(Ref 41)

Ex 14

$P = X^3 + pX + q \in \mathbb{C}[X]$.

$A = P'(a)\,P'(b)\,P'(c)$.

$P'(x) = 3x^2 + p$

$$A = (3a^2+p)(3b^2+p)(3c^2+p)$$

$$= (9a^2b^2 + p3a^2 + p3b^2 + p^2)(3c^2+p)$$

$$= 27a^2b^2c^2 + 9a^2c^2p + 9b^2c^2p + 3c^2p^2 + 9a^2b^2p + p^2 3a^2 + p^2 3b^2 + p^3$$

$$= 27(a^2b^2c^2) + 9p\,(a^2c^2 + b^2c^2 + a^2b^2) + 3p^2(c^2+b^2+a^2) + p^3$$

$$= 27\,\sigma_3^2 + 9p\left[(ab+bc+ca)^2 - 2abc(a+b+c)\right] + 3p^2\left[(a+b+c)^2 - 2(ab+ac+bc)\right] + p^3$$

$$= 27\,\sigma_3^2 + 9p\left[\sigma_2^2 - 2\sigma_3\sigma_1\right] + 3p^2\left[\sigma_1^2 - 2\sigma_2\right] + p^3$$

or $X^3 + pX + q = X^3 - \sigma_1 X^2 + \sigma_2 X - \sigma_3$ donc $\sigma_1 = 0$

$\sigma_2 = p$

$\sigma_3 = -q$

$$A = 27q^2 + 9p^3 \quad - 6p^3 + p^3$$

$$A = 27q^2 + 4p^3.$$

or $A = 0$ ssi l'une des 3 nombres $P'(a)$, $P'(b)$ ou $P'(c)$ est nul