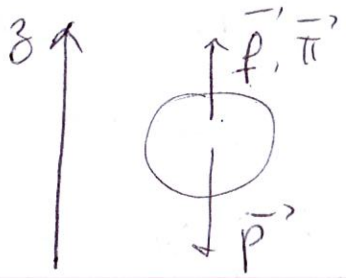
Exo 3

1) $\Sigma = \{\text{particule}\}$



BDF : $\vec{P} = -mg\,\vec{u_z}$

$\vec{f} = +6\pi R \eta v\,\vec{u_z}$

$\vec{\Pi} = \dfrac{m\mu' g}{\mu}\,\vec{u_z}$

PFD : $m\vec{a} = \vec{P} + \vec{f} + \vec{\Pi}$

$m\ddot{z} = 6\pi R\eta \dot{z} + \dfrac{m\mu' g}{\mu} - mg$

En stationnaire ~~$v_0 \Rightarrow$~~ $0 = 6\pi\eta R v_0 + \dfrac{m\mu' g}{\mu} - mg$

ie $\boxed{v_0 = \left(1 - \dfrac{\mu'}{\mu}\right)\dfrac{mg}{6\pi\eta R}}$

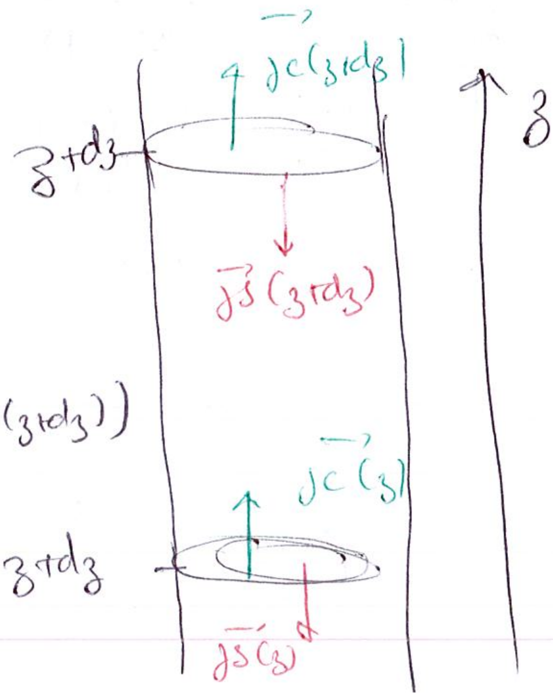
• $\vec{S}\cdot\vec{j_S} = \dfrac{\delta N}{\delta t} = \dfrac{c(z)\,S\,v_0\,dt}{dt}$

$\Leftrightarrow \boxed{j_S = c(z)\,v_0}$

2) Loi de Fick : $\vec{j_c} = -D\,\vec{\mathrm{grad}}(n)$

$\boxed{\vec{j_c} = -D\dfrac{\partial n}{\partial z}\vec{u_z}} > 0$

Car la sédimentation entraine les particules vers le bas i.e. $\dfrac{\partial n}{\partial z} < 0$



3) Soit $N = N_a \times c$

• $dN = N(t+dt) - N(t) = S\,dt\,dz\,N_a\dfrac{\partial c}{\partial t}$

• $\delta N_{éch} = \delta N_{entrant} - \delta N_{sortant}$

$= S\,dt\left(j_c(z) + j_S(z+dz) - j_S(z) - j_c(z+dz)\right)$

$= S\,dt\left(-\dfrac{\partial j_c}{\partial z} + \dfrac{\partial j_S}{\partial z}\right)dz$

Conservation de la matière

$$\boxed{\frac{dc}{dt} = \frac{S\,dt\,dz}{Na}\left(\frac{\partial j_S}{\partial z} - \frac{\partial j_c}{\partial z}\right)}$$

