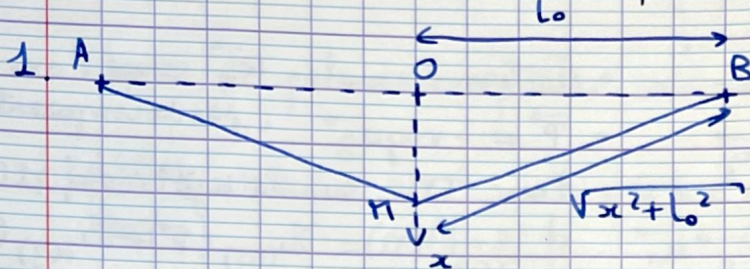


Exercice 4 (TD1 mécanique)



Système { funambule (m) } conservatif.

$$E_{p, \text{grav}}(x) = -mgx$$

$$E_{p, \text{el}}(x) = 2 \times \frac{1}{2} (2k) (\sqrt{x^2 + L_0^2} - L_0)^2$$

$$= 2kL_0^2 \left(\sqrt{1 + \left(\frac{x}{L_0}\right)^2} - 1 \right)$$

$$\approx 2kL_0^2 \left(\left(\frac{x}{L_0}\right)^2 \right)^2 \quad (\text{DL à l'ordre 2 car } x \ll L_0)$$

$$= \frac{kx^4}{2L_0^2}$$

$$\text{donc } E_p(x) = \frac{kx^4}{2L_0^2} - mgx$$

$$\text{À l'équilibre : } \frac{dE_p}{dx}(x_e) = 0$$

$$\text{donc } \frac{2kx_e^3}{L_0^2} = mg \quad \text{donc } x_e = \left(\frac{mgL_0^2}{2k} \right)^{1/3}$$

2. Au voisinage de l'équilibre : $\neq 0$

$$E_m \approx \frac{1}{2} m \dot{x}^2 + E_p(x_e) + \frac{1}{2} \frac{d^2 E_p}{dx^2}(x_e) (x - x_e)^2$$

$$\text{donc } \frac{dE_m}{dt} = 0 = m \ddot{x} + \ddot{x} (x - x_e) \frac{d^2 E_p}{dx^2}(x_e)$$

En posant $X = x - x_e$, on a :

$$\ddot{X} + \frac{1}{m} \frac{d^2 E_p}{dx^2}(x_e) X = 0 \quad \text{d'où } T = 2\pi \times \sqrt{m} \times \frac{1}{\sqrt{\frac{d^2 E_p}{dx^2}(x_e)}}$$

$$\text{or } \frac{d^2 E_p}{dx^2}(x_e) = \frac{6kx_e^2}{L_0^2} = \frac{6k}{L_0^2} \left(\frac{mgL_0^2}{2k} \right)^{2/3} = 6 \times \left(\frac{mg}{2} \right)^{2/3} \times \frac{k^{1/3}}{L_0^{2/3}}$$

$$\text{donc } T = 2\pi \times \sqrt{m} \times \left(\frac{2L_0}{mg} \right)^{1/3} \times \frac{1}{\sqrt{6} k^{1/6}} = \frac{2\pi}{\sqrt{6}} \sqrt{\frac{m}{k}} \times \left(\frac{2L_0 k}{mg} \right)^{1/3} = T$$

