Ex 8 $\cos^4 x + \sin^4 x = \left(\cos^2 x + \sin^2 x\right)^2 - 2\cos^2 x \sin^2 x = 1 - 2\cos^2 x \sin^2 x$

$$\cos^2 x \sin^2 x = \left(\frac{e^{ix} + e^{-ix}}{2}\right)^2 \left(\frac{e^{ix} - e^{-ix}}{2i}\right)^2$$

$$= -\frac{1}{16}\left[e^{2ix} + 2 + e^{-2ix}\right]\left[e^{2ix} - 2 + e^{-2ix}\right]$$

$$= -\frac{1}{16}\left[e^{4ix} \cancel{-2e^{2ix}} + 1 + \cancel{2e^{2ix}} - 4 + \cancel{2e^{-2ix}} + 1 \cancel{-2e^{-2ix}} + e^{-4ix}\right]$$

$$= -\frac{1}{16}\left[e^{4ix} + e^{-4ix} - 2\right] = -\frac{1}{16}\left[2\cos(4x) - 2\right]$$

$$= -\frac{1}{8}\cos(4x) + \frac{1}{8}$$

donc $\cos^4 x + \sin^4 x = 1 - 2\left(-\frac{1}{8}\cos(4x) + \frac{1}{8}\right)$

$$= 1 + \frac{1}{4}\cos(4x) - \frac{1}{4} = \frac{3}{4} + \frac{1}{4}\cos(4x)$$

donc $\boxed{\cos^4 x + \sin^4 x = \frac{3}{4} + \frac{1}{4}\cos(4x)}$

Ex 9

$$\cos^4 x \times \sin^2 x = \left(\frac{e^{ix}+e^{-ix}}{2}\right)^4 \times \left(\frac{e^{ix}-e^{-ix}}{2i}\right)^2$$

$$= -\frac{1}{2^6}\left(e^{4ix}+4e^{2ix}+6+4e^{-2ix}+e^{-4ix}\right)\left(e^{2ix}-2+e^{-2ix}\right)$$

$$= -\frac{1}{2^6}\Big(e^{6ix}-2e^{4ix}+e^{2ix}+4e^{4ix}-8e^{2ix}+4+6e^{2ix}-12+6e^{-2ix}$$
$$+4-8e^{-2ix}+4e^{-4ix}+e^{-2ix}-2e^{-4ix}+e^{-6ix}\Big)$$

$$= -\frac{1}{2^6}\left(e^{6ix}+2e^{4ix}-e^{2ix}-4-e^{-2ix}+2e^{-4ix}+e^{-6ix}\right) \qquad 2^4=16$$

$$= -\frac{1}{2^6}\left(2\cos 6x+4\cos 4x-2\cos 2x-4\right)$$

$$= -\frac{1}{32}\cos 6x-\frac{1}{16}\cos 4x+\frac{1}{32}\cos 2x+\frac{1}{16}.$$

1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 ·

$$\sin^6 x = \left(\frac{e^{ix}-e^{-ix}}{2i}\right)^6 = -\frac{1}{2^6}\left(e^{6ix}-6e^{4ix}+15e^{2ix}-20+15e^{-2ix}-6e^{-4ix}+e^{-6ix}\right)$$

$$= -\frac{1}{2^6}\left(e^{6ix}+e^{-6ix}-6\left(e^{4ix}+e^{-4ix}\right)+15\left(e^{2ix}+e^{-2ix}\right)-20\right)$$

$$= -\frac{1}{2^6}\left(2\cos(6x)-12\cos(4x)+30\cos(2x)-20\right)$$

$$= -\frac{1}{32}\cos(6x)+\frac{3}{16}\cos(4x)-\frac{15}{32}\cos(2x)+\frac{5}{16}.$$

