac{\omega}{\omega_c} \gg 1$

$\underline{H}_{HF} = H_0 \longrightarrow G_{HF} = H_0.$

réel positif

$\boxed{\varphi_{HF} = 0}$

$\longrightarrow \boxed{G_{dB_{BF}} = \underbrace{20 \log(H_0)}_{\text{constante}}.}$

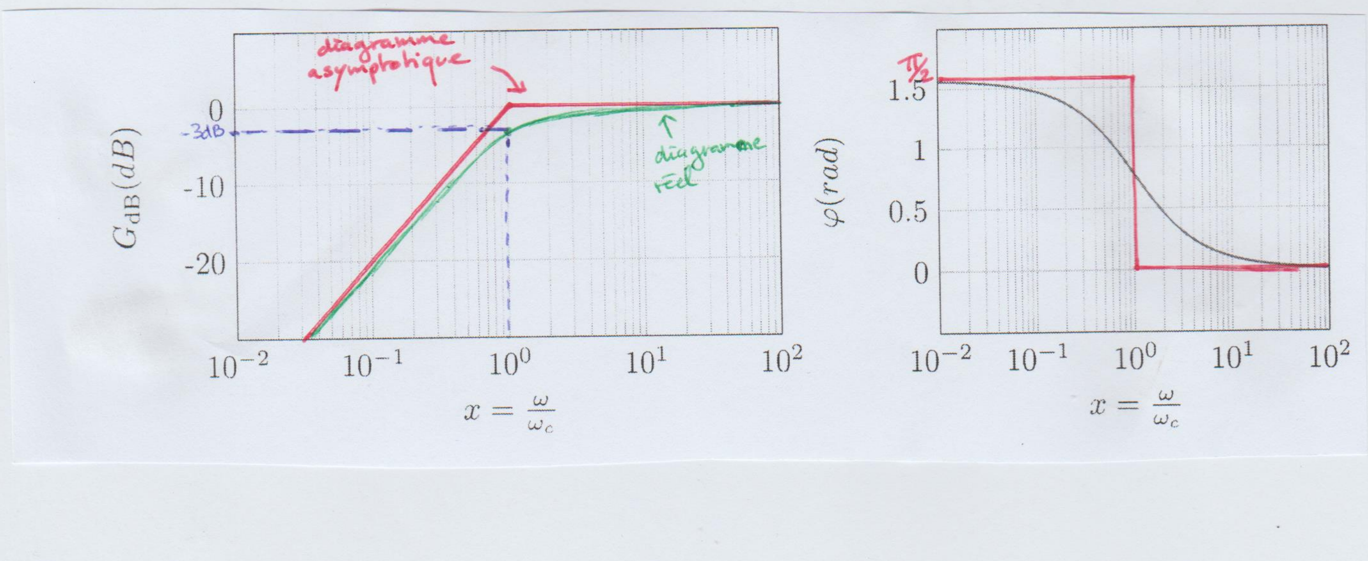
En $x = 1$: $G_{dB_{BF}}(x=1) = G_{dB_{HF}}(x=1) = 20 \cdot \log(H_0)$

- Avec $H_0 = 1$, on a $20 \cdot \log(H_0) = 0$ dB.

Q2 :

Allure du diagramme réel : on cherche la valeur réelle du gain en dB pour $x = 1$:

$$G_{dB}(x=1) = 20 \cdot \log(|\underline{H}(x=1)|) = 20 \cdot \log\left(\frac{1}{\sqrt{1^2+1^2}}\right) = -10 \cdot \log(2) = -3\,\text{dB}$$



SF6

Q1 a. $e(t) = E_m \cdot \cos(2\pi \cdot f \cdot t)$ avec $E_m = 4V$, $f = 2kHz$

$\downarrow$ filtre linéaire → la fréquence ne change pas

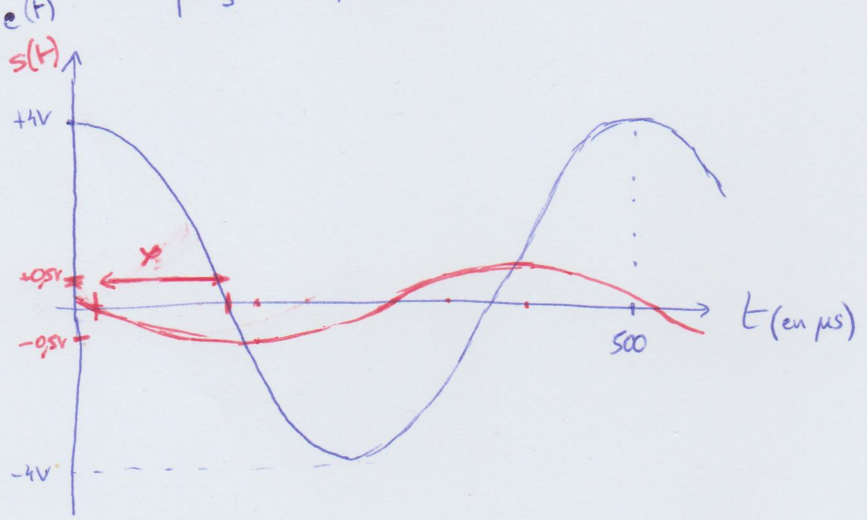
$s(t) = S_m \cdot \cos(2\pi \cdot f \cdot t + \varphi_s)$ avec

