Exercice 1

Q1 : $\underline{e}(t) = E.\sqrt{2}.e^{j\omega t}$ et $\underline{i}(t) = I.\sqrt{2}\,e^{j\omega\psi}.e^{j\omega t} = \underline{I}.e^{j\omega t}$

Q2 $U_{eff} = \sqrt{\langle u^2 \rangle} = \sqrt{\frac{1}{T}\int_0^T U_m^2 . \cos^2(\omega t+\psi)\,dt} = \sqrt{\frac{U_m^2}{T}\int_0^T \left(\frac{1}{2}+\frac{1}{2}\cos(2\omega t+2\psi)\right)dt}$

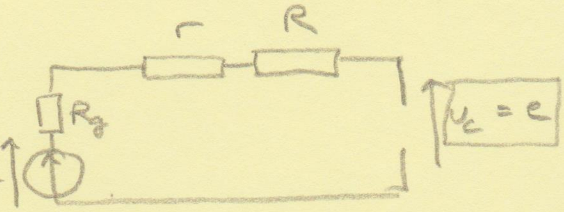
$\hookrightarrow \boxed{U_{eff} = \frac{U_m}{\sqrt{2}}}$

Q3 :
bobine : $\underline{Z}_L = jL\omega$
résistor : $\underline{Z}_R = R$
condensateur : $\underline{Z}_C = \frac{1}{jC\omega}$

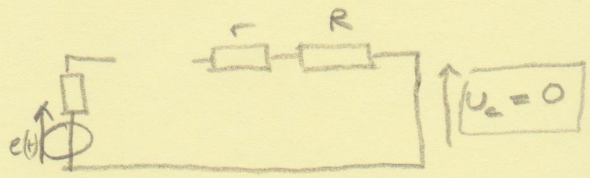
Q4

	BF	HF
bobine	fil	circuit ouvert
résistor	R	R
condensateur	circuit ouvert	fil

$\nearrow$ BF



$\searrow$ HF



Q5. $\underline{U}_c = \dfrac{\frac{1}{jC\omega}}{\frac{1}{jC\omega} + jL\omega + R_g + r + R}\underline{E}$ (pont diviseur de tension)

$$\boxed{\underline{U}_c = \frac{\underline{E}}{1 - LC\omega^2 + jC(R_g+r+R)\omega}}$$

$A = \underline{E} = E.\sqrt{2}$

$\omega_0 = \frac{1}{\sqrt{LC}}$

$Q = \frac{1}{R_g+r+R} \times \sqrt{\frac{L}{C}}$

Q6 : $U_{cE} = \frac{U_c}{\sqrt{2}} = \frac{|\underline{U}_c|}{\sqrt{2}} = \dfrac{E}{\sqrt{\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(\frac{1}{Q}\frac{\omega}{\omega_0}\right)^2}}$

Q7 : $U_{cE} = \dfrac{E}{\sqrt{(1-x^2)^2 + \left(\frac{1}{Q}x\right)^2}}$

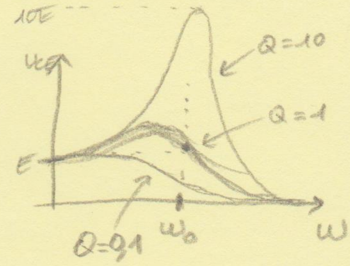
$\frac{dU_{cE}}{dx} = 0 \iff -4x.(1-x^2) + \frac{2}{Q^2}x = 0 \Rightarrow x = 0$ ou $x = \sqrt{\underbrace{1 - \frac{1}{2Q^2}}_{\text{positif seulement si } Q > \frac{1}{\sqrt{2}}}}$

$\Rightarrow \boxed{Q_{min} = \frac{1}{\sqrt{2}}}$ et $\boxed{x_r = \sqrt{1 - \frac{1}{2Q^2}}}$

$$\boxed{\omega_r = \omega_0 \cdot x_r = \omega_0 \cdot \sqrt{1 - \frac{1}{2Q^2}} \leq \omega_0}$$

Q8 : $\boxed{x=1}$ $\quad U_{CE}(\omega = \omega_0) = \dfrac{E}{\sqrt{(1-1^2)^2 + \frac{1}{Q^2} \times 1^2}} = Q.E.$

Q9



Q10 :

$$\underline{Z} = R + r + R_G + jL\omega + \frac{1}{jC\omega}$$

$$\underline{Z} = R_0 + j\frac{L}{R_0}\omega + \frac{1}{jCR_0\omega} \qquad \text{avec } R_0 = R + r + R_G$$

$$\text{or } \frac{L}{R_0} = \frac{Q}{\omega_0} \text{ et } R_0 C = \frac{1}{Q\omega_0}$$

$$\hookrightarrow \boxed{\underline{Z} = R_0 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