4) En régime permanent

$$\frac{S\,dt\,dz}{Na}\left(\frac{\partial j_S}{\partial z} - \frac{\partial j_c}{\partial z}\right) = 0$$

ie $\dfrac{\partial j_S}{\partial z} = \dfrac{\partial j_c}{\partial z}$

$$\frac{dc}{dz} v_0 = -D\frac{d^2c}{dz^2}$$

Soit $X = \dfrac{dc}{dz}$ ie $\dfrac{dX}{dz} + \dfrac{v_0}{D}X = 0$

$$X(z) = Ae^{-z/\delta} \quad \text{avec } \delta = \frac{D}{v_0}$$

$$\Rightarrow \quad c(z) = -\delta A e^{-z/\delta} + B$$

$c(0) = -\delta A + B = C_0 \Leftrightarrow B = C_0 + \delta A$

$j_S(0) = j_c(0) \Leftrightarrow v_0 C_0 = -D\dfrac{\partial c}{\partial z} \Leftrightarrow \dfrac{\partial c}{\partial z} = -\dfrac{v_0 C_0}{D} = A$

Car $\vec{j_S}(0) + \vec{j_c}(0) = \vec{0}$

$. B = C_0 - \dfrac{D}{V_0} \times \dfrac{V_0}{D} C_0 = 0$

$$C(z) = \frac{V_0 C_0 D}{V_0 D} e^{-z/\delta} = \boxed{C_0 e^{-z/\delta} = c(z)}$$

4) a) $h = \dfrac{k_B T}{D}$ ie $[h] = \dfrac{m^2\,kg\,s^{-1}}{s^2\,m^2} = kg\,s^{-1}$

b) Si $T = cst$ alors D cst ie δ cst donc $\begin{cases} z \nearrow \to c(z) \searrow \\ z \searrow \to c(z) \nearrow \end{cases}$

c) Si $z = cst$ alors $\begin{cases} T \nearrow,\ D \nearrow,\ \delta \nearrow,\ z/\delta \searrow\ e^{-z/\delta} \nearrow \\ T \searrow,\ D \searrow,\ \delta \searrow,\ z/\delta \nearrow\ e^{-z/\delta} \searrow \end{cases}$

Correction TD : TH02

Exercice 1 : (suite)

2.

Pour Σ_1' = récipient 1

On avait

$$V_1 \frac{dC_1}{dt} = \frac{-DN_A S}{L} \Delta c \quad (1)$$

Pour Σ_2 : récipient 2 :

Entre t et $t+dt$:

$$dn_2 = n_2(t+dt) - n_2(t)$$
$$= V_2 C_2(t+dt) - V_2 C_2(t)$$

d'où $$V_2 \frac{dC_2}{dt} = \frac{DN_A S}{L} \Delta c \quad (2)$$

$$\frac{(2)}{V_2} - \frac{(1)}{V_1} : \frac{dC_2}{dt} - \frac{dC_1}{dt} = \frac{DN_A S}{L V_1} \Delta c + \frac{DN_A S}{L V_2} \Delta c$$

$$\Leftrightarrow \frac{d\Delta c}{dt} = -\Delta c \left(\frac{DN_A S}{L V_1} + \frac{DN_A S}{L V_2} \right)$$

$$\Leftrightarrow \frac{d\Delta c}{dt} + \Delta c \left(\frac{DN_A S (V_1 + V_2)}{L V_1 V_2} \right) = 0$$

On pose $\tau = \dfrac{L V_1 V_2}{DN_A S (V_1 + V_2)}$

On a alors : $\Delta c(t) = A \exp\left(\frac{-t}{\tau}\right)$

On prend les CI :

$\Delta c(0) = A = c_{0_1} - c_{0_2}$

Donc $\Delta c(t) = (c_{0_1} - c_{0_2}) \exp\left(\frac{-t}{\tau}\right)$

3.

On a $\Delta c(t_0) = \underbrace{(c_{0_1} - c_{0_2})}_{\Delta c_0} \exp\left(\frac{-t}{\tau}\right) = \frac{\Delta c_0}{10}$

Donc $t_0 = \tau \ln(10)$