

Cahier de calcul

— réponses —



Margarita philosophica (La perle philosophique), Gregor REISCH (1508)

Cette gravure, extraite d'un manuel d'université de l'époque, représente Arithmetica, allégorie des mathématiques, arbitrant une compétition entre Boèce, qui utilise les chiffres indo-arabes, et Pythagore, qui utilise un boulier.

Ce cahier de calcul a été écrit collectivement.

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Sommaire

| | |
|---|----|
| 1. Fractions..... | 1 |
| 2. Puissances..... | 2 |
| 3. Calcul littéral..... | 3 |
| 4. Racines carrées..... | 4 |
| 5. Expressions algébriques..... | 5 |
| 6. Équations du second degré..... | 6 |
| 7. Exponentielle et Logarithme..... | 7 |
| 8. Trigonométrie..... | 8 |
| 9. Dérivation..... | 10 |
| 10. Primitives..... | 11 |
| 11. Calcul d'intégrales..... | 13 |
| 12. Intégration par parties..... | 14 |
| 13. Changements de variable..... | 15 |
| 14. Intégration des fractions rationnelles..... | 16 |
| 15. Systèmes linéaires..... | 17 |
| 16. Nombres complexes..... | 18 |
| 17. Trigonométrie et nombres complexes..... | 19 |
| 18. Sommes et produits..... | 20 |
| 19. Coefficients binomiaux..... | 21 |
| 20. Manipulation des fonctions usuelles..... | 22 |
| 21. Suites numériques..... | 23 |
| 22. Développements limités..... | 24 |
| 23. Arithmétique..... | 25 |
| 24. Polynômes..... | 26 |
| 25. Décomposition en éléments simples..... | 27 |
| 26. Calcul matriciel..... | 28 |
| 27. Algèbre linéaire..... | 30 |
| 28. Équations différentielles..... | 31 |
| 29. Séries numériques..... | 32 |
| 30. Structures euclidiennes..... | 33 |
| 31. Groupes symétriques..... | 34 |
| 32. Déterminants..... | 35 |
| 33. Fonctions de deux variables..... | 36 |

Fiche n° 1. Fractions

Réponses

| | | | | | |
|-------------|----------------------|-------------|-------------------------|--------------|-----------------------------------|
| 1.1 a)..... | $\frac{4}{5}$ | 1.3 c)..... | $\frac{-10}{3}$ | 1.7 | $\frac{n^3 + n}{n + 1}$ |
| 1.1 b)..... | 2^5 | 1.3 d)..... | 1 000 | 1.8 a)..... | $4 + \frac{5}{6}$ |
| 1.1 c)..... | 3 | 1.4 | $\frac{16}{35}$ | 1.8 b)..... | $1 + \frac{1}{k - 1}$ |
| 1.1 d)..... | $-2 \times 3^{3k-2}$ | 1.5 a)..... | 2 022 | 1.8 c)..... | $3 + \frac{5}{x - 2}$ |
| 1.2 a)..... | $\frac{1}{6}$ | 1.5 b)..... | $\frac{1}{2}$ | 1.9 | 2t |
| 1.2 b)..... | $\frac{7}{15}$ | 1.5 c)..... | 1 | 1.10 a)..... | $\frac{3}{5} > \frac{5}{9}$ |
| 1.2 c)..... | 9 | 1.5 d)..... | 2 | 1.10 b)..... | $\frac{12}{11} > \frac{10}{12}$ |
| 1.2 d)..... | $\frac{1}{9}$ | 1.6 a)..... | $\frac{-1}{n(n + 1)^2}$ | 1.10 c)..... | $\frac{125}{25} = \frac{105}{21}$ |
| 1.3 a)..... | 247 | 1.6 b)..... | $-\frac{ab}{a - b}$ | 1.11 | Non |
| 1.3 b)..... | $\frac{203}{24}$ | 1.6 c)..... | $\frac{3}{2}n$ | 1.12 | A > B |

