

Divisions euclidiennes - Chap 18

Ex:

$$\begin{array}{r|l}
 X^4 + 2X^3 - X^2 + X - 4 & X^2 + 3X - 5 \\
 \underline{X^4 + 3X^3 - 5X^2} & X^2 - X + 7 \\
 -X^3 + 4X^2 + X & \\
 \underline{-X^3 - 3X^2 + 5X} & \\
 7X^2 - 4X - 4 & \\
 \underline{7X^2 + 21X - 35} & \\
 -25X + 31 &
 \end{array}$$

donc $X^4 + 2X^3 - X^2 + X - 4 = (X^2 + 3X - 5)(X^2 - X + 7) + (-25X + 31)$
 on a bien $\deg(-25X + 31) < \deg(X^2 + 3X - 5)$

Ev $X^2 + 3X - 1 = 0 \cdot X(X^3 - X^2 + 2X) + X^2 + 3X - 1$
 on a bien $\deg(X^2 + 3X - 1) < \deg(X^3 - X^2 + 2X)$

Ev

$$\begin{array}{r|l}
 2X^4 + 5X^3 - X^2 + 2X + 1 & 2X^2 - 3X + 1 \\
 \underline{2X^4 - 3X^3 + X^2} & X^2 + 4X + 5 \\
 8X^3 - 2X^2 + 2X & \\
 \underline{8X^3 - 12X^2 + 4X} & \\
 10X^2 - 2X + 1 & \\
 \underline{10X^2 - 15X + 5} & \\
 13X - 4 &
 \end{array}$$

donc $2X^4 + 5X^3 - X^2 + 2X + 1 = (2X^2 - 3X + 1)(X^2 + 4X + 5) + (13X - 4)$
 on a bien $\deg(13X - 4) < \deg(2X^2 - 3X + 1)$

Ev

$$\begin{array}{r|l}
 3X^5 - 2X^4 + \frac{1}{3}X^3 - X + 5 & X^3 + 5X^2 + 1 \\
 \underline{3X^5 + 15X^4 + 3X^2} & 3X^2 - 17X + \frac{256}{3} \\
 -17X^4 + \frac{1}{3}X^3 - 3X^2 - X & \\
 \underline{-17X^4 - 85X^3 - 17X} & \\
 \frac{256}{3}X^3 - 3X^2 + 16X + 5 & \\
 \underline{\frac{256}{3}X^3 + \frac{1280}{3}X^2 + \frac{256}{3}} & \\
 -\frac{1280}{3}X^2 + 16X - \frac{241}{3} &
 \end{array}$$

donc $3X^5 - 2X^4 + \frac{1}{3}X^3 - X + 5 = (X^3 + 5X^2 + 1)(3X^2 - 17X + \frac{256}{3}) - \frac{1280}{3}X^2 + 16X - \frac{241}{3}$
 et $\deg(-\frac{1280}{3}X^2 + 16X - \frac{241}{3}) < \deg(X^3 + 5X^2 + 1)$