

Ex 13
(Ex 38)

$$P = X^3 - 3X - 1 = 0 \quad \text{donc} \begin{cases} \sigma_1 = 0 \\ \sigma_2 = -3 \\ \sigma_3 = 1 \end{cases}$$

$$\sigma P = X^3 - \sigma_1 X^2 + \sigma_2 X - \sigma_3 = 0$$

$$\bullet a^2 + b^2 + c^2 = \sigma_1^2 - 2\sigma_2 = 0 - 2(-3) = 6$$

$$\bullet a^3 + b^3 + c^3 = \sigma_1 (\sigma_1^2 - 2\sigma_2) - \sigma_1 \sigma_2 + 3\sigma_3$$

$$x^3 = 1 + 3x$$

$$x^4 = x + 3x^2$$

$$\text{ou } a^3 + b^3 + c^3 = 3\sigma_3 = 3.$$

$$\bullet a^4 + b^4 + c^4 = 1 + 3a + 1 + 3b + 1 + 3c = 3 + 3(a+b+c) = 3$$

$$= a + 3a^2 + b + 3b^2 + c + 3c^2$$

$$= a + b + c + 3(a^2 + b^2 + c^2) = 3 + 3 \times 6 = 18.$$

$$\bullet \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{ab+ac+bc}{abc} = \frac{-3}{1} = -3.$$

$$\bullet \frac{1}{(a^3-1)^2} + \frac{1}{(b^3-1)^2} + \frac{1}{(c^3-1)^2} = \frac{1}{(3a)^2} + \frac{1}{(3b)^2} + \frac{1}{(3c)^2} = \frac{1}{9} \left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} \right)$$

$$= \frac{b^2c^2 + a^2b^2 + b^2c^2}{9 a^2 b^2 c^2}$$

$$= \frac{(ab+bc+ac)^2 - 2abc(a+b+c)}{9 a^2 b^2 c^2} = \frac{9+0}{9} = 1.$$

$$\text{car } (ab+bc+ac)^2 - 2abc(a+b+c)$$

$$= (ab)^2 + \underline{ab^2c} + \underline{a^2bc} + \underline{ab^2c} + (bc)^2 + abc^2 + \underline{a^2bc} + \underline{ab^2c} + (ac)^2$$

$$- 2(\underline{ab^2c} + \underline{a^2bc} + \underline{ab^2c})$$

$$= b^2c^2 + a^2b^2 + b^2c^2$$