Fiche n° 2. Puissances

Réponses

2.1 a)..... 10^8

2.1 b)..... 10^{15}

2.1 c)..... 10^2

2.1 d)..... 10^{-2}

2.1 e)..... 10^4

2.1 f)..... 10^{-8}

2.2 a)..... 15^4

2.2 b)..... 5^{-6}

2.2 c)..... 2^7

2.2 d)..... $(-7)^{-2}$

2.2 e)..... 3^5

2.2 f)..... 3^{28}

2.3 a)..... $2^{-4} \cdot 3^{-1}$

2.3 b)..... $2^{21} \cdot 3$

2.3 c)..... 2

2.3 d)..... $2^{38} \cdot 3^{26}$

2.4 a)..... 8

2.4 b)..... 11

2.4 c)..... 3^{10}

2.4 d)..... $2^6 \cdot 5$

2.5 a)..... $\frac{x}{x+1}$

2.5 b)..... $\frac{1}{x-2}$

2.5 c)..... $\frac{2x}{x+1}$

2.5 d)..... $\frac{2}{x-2}$

Fiche n° 3. Calcul littéral

Réponses

3.1 a) $8x^3 - 6x^2 + \frac{3}{2}x - \frac{1}{8}$

3.1 b) $x^5 - 2x^4 + x^3 - x^2 + 2x - 1$

3.1 c) $x^5 - x^3 + x^2 - 1$

3.1 d) $x^5 + 2x^4 + x^3 - x^2 - 2x - 1$

3.1 e) $x^5 - x^3 - x^2 + 1$

3.1 f) $x^4 + x^2 + 1$

3.2 a) $-2 + 12x - 17x^2 + 8x^3 - 3x^4$

3.2 b) $-28 + 21x$

3.2 c) $2 - x + x^3 - x^4 - x^5$

3.2 d) $-1 - 3x - 3x^2 + x^3$

3.2 e) $1 + x^4$

3.2 f) $1 + 2x + 3x^2 + 2x^3 + x^4$

3.3 a) $-6(6x + 7)$

3.3 b) $4(5x + 4)(-5x + 1)$

3.3 c) $2(3x - 4)(10x + 3)$

3.3 d) $-8(x + 1)(x + 16)$

3.4 a) $(x - 1)^2$

3.4 b) $(x + 2)^2$

3.4 c) $(x + 1)(x + 2)$

3.4 d) $3\left(x + \frac{7 - \sqrt{37}}{6}\right)\left(x + \frac{7 + \sqrt{37}}{6}\right)$

3.4 e) $2\left(x + \frac{3 - \sqrt{233}}{4}\right)\left(x + \frac{3 + \sqrt{233}}{4}\right)$

