

Chap 1
Ex A

$$\textcircled{1} \sum_{k=0}^n (n-k)^2 = \sum_{j=0}^n j^2 = \frac{n(n+1)(2n+1)}{6}$$

$j = n-k$

$$\sum_{k=0}^n (n-k+1) = \sum_{j=1}^{n+1} j = \frac{(n+1)(n+2)}{2}$$

$j = n-k+1$

$$\sum_{k=2}^{10} 2^k = \frac{2^2 - 2^{11}}{1-2} = \frac{2^2 - 2^{11}}{-1} = 2^{11} - 2^2 = 2^2(2^9 - 1)$$

$$\sum_{k=2}^n q^k = \frac{q^2 - q^{n+1}}{1-q} = q^2 \frac{1-q^{n-1}}{1-q} \text{ si } q \neq 1 \text{ et } \sum_{k=2}^n q^k = (n-2+1) = n-1 \text{ sinon}$$

$$\sum_{k=i}^j q^k = \frac{q^i - q^{j+1}}{1-q} = q^i \frac{(1-q^{j+1-i})}{1-q} \text{ si } q \neq 1 \text{ et } j-i+1 \text{ sinon}$$

Ex A'

$$\textcircled{1} A = 1+2+\dots+500 = \sum_{k=1}^{500} k = \frac{500 \times 501}{2}$$

$$\textcircled{2} B = 6+8+10+\dots+150 = \sum_{k=3}^{75} 2k = 2 \sum_{k=3}^{75} k = 2 \left(\sum_{k=1}^{75} k - \sum_{k=1}^2 k \right) = 2 \left(\frac{75 \times 76}{2} - 3 \right)$$

par linéarité

$$\textcircled{3} C = 8+13+18+\dots+1998+2003$$

$$= \sum_{k=0}^{399} (8+5k) = \sum_{k=0}^{399} 8 + 5 \sum_{k=0}^{399} k = 8 \times (399-0+1) + 5 \times \frac{399 \times 400}{2}$$

$$\textcircled{4} D = 5+10+20+40+\dots+5120 = \sum_{k=0}^{10} 5 \times 2^k \text{ car } 5120 \div 5 = 1024 \text{ et } 1024 = 2^{10}$$

$$D = 5 \times \sum_{k=0}^{10} 2^k = 5 \times \frac{2^0 - 2^{11}}{1-2} = -5 \times (1-2^{11}) = 5(2^{11}-1)$$

$$\textcircled{5} E = 2^2 + 2^5 + 2^8 + \dots + 2^{20} = \sum_{k=0}^6 2^2 \times (2^3)^k = 4 \sum_{k=0}^6 8^k = 4 \times \frac{1-8^7}{1-8}$$

$$E = 4 \times \frac{8^7 - 1}{7}$$