pour lecture graphique

$$\boxed{\begin{cases} S_m = E_m \times \underset{\text{gain}}{G(f)} = E_m \times 10^{\frac{G_{dB}(f)}{20}} = E_m \times \dfrac{x}{\sqrt{1+x^2}} \\ \varphi_s = \underbrace{\cancel{\varphi_e}}_{=0 \text{ ici}} + \underbrace{\varphi(f)}_{\text{déphasage}} = \dfrac{\pi}{2} - \arctan(x) \end{cases}}$$

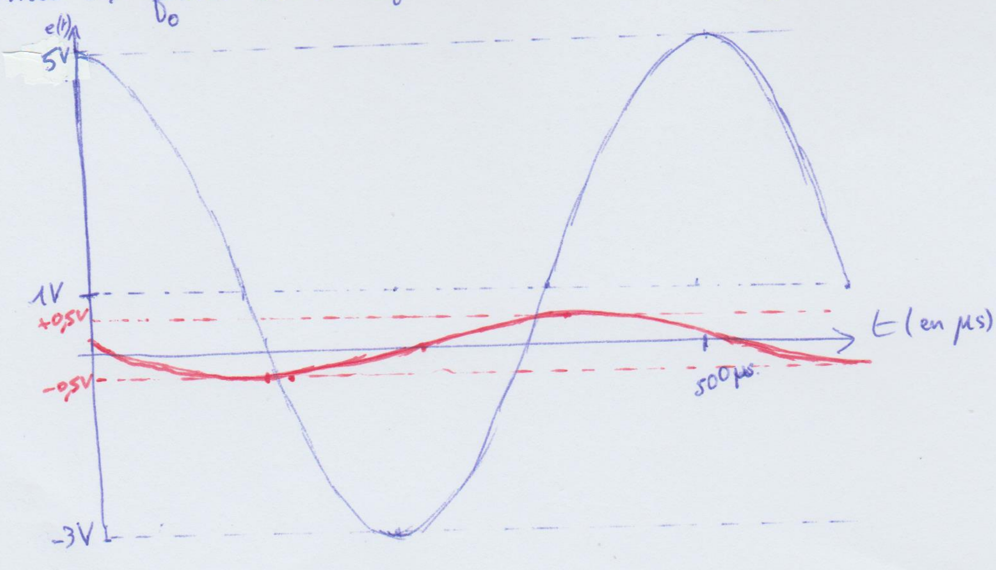
$\left(x = \frac{2\pi f}{\omega_c} = 0{,}13\right)$ prendre la valeur exacte pour les AN.

AN $\Rightarrow$ $\begin{cases} S_m = 4V \times \dfrac{0{,}13}{\sqrt{1+(0{,}13)^2}} = \underline{0{,}50V} \\ \varphi_s = 1{,}4 \text{ rad.} \end{cases}$



Q1b : $e(t) = E_0 + E_m \cdot \cos(2\pi f \cdot t)$ avec $\begin{cases} E_0 = 1V \\ E_m = 4V \\ f = 2kHz \end{cases}$

Même raisonnement et même résultat pour la composante à la fréquence f.
Par contre la composante continue E_0 est totalement supprimée par le filtre car continue $\Rightarrow f_0 = 0$ donc $x_0 = 0$. Dans ce cas le gain vaut 0.