3.4 f) $-5(x - 1)\left(x - \frac{1}{5}\right)$

3.5 a) $(x + y - z)(x + y + z)$

3.5 b) $(14x + 3y)(-12x + 3y)$

3.5 c) $(x + 1)(y + 1)$

3.5 d) $(x - 1)(y - 1)$

3.5 e) $(x + y)(x + 1)^2$

3.5 f) $(a^2 + b^2)(y - 4x^2)(y + 4x^2)$

3.6 a) $(x - 1)(x + 1)(x^2 + 1)$

3.6 b) $-8(x^2 + 1)(x - 4)(x + 4)$

3.6 c) $(x^2 + x + 1)(x^2 - x + 1)$

3.6 d) $(a^2 + b^2)(c^2 + d^2)$

3.6 e) $(a^2 + b^2 + c^2 + d^2)(p^2 + q^2 + r^2 + s^2)$

Fiche n° 4. Racines carrées

Réponses

| | | | | | |
|--------------|------------------------------------|--------------|---|--------------|--|
| 4.1 a)..... | $\boxed{5}$ | 4.3 a) | $\boxed{2 - \sqrt{2} - \sqrt{3} + \frac{1}{2}\sqrt{6}}$ | 4.5 c)..... | $\boxed{1 + \sqrt{x-1}}$ |
| 4.1 b)..... | $\boxed{\sqrt{3} - 1}$ | 4.3 b) | $\boxed{3 - 2\sqrt{2}}$ | 4.5 d) | $\boxed{\frac{1}{2} \frac{1}{x-1}}$ |
| 4.1 c)..... | $\boxed{-\sqrt{3} + 2}$ | 4.3 c)..... | $\boxed{1 - \sqrt{10} + \sqrt{15}}$ | 4.5 e) | $\boxed{\frac{x(x-2)}{(x-1)\sqrt{x-1}}}$ |
| 4.1 d)..... | $\boxed{\sqrt{7} - 2}$ | 4.3 d).... | $\boxed{\sqrt{15} + \sqrt{10} - \sqrt{6} - 2}$ | 4.5 f) | $\boxed{-4(x-1)^2}$ |
| 4.1 e)..... | $\boxed{\pi - 3}$ | 4.3 e) | $\boxed{-(\sqrt{2} + \sqrt{3})}$ | 4.6 a)..... | $\boxed{\sqrt{2}}$ |
| 4.1 f)..... | $\boxed{ 3 - a }$ | 4.3 f).... | $\boxed{-\frac{3 + \sqrt{2} + \sqrt{3} + \sqrt{6}}{2}}$ | 4.6 b) | $\boxed{2\sqrt{2}}$ |
| 4.2 a) | $\boxed{20}$ | 4.3 g)..... | $\boxed{2\sqrt{2}}$ | 4.7 a)..... | $\boxed{-11 + 5\sqrt{5}}$ |
| 4.2 b) | $\boxed{9 + 4\sqrt{5}}$ | 4.3 h) | $\boxed{50 - 25\sqrt{3}}$ | 4.7 b)..... | $\boxed{1 + \sqrt{2}}$ |
| 4.2 c)..... | $\boxed{1 + \sqrt{3}}$ | 4.4 | $\boxed{\frac{\sqrt{2} + 2 - \sqrt{6}}{4}}$ | 4.7 c)..... | $\boxed{1 + \sqrt{2}}$ |
| 4.2 d)..... | $\boxed{3 + \sqrt{2}}$ | 4.5 a)..... | $\boxed{\frac{x}{\sqrt{x-1}}}$ | 4.7 d)..... | $\boxed{\sqrt{3}}$ |
| 4.2 e)..... | $\boxed{12\sqrt{7}}$ | 4.5 b)..... | $\boxed{x - \sqrt{x^2 - 1}}$ | 4.7 e)..... | $\boxed{1 + \sqrt{5}}$ |
| 4.2 f)..... | $\boxed{12}$ | | | 4.7 f) | $\boxed{\ln(1 + \sqrt{2})}$ |
| 4.2 g)..... | $\boxed{9 - \frac{10}{3}\sqrt{2}}$ | | | 4.8 | $\boxed{1}$ |
| 4.2 h) | $\boxed{10}$ | | | | |

