Formules de trigonométrie circulaire

$(\cos t)^2 + (\sin t)^2 = 1.$

$\cos(a+b) = \cos a \cos b - \sin a \sin b,$
$\cos(a-b) = \cos a \cos b + \sin a \sin b,$
$\sin(a+b) = \sin a \cos b + \cos a \sin b,$
$\sin(a-b) = \sin a \cos b - \cos a \sin b,$

$$\tan(a+b) = \frac{\tan a + tanb}{1 - \tan a \tan b},$$
$$\tan(a-b) = \frac{\tan a - tanb}{1 + \tan a \tan b}.$$

$\cos(2a) = (\cos a)^2 - (\sin a)^2 = 2(\cos a)^2 - 1 = 1 - 2(\sin a)^2,$
$\sin(2a) = 2 \sin a \cos a,$
$$\tan(2a) = \frac{2 \tan a}{1 - (\tan a)^2}.$$

$$(\cos a)^2 = \frac{\cos(2a) + 1}{2},$$
$$(\sin a)^2 = \frac{1 - \cos(2a)}{2}.$$

En posant $u = \tan(\frac{t}{2})$,
$$\cos t = \frac{1-u^2}{1+u^2}$$
$$\sin t = \frac{2u}{1+u^2}$$
$$\tan t = \frac{2u}{1-u^2}.$$

$2 \cos a . \cos b = \cos(a+b) + \cos(a-b),$
$2 \sin a . \sin b = \cos(a-b) - \cos(a+b),$
$2 \sin a . \cos b = \sin(a+b) + \sin(a-b).$

$$\cos p + \cos q = 2 \cos \frac{p+q}{2} \cos \frac{p-q}{2},$$
$$\cos p - \cos q = -2 \sin \frac{p+q}{2} \sin \frac{p-q}{2},$$
$$\sin p + \sin q = 2 \sin \frac{p+q}{2} \cos \frac{p-q}{2},$$
$$\sin p - \sin q = 2 \sin \frac{p-q}{2} \cos \frac{p+q}{2}.$$

$$\frac{d(\sin x)}{dx} = \cos x,$$
$$\frac{d(\cos x)}{dx} = -\sin x,$$
$$\frac{d(\tan x)}{dx} = 1 + (\tan x)^2 = \frac{1}{(\cos x)^2}.$$

Formules de trigonométrie hyperbolique

$\mathrm{ch}(t) = \dfrac{e^t + e^{-t}}{2}, \quad \mathrm{sh}(t) = \dfrac{e^t - e^{-t}}{2}.$
$\mathrm{ch}(t) + \mathrm{sh}(t) = e^t,$
$\mathrm{ch}(t) - \mathrm{sh}(t) = e^{-t}.$

$(\mathrm{ch}(t))^2 - (\mathrm{sh}(t))^2 = 1.$

$$\left.\begin{array}{l} \mathrm{ch}(a+b) = \mathrm{ch}(a)\mathrm{ch}(b) + \mathrm{sh}(a)\mathrm{sh}(b), \\ \mathrm{sh}(a+b) = \mathrm{sh}(a)\mathrm{ch}(b) + \mathrm{ch}(a)\mathrm{sh}(b), \\ (\mathrm{ch}(a))^2 = \dfrac{\mathrm{ch}(2a)+1}{2} \text{ et } (\mathrm{sh}(a))^2 = \dfrac{\mathrm{ch}(2a)-1}{2}. \end{array}\right\}$$ **Ces relations sont hors programme.**

$\dfrac{d(\mathrm{sh}(x))}{dx} = \mathrm{ch}(x),$
$\dfrac{d(\mathrm{ch}(x))}{dx} = \mathrm{sh}(x),$
$\dfrac{d(\mathrm{th}(x))}{dx} = 1 - (\mathrm{th}(x))^2 = \dfrac{1}{(\mathrm{ch}(x))^2}.$

Applications réciproques

$\mathrm{Asin} : [-1,1] \longrightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]. \quad \dfrac{d(\mathrm{Asin}(t))}{dt} = \dfrac{1}{\sqrt{1-t^2}}.$

$\mathrm{Acos} : [-1,1] \longrightarrow [0,\pi]. \quad \dfrac{d(\mathrm{Acos}(t))}{dt} = \dfrac{-1}{\sqrt{1-t^2}}.$

$\mathrm{Atan} : \mathbb{R} \longrightarrow]-\frac{\pi}{2}, \frac{\pi}{2}[. \quad \dfrac{d(\mathrm{Atan}(t))}{dt} = \dfrac{1}{1+t^2}.$

La suite est hors programme.

$\mathrm{Argsh} : \mathbb{R} \longrightarrow \mathbb{R}. \quad \mathrm{Argsh}(x) = \ln(x + \sqrt{1+x^2}). \quad \dfrac{d(\mathrm{Argsh}(t))}{dt} = \dfrac{1}{\sqrt{1+t^2}}.$

$\mathrm{Argch} : [1,+\infty[\longrightarrow \mathbb{R}_+. \quad \mathrm{Argch}(x) = \ln(x + \sqrt{x^2-1}). \quad \dfrac{d(\mathrm{Argch}(t))}{dt} = \dfrac{1}{\sqrt{t^2-1}}.$

$\mathrm{Argth} :]-1,1[\longrightarrow \mathbb{R}. \quad \mathrm{Argth}(x) = \dfrac{1}{2}\ln\dfrac{1+x}{1-x}. \quad \dfrac{d(\mathrm{Argth}(t))}{dt} = \dfrac{1}{1-t^2}.$