Ex 10

$$\cos(2x) = 2\cos^2 x - 1$$

$$\cos(3x) = \mathrm{Re}\left(\cos(3x)+i\sin(3x)\right) \overset{\text{Moivre}}{=} \mathrm{Re}\left(\left(\cos(x)+i\sin(x)\right)^3\right)$$
$$= \mathrm{Re}\left(\cos^3 x+3i\cos^2 x\sin x-3\cos x\sin^2 x-i\sin^3 x\right)$$
$$= \cos^3 x-3\cos x\left(\sin^2 x\right) = \cos^3 x-3\cos x\left(1-\cos^2 x\right)$$
$$= \cos^3 x-3\cos x+3\cos^3 x = 4\cos^3 x-3\cos x.$$

cours

$$\cos(4x) = \mathrm{Re}\left(\cos(4x)+i\sin(4x)\right) \overset{\text{Moivre}}{=} \mathrm{Re}\left(\left(\cos(x)+i\sin(x)\right)^4\right)$$
$$= \mathrm{Re}\left(\cos^4 x+4i\cos^3 x\sin x-6\cos^2 x\sin^2 x-4i\cos x\sin^3 x+\sin^4 x\right)$$
$$= \cos^4 x-6\cos^2 x\sin^2 x+\sin^4 x$$
$$= \cos^4 x-6\cos^2 x\left(1-\cos^2 x\right)+\left(1-\cos^2 x\right)^2$$
$$= \cos^4 x-6\cos^2 x+6\cos^4 x+\left(1+\cos^4 x-2\cos^2 x\right)$$
$$= 8\cos^4 x-8\cos^2 x+1$$

Moivre

$$\cos(5x) = \mathrm{Re}\left(\cos(5x)+i\sin(5x)\right) = \mathrm{Re}\left((\cos x + i\sin x)^5\right)$$
$$= \mathrm{Re}\left(\cos^5 x + 5i\cos^4 x\sin x - 10\cos^3 x\sin^2 x - 10i\cos^2 x\sin^3 x + 5\cos x\sin^4 x + i\sin^5 x\right)$$
$$= \cos^5 x - 10\cos^3 x\sin^2 x + 5\cos x\sin^4 x$$
$$= \cos^5 x - 10\cos^3 x(1-\cos^2 x) + 5\cos x(1-\cos^2 x)^2$$
$$= \cos^5 x - 10\cos^3 x + 10\cos^5 x + 5\cos x(1+\cos^4 x - 2\cos^2 x)$$
$$= 11\cos^5 x - 10\cos^3 x + 5\cos x + 5\cos^5 x - 10\cos^3 x$$
$$= 16\cos^5 x - 20\cos^3 x + 5\cos x$$

Ex 6 Soit $M(z)$

① $|1+z| = a$ ssi $|z-(-1)| = a$ ssi $M \in \mathcal{C}(\Omega, a)$ où $\Omega(-1)$ et $a \geq 0$

② $0 \leq a \leq |1-z| \leq b$ ssi $0 \leq a \leq |z-1| \leq b$ ssi M appartient à la couronne centrée en $\Omega(1)$ et de rayon intérieur a et de rayon extérieur b.

③ $|z| = |1-z| = 1$ ssi $\begin{cases} |z| = 1 \\ |z-1| = 1 \end{cases}$ ssi $M \in \mathcal{C}(0,1) \cap \mathcal{C}(\Omega,1)$ où $\Omega(1)$.

Déterminons les points d'intersection de $\mathcal{C}(0,1) \cap \mathcal{C}(\Omega,1)$.

Soit $M(x,y)$ alors $M \in \mathcal{C}(0,1)$ ssi $x^2+y^2=1$
$M \in \mathcal{C}(\Omega,1)$ ssi $(x-1)^2+y^2=1$

Donc $M \in \mathcal{C}(0,1) \cap \mathcal{C}(\Omega,1)$ ssi $\begin{cases} x^2+y^2=1 \\ (x-1)^2+y^2=1 \end{cases}$

ssi $\begin{cases} x^2+y^2=1 \\ x^2+1-2x+y^2=1 \end{cases}$

ssi $\begin{cases} x^2+y^2=1 \\ 1-2x=0 \end{cases}$

ssi $\begin{cases} y^2 = \frac{3}{4} \\ x = \frac{1}{2} \end{cases}$ ssi $x=\frac{1}{2}$ et $y=\frac{\sqrt{3}}{2}$ ou $x=\frac{1}{2}$ et $y=-\frac{\sqrt{3}}{2}$

Donc $M \in \mathcal{C}(0,1) \cap \mathcal{C}(\Omega,1)$ ssi $z \in \left\{\frac{1}{2}+i\frac{\sqrt{3}}{2}, \frac{1}{2}-i\frac{\sqrt{3}}{2}\right\}$

④ $1+|z| \leq a$ ssi $|z| \leq a-1$ ssi $M \in \mathcal{C}(0,a-1)$ où $a-1 \geq 0$