Fiche n° 5. Expressions algébriques

Réponses

5.1 a) $7a^2 + 12a + 7$

5.1 b) $a^2 - a - 1$

5.1 c) $4a^2 - a - 3$

5.1 d) $-a^2 + 1$

5.2 a) $8 + 6i$

5.2 b) $8 - 6i$

5.2 c) $18 - 26i$

5.2 d) $-9 - 46i$

5.3 a) $39 - 18i$

5.3 b) 2197

5.3 c) $-4 + 43i\sqrt{5}$

5.3 d) 1

5.4 a) 3

5.4 b) 1

5.4 c) 1

5.4 d) 0

5.4 e) -1

5.4 f) 31

5.5 a) $a^2 + 2$

5.5 b) $a^3 + 3a$

5.5 c) $a^4 + 4a^2 + 2$

5.6 a) $a^2 - 2b$

5.6 b) $ab - 3c$

5.6 c) $a^3 - 3ab + 3c$

5.6 d) $ab - c$

5.6 e) ac

5.6 f) $-2ac + b^2$

5.7 a) $a^2b - ac - 2b^2$

5.7 b) ... $a^4 - 4a^2b + 4ac + 2b^2$

5.7 c) 0

5.7 d) 1

5.7 e) a

Fiche n° 6. Équations du second degré

Réponses

- 6.1 a) $3, 3$
- 6.1 b) $-1/3, -1/3$
- 6.1 c) $2, -6$
- 6.1 d) $2, 3$
- 6.1 e) $0, \text{ donc } 5$
- 6.1 f) $0, \text{ donc } -3/2$
- 6.1 g) \emptyset
- 6.1 h) $1 \text{ donc } -5$
- 6.1 i) $1 \text{ donc } 8/3$
- 6.1 j) $-1 \text{ donc } -19/5$
- 6.2 a) $6, 7$
- 6.2 b) $-3, -5$
- 6.2 c) $-7, -11$
- 6.2 d) $-3, 11$
- 6.2 e) a, b
- 6.2 f) $a - b, a + b$
- 6.3 a) $2/3$
- 6.3 b) $-2/7$
- 6.3 c) $-1/m$
- 6.3 d) $2m/(m + 3)$
- 6.4 a) $1 \text{ donc } (a - b)/(b - c)$
- 6.4 b) $1 \text{ donc } c(a - b)/(a(b - c))$
- 6.4 c) $m \text{ donc } -(m + a + b)$
- 6.4 d) $m \text{ donc } m(a - b)/(b - c)$
- 6.4 e) $m \text{ donc } ab/m$
- 6.4 f) $a + b \text{ puis } 2ab/(a + b).$
- 6.5 a) $x^2 - 22x + 117 = 0$
- 6.5 b) $x^2 - 6x - 187 = 0$
- 6.5 c) $x^2 - 4x + 1 = 0$
- 6.5 d) $x^2 - 2mx + 3 = 0$
- 6.5 e) $2x^2 - (4m + 1)x + (2m^2 + m - 15) = 0$
- 6.5 f) $m^2x^2 + (m - 2m^2)x + (m^2 - m - 2) = 0$
- 6.6 a) $m = -3/4 \text{ et } x = 3/4$
- 6.6 b) ... $m = -1 \text{ et } x = -2, \text{ ou } m = 7 \text{ et } x = 2/3$
- 6.6 c) $m = 1 \text{ et } x = -1 \text{ ou } m = -1 \text{ et } x = 1$
- 6.7 a) $a = 2 \text{ et } b = 3$
- 6.7 b) $a = -2 \text{ et } b = 1$
- 6.7 c) $a = -3 \text{ et } b = 5$
- 6.7 d) $a = 1/2 \text{ et } b = 8$
- 6.7 e) $a = 1 \text{ et } b = 3\sqrt{7}$
- 6.8 a) $] -\infty, 1] \cup [\sqrt{2}, +\infty[$
- 6.8 b) $[-3, 5]$
- 6.8 c) $] -\infty, -1] \cup [2/3, +\infty[$
- 6.8 d) $] -\infty, -1/2[\cup [4, +\infty[$

Fiche n° 7. Exponentielle et Logarithme

Réponses

| | | | | | |
|-------------|----------------------------------|-------------|-------------------|--------------|-------------------------------|
| 7.1 a)..... | $4 \ln 2$ | 7.5 b)..... | $\frac{1}{2}$ | 7.8 a)..... | \mathbb{R} |
| 7.1 b)..... | $9 \ln 2$ | 7.5 c)..... | $\frac{1}{3}$ | 7.8 b)..... | ok |
| 7.1 c)..... | $-3 \ln 2$ | 7.5 d)..... | $\frac{1}{9}$ | 7.8 c)..... | 1 |
| 7.1 d)..... | $\frac{1}{2} \ln 2$ | 7.5 e)..... | $-\frac{1}{2}$ | 7.8 d)..... | -1 |
| 7.1 e)..... | $3 \ln 2$ | 7.5 f)..... | $\frac{3}{2}$ | 7.9 a)..... | $x + \ln 2$ |
| 7.1 f)..... | $2 \ln 2 + 2 \ln 3$ | 7.6 a)..... | -2 | 7.9 b)..... | $\frac{e^x}{\sqrt{1+x}}$ |
| 7.2 a)..... | $-\ln 3 - 2 \ln 2$ | 7.6 b)..... | $\frac{1}{\ln 2}$ | 7.9 c)..... | $\ln x - 1 $ |
| 7.2 b)..... | $2 \ln 3 - 2 \ln 2$ | 7.6 c)..... | -17 | 7.9 d)..... | $-\frac{1}{1+x}$ |
| 7.2 c)..... | $\ln 3 + 11 \ln 2$ | 7.6 d)..... | 1 | 7.9 e)..... | $e^{x \ln(1+x)}$ |
| 7.2 d)..... | $3 \ln 5 + 2 \ln 2$ | 7.6 e)..... | -1 | 7.10 a)..... | $x \geq \frac{\ln 12 + 5}{3}$ |
| 7.2 e)..... | $-2 \ln 5 + 4 \ln 2$ | 7.6 f)..... | e | 7.10 b)..... | $x \in [0, 1[$ |
| 7.2 f)..... | $2 \ln 5 - 2 \ln 2$ | 7.7 a)..... | impaire | 7.10 c)..... | $x \geq \frac{2}{e}$ |
| 7.3..... | $-2 \ln 2 - 2 \ln 5$ | 7.7 b)..... | impaire | 7.10 d)..... | $x \geq -\frac{1}{12}$ |
| 7.4 a)..... | $\frac{25}{8} \ln(\sqrt{2} - 1)$ | 7.7 c)..... | impaire | 7.10 e)..... | \emptyset |
| 7.4 b)..... | $17 + 12\sqrt{2}$ | 7.7 d)..... | impaire | 7.10 f)..... | $\frac{-13 - \sqrt{273}}{2}$ |
| 7.4 c)..... | 0 | | | | |
| 7.4 d)..... | 0 | | | | |
| 7.5 a)..... | 8 | | | | |

