

10 5. d)

* Inversión de $P = \begin{pmatrix} 2 & -1 & 0 \\ -1 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

$$P^2 = \begin{pmatrix} 2 & -1 & 0 & | & 1 & 0 & 0 \\ -1 & -1 & 1 & | & 0 & 1 & 0 \\ 1 & 1 & 1 & | & 0 & 0 & 1 \end{pmatrix}$$

$${}^2L \begin{pmatrix} 1 & 1 & 1 & | & 0 & 0 & 1 \\ -1 & -1 & 1 & | & 0 & 1 & 0 \\ 2 & -1 & 0 & | & 1 & 0 & 0 \end{pmatrix} \begin{matrix} \uparrow \\ \downarrow \end{matrix}$$

$${}^2L \begin{pmatrix} 1 & 1 & 1 & | & 0 & 0 & 1 \\ 0 & 0 & 2 & | & 0 & 1 & 1 \\ 0 & -3 & -2 & | & 1 & 0 & -2 \end{pmatrix} \begin{matrix} \\ L_2 \leftarrow L_2 + L_1 \\ L_3 \leftarrow L_3 - 2L_1 \end{matrix}$$

$${}^2L \begin{pmatrix} 1 & 1 & 1 & | & 0 & 0 & 1 \\ 0 & 1 & 2/3 & | & -1/3 & 0 & 2/3 \\ 0 & 0 & 1 & | & 0 & 1/2 & 1/2 \end{pmatrix} \begin{matrix} \\ L_2 \leftarrow L_2 / -3 \\ L_3 \leftarrow L_2 / 2 \end{matrix}$$

$${}^2L \begin{pmatrix} 1 & 1 & 0 & | & 0 & -1/2 & 1/2 \\ 0 & 1 & 0 & | & -1/3 & -1/3 & 1/3 \\ 0 & 0 & 1 & | & 0 & 1/2 & 1/2 \end{pmatrix} \begin{matrix} L_1 \leftarrow L_1 - L_3 \\ L_2 \leftarrow L_2 - \frac{2}{3}L_3 \end{matrix}$$

$${}^2L \begin{pmatrix} 1 & 0 & 0 & | & 1/3 & -1/6 & 1/6 \\ 0 & 1 & 0 & | & 1/3 & -1/3 & 1/3 \\ 0 & 0 & 1 & | & 0 & 1/2 & 1/2 \end{pmatrix} \begin{matrix} \\ \\ L_1 \leftarrow L_1 - L_2 \end{matrix}$$

Ainsi, P est inversible

$$\& P^{-1} = \begin{pmatrix} 1/3 & -1/6 & 1/6 \\ -1/3 & -1/3 & 1/3 \\ 0 & 1/2 & 1/2 \end{pmatrix}$$

$$P^{-1} = \frac{1}{6} \begin{pmatrix} 2 & -1 & 1 \\ -2 & -2 & 2 \\ 0 & 3 & 3 \end{pmatrix}$$

* Calcul de $P^{-1}AP$

$$P^{-1}A = \frac{1}{6} \begin{pmatrix} 2 & -1 & 1 \\ -2 & -2 & 2 \\ 0 & 3 & 3 \end{pmatrix} \begin{pmatrix} 11 & -5 & 5 \\ -5 & 3 & -3 \\ 5 & -3 & 3 \end{pmatrix}$$

$$= \frac{1}{6} \begin{pmatrix} 32 & -16 & 16 \\ -2 & -2 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$P^{-1}A = \frac{1}{3} \begin{pmatrix} 16 & -8 & 8 \\ -1 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$P^{-1}AP = \frac{1}{3} \begin{pmatrix} 16 & -8 & 8 \\ -1 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2 & -1 & 0 \\ -1 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \frac{1}{3} \begin{pmatrix} 48 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$P^{-1}AP = \begin{pmatrix} 16 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$