Q11 : $\boxed{\underline{I} = \dfrac{\underline{E}}{\underline{Z}} = \dfrac{E\cdot\sqrt{2}}{R_0\cdot\left(1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right)}}$

Q12 : $I_e = |\underline{I}_e| = \dfrac{I}{\sqrt{2}} = \dfrac{E/R_0}{\sqrt{1 + Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}}$ $\qquad A' = E/R_0$

$B = Q$

↳ mesure de la tension aux bornes de la résistance R, u_R, puis

$$i(t) = \frac{u_R(t)}{R} \Rightarrow I_e = \frac{U_{Re}}{R}$$

Q13 : $\dfrac{dI_e}{d\omega} = 0 \Rightarrow \omega = \omega_0$

$$\hookrightarrow \boxed{\omega_r' = \omega_0} \text{ et } I_{max} = \frac{E}{R_0}$$

Q14 $I_e = \dfrac{I_{max}}{\sqrt{2}} \Rightarrow 1 + Q^2\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)^2 = 2 \Rightarrow \left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)^2 = \dfrac{1}{Q^2}$

$\xrightarrow{\text{si } \omega > \omega_0} \dfrac{\omega_1}{\omega_0} - \dfrac{\omega_0}{\omega_1} = \dfrac{1}{Q}$ *

$\xrightarrow{\text{si } \omega < \omega_0} \dfrac{\omega_2}{\omega_0} - \dfrac{\omega_0}{\omega_2} = -\dfrac{1}{Q}$ **

$$\Rightarrow \begin{cases} * \ \omega_1^2 - \omega_0^2 - \dfrac{\omega_0\omega_1}{Q} = 0 \Rightarrow \omega_1 = \dfrac{\omega_0}{2Q}\left(1 + \sqrt{1+Q^2}\right) \\ ** \ \omega_2^2 - \omega_0^2 + \dfrac{\omega_0\omega_2}{Q} = 0 \Rightarrow \omega_2 = \dfrac{\omega_0}{2Q}\left(-1 + \sqrt{1+Q^2}\right) \end{cases} \Rightarrow \boxed{\Delta\omega = \omega_1 - \omega_2 = \frac{\omega_0}{Q}}$$

(on garde les solutions physiques)

Q15 : (1) → U_{CE}

(2) → I_c

comparaison à $f = 0$

$I_c(0) = 0$ et $U_{CE}(0) = E$.

Q16 : • $E = 5V$ ($= U_{CE}(0)$)

• $f_0 = 1{,}6\ kHz$ (pic de résonance en intensité)

• $I_{max} = 9\ mA$

• $Q = \dfrac{U_{CE}(f_0)}{E} = \dfrac{9}{5} = 1{,}8$

• bande passante : sur courbe → on cherche $I = \dfrac{I_{max}}{\sqrt{2}} = 6{,}3\ mA$

↳ $\left.\begin{cases} f_1 = 1{,}2\ kHz \\ f_2 = 2{,}1\ kHz \end{cases}\right\} \Delta f = 0{,}9\ kHz$

(on retrouve $\dfrac{f_0}{Q} = \dfrac{1{,}6}{1{,}8} \approx 0{,}9\ kHz$)

Q17 : $I_{max} = \dfrac{E}{R_0} \Rightarrow R + R_g + r = \dfrac{E}{I_{max}}$

↳ $\boxed{r = \dfrac{E}{I_{max}} - (R + R_g)}$

AN : $r = \dfrac{5}{9 \times 10^{-3}} - 530 \approx \underline{26\ \Omega}$

$Q = \dfrac{1}{R_0} \times \sqrt{\dfrac{L}{C}} \Rightarrow \boxed{L = C \times Q^2 \times R_0^2}$

$L = 0{,}1 \times 10^{-6} \times (1{,}8)^2 \times \left(\dfrac{5}{9 \cdot 10^{-3}}\right)^2$

AN : $\underline{L = 0{,}1\ H}$.