Fiche n° 8. Trigonométrie

Réponses

8.1 a)..... 0

8.1 b)..... 0

8.1 c)..... $-1 - \sqrt{3}$

8.1 d)..... $-\frac{1}{2}$

8.2 a)..... 0

8.2 b)..... $-\sin x$

8.2 c)..... $2 \cos x$

8.2 d)..... $-2 \cos x$

8.3 a)..... $\frac{\sqrt{6} - \sqrt{2}}{4}$

8.3 b)..... $\frac{\sqrt{6} + \sqrt{2}}{4}$

8.3 c)..... $\frac{\sqrt{6} - \sqrt{2}}{4}$

8.3 d)..... $\frac{\sqrt{3} - 1}{\sqrt{3} + 1}$

8.4 a)..... $-\sin x$

8.4 b)..... $\frac{1}{\cos x}$

8.4 c)..... 0

8.4 d)..... $4 \cos^3 x - 3 \cos x$

8.5 a)..... $\frac{\sqrt{2} + \sqrt{2}}{2}$

8.5 b)..... $\frac{\sqrt{2} - \sqrt{2}}{2}$

8.6 a)..... $\tan x$

8.6 b)..... 2

8.6 c)..... $8 \cos^4 x - 8 \cos^2 x + 1$

8.7 a)..... $\left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}$

8.7 a)..... $\left\{ -\frac{\pi}{3}, \frac{\pi}{3} \right\}$

8.7 a)..... $\left\{ \frac{\pi}{3} + 2k\pi, k \in \mathbb{Z} \right\} \cup \left\{ -\frac{\pi}{3} + 2k\pi, k \in \mathbb{Z} \right\}$

8.7 b)..... $\left\{ \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$

8.7 b)..... $\left\{ \frac{-2\pi}{3}, \frac{-\pi}{3} \right\}$

8.7 b)..... $\left\{ \frac{4\pi}{3} + 2k\pi, k \in \mathbb{Z} \right\} \cup \left\{ \frac{5\pi}{3} + 2k\pi, k \in \mathbb{Z} \right\}$

8.7 c)..... $\left\{ \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$

8.7 c)..... $\left\{ -\frac{5\pi}{6}, -\frac{\pi}{6} \right\}$

8.7 c)..... $\left\{ \frac{7\pi}{6} + 2k\pi, k \in \mathbb{Z} \right\} \cup \left\{ \frac{11\pi}{6} + 2k\pi, k \in \mathbb{Z} \right\}$

8.7 d)..... $\left\{ \frac{\pi}{4}, \frac{5\pi}{4} \right\}$

8.7 d)..... $\left\{ -\frac{3\pi}{4}, \frac{\pi}{4} \right\}$

8.7 d)..... $\left\{ \frac{\pi}{4} + k\pi, k \in \mathbb{Z} \right\}$

8.7 e)..... $\left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$

8.7 e)..... $\left\{ -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4} \right\}$

8.7 e)..... $\left\{ \frac{\pi}{4} + k\frac{\pi}{2}, k \in \mathbb{Z} \right\}$

8.7 f)..... $\left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$

8.7 f)..... $\left\{ -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6} \right\}$

8.7 f)..... $\left\{ \frac{\pi}{6} + k\pi, k \in \mathbb{Z} \right\} \cup \left\{ \frac{5\pi}{6} + k\pi, k \in \mathbb{Z} \right\}$

8.7 g)..... $\left\{ \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12} \right\}$

8.7 g)..... $\left\{ -\frac{11\pi}{12}, -\frac{\pi}{12}, \frac{\pi}{12}, \frac{11\pi}{12} \right\}$

8.7 g)..... $\left\{ \frac{\pi}{12} + k\pi, k \in \mathbb{Z} \right\} \cup \left\{ \frac{11\pi}{12} + k\pi, k \in \mathbb{Z} \right\}$