SF6

Q1c : $e(t) = E_1 \cdot \cos(2\pi f_1 \cdot t) + E_2 \cdot \cos(2\pi \cdot f_2 \cdot t) + E_3 \cdot \cos(2\pi \cdot f_3 \cdot t)$

avec
$\begin{cases} E_1 = E_2 = E_3 = 1V \\ f_1 = 50\,Hz \\ f_2 = 1000\,Hz \\ f_3 = 10000\,Hz \end{cases}$

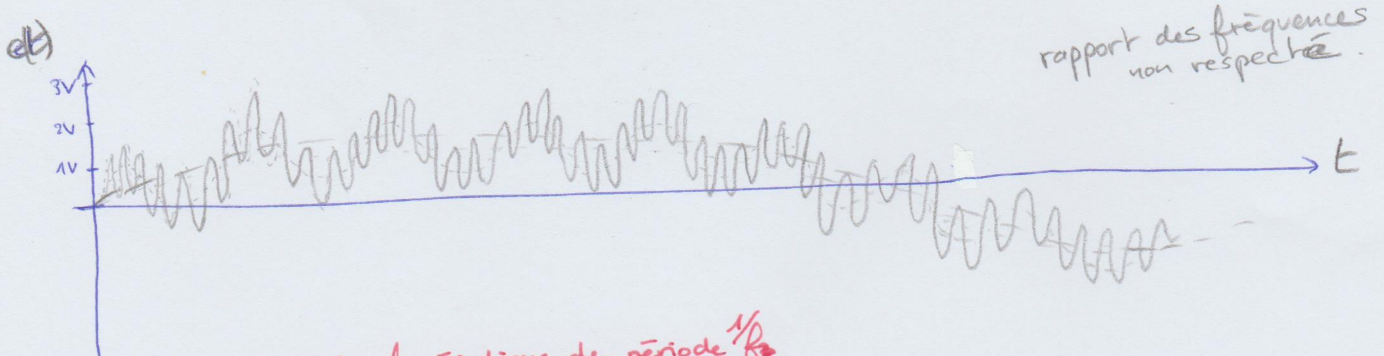
$\Downarrow$ filtre passe haut $f_c = 1,6 \times 10^4\,Hz$

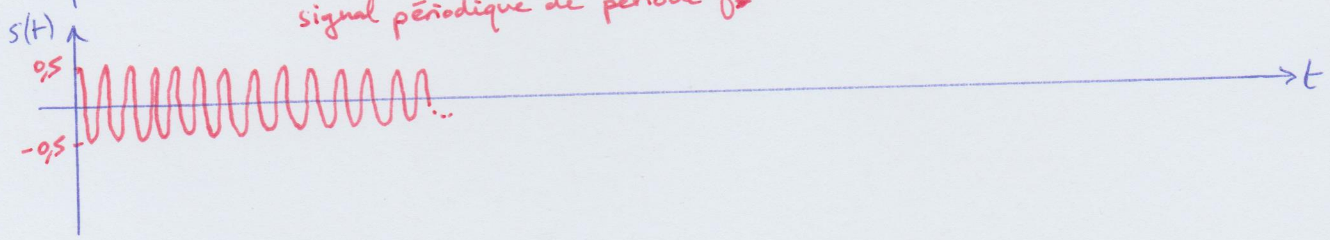
$$s(t) = S_1 \cdot \cos(2\pi f_1 \cdot t + \varphi_1) + S_2 \cdot \cos(2\pi f_2 \cdot t + \varphi_2) + S_3 \cdot \cos(2\pi f_3 \cdot t + \varphi_3)$$

avec $\begin{cases} S_1 = E_1 \times G(f_1) = 1 \times \dfrac{f_1/f_c}{\sqrt{1+(f_1/f_c)^2}} = \underline{0{,}003\,V} \\ \varphi_1 = \varphi(f_1) = \dfrac{\pi}{2} - \arctan\left(\dfrac{f_1}{f_c}\right) = 1{,}57 \text{ rad} \simeq \dfrac{\pi}{2} \end{cases}$ ← négligeable.

$\begin{cases} S_2 = E_2 \times G(f_2) = \underline{0{,}063\,V} \\ \varphi_2 = \varphi(f_2) = 1{,}51 \text{ rad.} \end{cases}$ ← légère ondulation du signal à la fréquence f_2

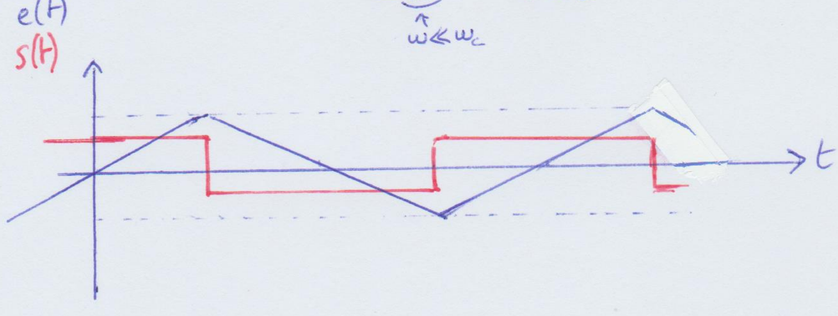
$\begin{cases} S_3 = E_3 \times G(f_3) = \underline{0{,}53\,V} \\ \varphi_3 = \varphi(f_3) = 1 \text{ rad } (\approx \frac{\pi}{3}) \end{cases}$ on retrouve principalement le signal à la fréquence f_3 en sortie (mais atténué)



