M2 | 4.4 : $\mathcal{E}_c = m\dfrac{R^2\dot{\theta}^2}{2}$, $\mathcal{E}_{pp} = mgR(1-\cos\theta)$,

$\vec{F}_{ie} = m r w^2 \vec{u}_r \Rightarrow \mathcal{E}_{p_{ie}} = -m\dfrac{r^2w^2}{2} = -m\,(R\sin\theta)^2\dfrac{w^2}{2}$

$$\Rightarrow \begin{cases} \mathcal{E}_p = mgR(1-\cos\theta) - m\dfrac{R^2w^2}{2}\sin^2\theta \\ \dfrac{d\mathcal{E}_p}{d\theta} = mgR\sin\theta - m\dfrac{R^2w^2}{2}\cdot 2\sin\theta\cdot\cos\theta \\ \dfrac{d^2\mathcal{E}_p}{d\theta^2} = mgR\cos\theta - mR^2w^2(\cos^2\theta - \sin^2\theta) \end{cases}$$

* À l'équilibre dans R' : $\dfrac{d\mathcal{E}_p}{d\theta} = 0$

$$mgR\sin\theta_{eq}\left(1 - \frac{Rw^2}{g}\cos\theta_{eq}\right) = 0$$

Donc : $\boxed{\theta_{eq}=0}$, $\boxed{\theta_{eq}=\pi}$, et si $Rw^2 > g$, $\theta_{eq} = \pm\arccos\left(\dfrac{g}{Rw^2}\right)$

* Stabilité de $\theta_{eq}=0$

$\dfrac{d^2\mathcal{E}_p}{d\theta^2}(\theta_{eq}=0) = mgR - mR^2w^2$ >0 si $g > Rw^2$ (stable) ; <0 si $g < Rw^2$ (instable)

* stab. de $\theta_{eq} = \pi$

$\dfrac{d^2\mathcal{E}_p}{d\theta^2}(\theta=\pi) = -mgR - mR^2w^2 < 0$ instable

* Si $Rw^2 > g$, stabilité de $\cos\theta_{eq} = \dfrac{g}{Rw^2}$:

$$\frac{d^2\mathcal{E}_p}{d\theta^2}(\theta=\theta_{eq}) = mgR\frac{g}{Rw^2} - mR^2w^2\left(\left(\frac{g}{Rw^2}\right)^2 - \left(1-\left(\frac{g}{Rw^2}\right)^2\right)\right)$$

$$= \frac{mg^2}{w^2} - \frac{2mg^2}{w^2} + mR^2w^2 = mR^2w^2\underbrace{\left(1 - \frac{g^2}{R^2w^4}\right)}_{>0}$$

stable