Q18 : $\underline{i}(t) = \dfrac{\underline{E}}{\underline{Z}}$

$$\Psi = \arg(\underline{I}) = \arg\left(\frac{\underline{E}}{\underline{Z}}\right) = \arg(\underline{E}) - \arg(\underline{Z})$$

$$\hookrightarrow \boxed{\Psi = 0 - \arctan\left(\frac{Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}{R_0}\right)} \qquad \underline{\Psi(\omega_0) = 0}$$

$$\boxed{\varphi = \arg(\underline{u}_c) \begin{cases} = 0 - \arctan\left(\dfrac{\frac{1}{Q}\frac{\omega}{\omega_0}}{1 - \left(\frac{\omega}{\omega_0}\right)^2}\right) & \text{si } \omega < \omega_0 \\ = 0 - \left(\arctan\left(\dfrac{\frac{1}{Q}\frac{\omega}{\omega_0}}{1 - \left(\frac{\omega}{\omega_0}\right)^2}\right) + \pi\right) & \text{si } \omega > \omega_0 \end{cases}}$$

Q5

$$\underline{\varphi(\omega_0) = -\frac{\pi}{2}}$$

Q19 : On mesure la tension aux bornes de R puis on utilise $\boxed{i(t) = \dfrac{u_R(t)}{R}}$.

Q20 : Résonance en intensité étudiée précédemment.

Q21 : On est à $\omega = \omega_0$

$\hookrightarrow \varphi = -\frac{\pi}{2}$ et $\Psi = 0$ donc (a) $\Rightarrow$ aux bornes de R

$\hookrightarrow$(b) $\hookrightarrow$ (a) (b) $\Rightarrow$ aux bornes de C

Déphasage : oscillo (a) : $\varphi_{XY} = 0$

oscillo (b) : $\varphi_{XY} = -\frac{\pi}{2}$ (X est en retard de $\frac{\pi}{2}$ par rapport à Y)

quart de période

Q22 : $I_{max} = \dfrac{E}{R + R_g + r}$ or $I_{max} = \dfrac{U_{R\,max}}{R}$

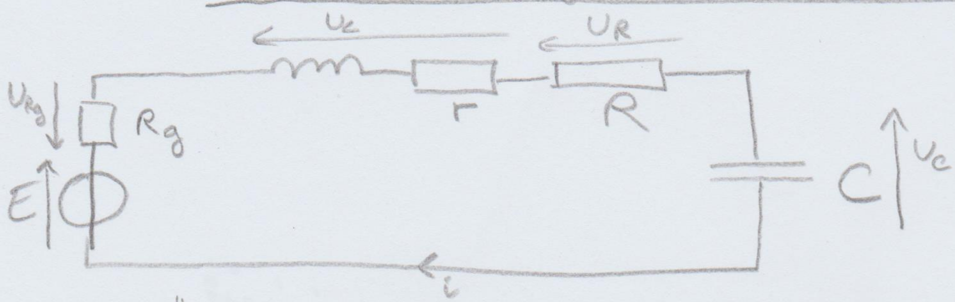
$$\hookrightarrow \boxed{r = R \times \left(\frac{E}{U_{R\,max}} - 1\right) - R_g} \qquad \underline{AN} : r = 50\,\Omega$$

$$U_{c\,e\,max} = \frac{1}{R_0} \times \sqrt{\frac{L}{C}} \times E \Rightarrow \boxed{L = C \times \left(\frac{U_{c\,e\,max} \times \sqrt{2}}{E \times \sqrt{2}}\right)^2 \times R_0^2}$$

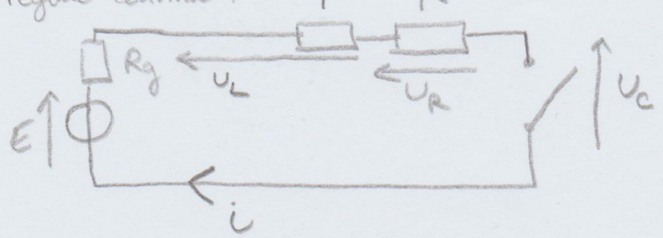
$\underline{AN}$: $L = 0{,}37$ H

Etude en régime stationnaire

Q23 :



↳ régime continu :



* circuit ouvert : $\boxed{i = 0}$

* bobine → fil et $r.i = 0$: $\boxed{u_L = 0}$

* $\boxed{U_R = R.i = 0}$

* $U_c = E - \underbrace{(R_g + r + R)\, i}_{0} = E$

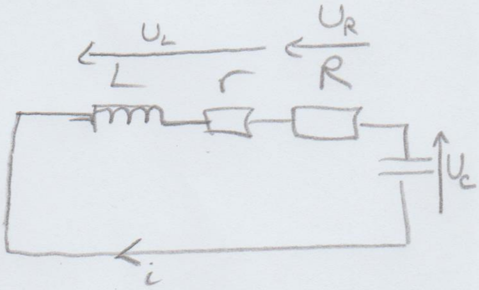
↳ $\boxed{U_c = E}$

Q24 : • loi des mailles : $0 = u_L + U_R + U_c$ (1)