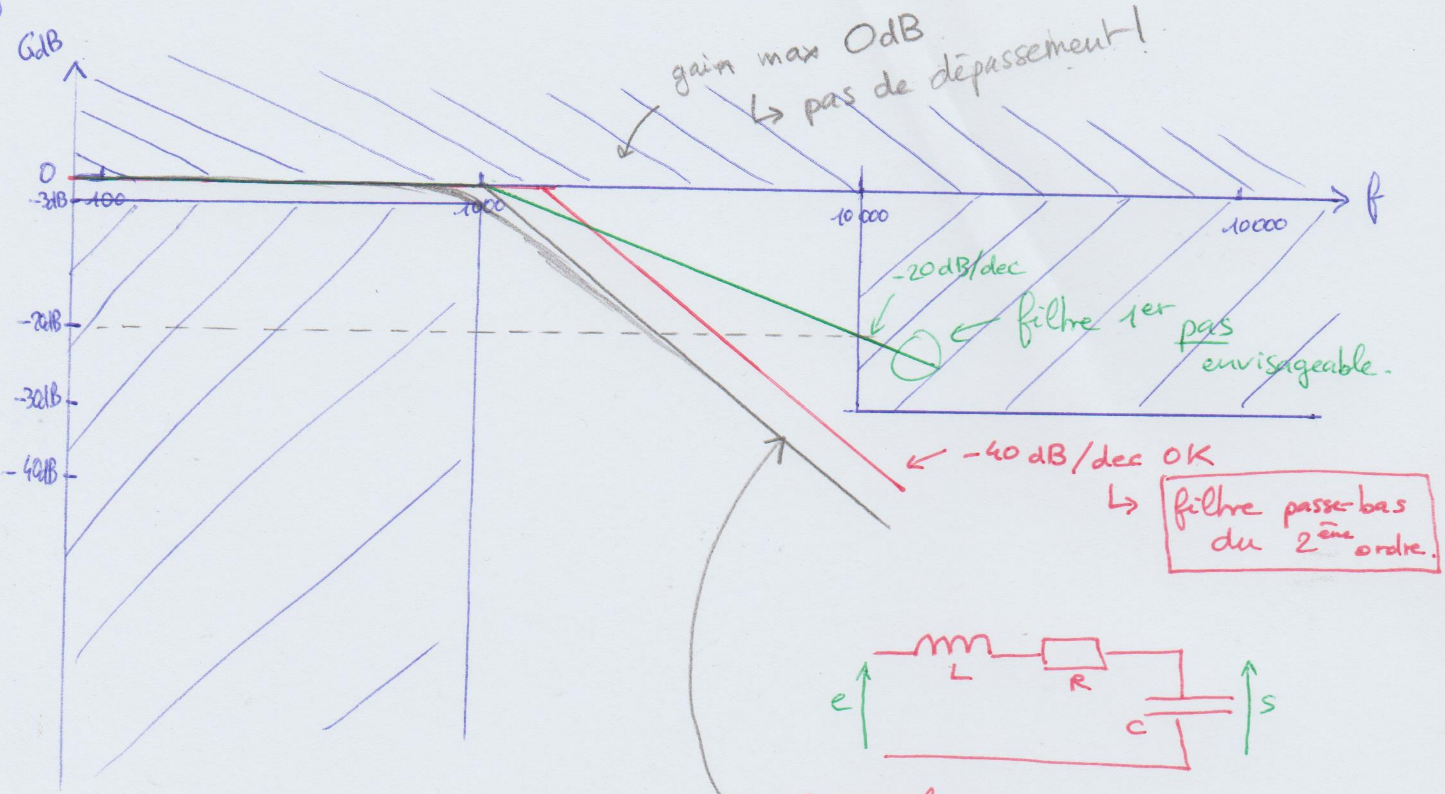


Q1d : A très basse Fréquence, le filtre passe haut se comporte comme un dérivateur :

$\underbrace{\underline{H}_{BF}}_{\omega \ll \omega_c} \simeq j\dfrac{\omega}{\omega_c}$ or en complexe $\times j\omega \Leftrightarrow$ dérivation



SF7



Par exemple :

Avec $Q = \frac{1}{\sqrt{2}}$ (on a alors $G_{dB}(\omega_0) = -3dB$)

et $\omega_0 = 2\pi . f_0 = 2\pi \times 1000$ Hz