• loi de comportement : bobine : $U_L = L.\frac{di}{dt} + r.i$ (2)

condensateur : $i = C\,\frac{dU_c}{dt}$ (3)

résistances : $u_R = R.i$ (4)



(2),(4) dans (1)

↳ $0 = L.\frac{di}{dt} + (r + R)\,i + u_c$

avec (3) → $0 = LC.\frac{d^2u_c}{dt^2} + (r+R).C.\frac{du_c}{dt} + u_c$

↳ $\boxed{\frac{d^2u_c}{dt^2} + \left(\frac{r+R}{L}\right).\frac{du_c}{dt} + \frac{1}{LC}\,u_c = 0}$ → $\omega_0 = \frac{1}{\sqrt{LC}}$

$Q = \frac{\sqrt{L}}{(R+r)\sqrt{C}}$

Q25 : bobine → continuité du courant

$i(0^+) = i(0^-) = 0$ ← Q23

condensateur → continuité de la tension

$u_c(0^+) = u_c(0^-) = E$

<u>Q26</u> : équation caractéristique :

$$r^2 + \frac{\omega_0}{Q'} r + \omega_0^2 = 0 \longrightarrow \text{discriminant : } \Delta = \frac{\omega_0^2}{Q'^2} - 4\omega_0^2$$

Régime pseudo périodique si $\Delta < 0$

$$\hookrightarrow \frac{\omega_0^2}{Q'^2} - 4\omega_0^2 < 0 \rightarrow \boxed{Q' > 0{,}5}$$

$$\frac{\sqrt{L}}{(R+R_g+r)\sqrt{C}} > 0{,}5$$

$$\boxed{2\sqrt{\frac{L}{C}} - (R_g + r) > R}$$

<u>Q27</u> Si $\Delta < 0 \Rightarrow r_{\frac{1}{2}} = -\frac{\omega_0}{2Q}\left(1 \pm j\sqrt{-1 + 4Q'^2}\right) = -\lambda \pm j\Omega$

solution de la forme indiquée.

$$\hookrightarrow \begin{cases} \lambda = \dfrac{\omega_0}{2Q} \\ \Omega = \dfrac{\omega_0}{2Q} \times \sqrt{4Q^2 - 1} = \omega_0 \times \sqrt{1 - \dfrac{1}{4Q^2}} \end{cases}$$

En $t=0$ $\begin{cases} u_c(0) = E = A \\ i(0) = C\dfrac{du_c}{dt}(0) = 0 = B\Omega \end{cases} \Rightarrow \boxed{\begin{matrix} A = E \\ B = 0 \end{matrix}}$

<u>Q28</u> : $T = 0{,}65\ \text{ms}$ (durée entre 2 maximums)

$$\hookrightarrow \Omega = \frac{2\pi}{T} = 9{,}7 \times 10^3\ \text{rad.s}^{-1}$$

<u>Q29</u> : $\delta = \ln\left(\frac{u_1}{u_2}\right) = \ln\left(\frac{u_c(T)}{u_c(2T)}\right) = \ln\left(\frac{e^{-\lambda T} . E \overbrace{\cos(\Omega T)}^{1}}{e^{-\lambda(2T)} . E . \underbrace{\cos(\Omega \times 2T)}_{1}}\right) = \ln\left(e^{\lambda . T}\right) = \lambda . T$

$$\hookrightarrow \boxed{\delta = \frac{\omega_0 . T}{2Q'}} \rightarrow \text{décrément logarithmique.}$$

<u>Q30</u> $Q' = \frac{\omega_0 . T}{2\delta} = \frac{\omega_0 \frac{2\pi}{\Omega}}{2\delta} = \frac{\pi}{\delta} \times \frac{1}{\sqrt{1 - \frac{1}{4Q^2}}}$

$$\hookrightarrow Q'^2 = \frac{\pi^2}{\delta^2} + \frac{1}{4} \longrightarrow \boxed{Q' = \frac{\pi}{\delta} \times \sqrt{1 + \frac{\delta^2}{4\pi^2}}}$$

et $\omega_0 = \dfrac{\Omega}{\sqrt{1 - \frac{1}{4Q^2}}}$

<u>AN</u> : $Q' = 2{,}4$

$\omega_0 = 9{,}9 \times 10^3\ \text{rad.s}^{-1}$

<u>Q31</u> $\omega_0 \simeq \Omega$ si $Q \gg 1$

approximation pas vérifiée mais ω_0 et Ω sont quand même proches.

<u>Q32</u> : $\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \boxed{L = \frac{1}{\omega_0^2 . C}}$ <u>AN</u> : $L = 0{,}1\ \text{H}$ et $Q' = 2{,}4$