8.7 h)..... $\left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2} \right\}$

8.7 h)..... $\left\{ -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6} \right\}$

- 8.7 h) $\left\{ \frac{\pi}{6} + k \frac{2\pi}{3}, k \in \mathbb{Z} \right\}$
- 8.7 i) $\left\{ \frac{\pi}{7}, \frac{13\pi}{7} \right\}$
- 8.7 i) $\left\{ -\frac{\pi}{7}, \frac{\pi}{7} \right\}$
- 8.7 i) $\left\{ \frac{\pi}{7} + 2k\pi, k \in \mathbb{Z} \right\} \cup \left\{ -\frac{\pi}{7} + 2k\pi, k \in \mathbb{Z} \right\}$
- 8.7 j) $\left\{ \frac{5\pi}{14}, \frac{9\pi}{14} \right\}$
- 8.7 j) $\left\{ \frac{5\pi}{14}, \frac{9\pi}{14} \right\}$
- 8.7 j) $\left\{ \frac{5\pi}{14} + 2k\pi, k \in \mathbb{Z} \right\} \cup \left\{ \frac{9\pi}{14} + 2k\pi, k \in \mathbb{Z} \right\}$
- 8.8 a) $\left[0, \frac{3\pi}{4} \right] \cup \left[\frac{5\pi}{4}, 2\pi \right]$
- 8.8 a) $\left[-\frac{3\pi}{4}, \frac{3\pi}{4} \right]$
- 8.8 b) $\left[\frac{\pi}{3}, \frac{5\pi}{3} \right]$
- 8.8 b) $\left[-\pi, -\frac{\pi}{3} \right] \cup \left[\frac{\pi}{3}, \pi \right]$
- 8.8 c) $\left[0, \frac{\pi}{6} \right] \cup \left[\frac{5\pi}{6}, 2\pi \right]$
- 8.8 c) $\left[-\pi, \frac{\pi}{6} \right] \cup \left[\frac{5\pi}{6}, \pi \right]$
- 8.8 d) $\left[0, \frac{\pi}{6} \right] \cup \left[\frac{5\pi}{6}, \frac{7\pi}{6} \right] \cup \left[\frac{11\pi}{6}, 2\pi \right]$
- 8.8 d) $\left[-\pi, -\frac{5\pi}{6} \right] \cup \left[-\frac{\pi}{6}, \frac{\pi}{6} \right] \cup \left[\frac{5\pi}{6}, \pi \right]$
- 8.8 e) $\left[\frac{\pi}{4}, \frac{\pi}{2} \right] \cup \left[\frac{5\pi}{4}, \frac{3\pi}{2} \right]$
- 8.8 e) $\left[-\frac{3\pi}{4}, -\frac{\pi}{2} \right] \cup \left[\frac{\pi}{4}, \frac{\pi}{2} \right]$
- 8.8 f) $\left[\frac{\pi}{4}, \frac{\pi}{2} \right] \cup \left[\frac{\pi}{2}, \frac{3\pi}{4} \right] \cup \left[\frac{5\pi}{4}, \frac{3\pi}{2} \right] \cup \left[\frac{3\pi}{2}, \frac{7\pi}{4} \right]$
- 8.8 f) $\left[-\frac{3\pi}{4}, -\frac{\pi}{2} \right] \cup \left[-\frac{\pi}{2}, -\frac{\pi}{4} \right] \cup \left[\frac{\pi}{4}, \frac{\pi}{2} \right] \cup \left[\frac{\pi}{2}, \frac{3\pi}{4} \right]$
- 8.8 g) $\left[0, \frac{3\pi}{4} \right] \cup \left[\frac{7\pi}{4}, 2\pi \right]$
- 8.8 g) $\left[-\frac{\pi}{4}, \frac{3\pi}{4} \right]$
- 8.8 h) $\left[0, \frac{3\pi}{8} \right] \cup \left[\frac{7\pi}{8}, \frac{11\pi}{8} \right] \cup \left[\frac{15\pi}{8}, 2\pi \right]$
- 8.8 h) $\left[-\pi, -\frac{5\pi}{8} \right] \cup \left[-\frac{\pi}{8}, \frac{3\pi}{8} \right] \cup \left[\frac{7\pi}{8}, \pi \right]$

Fiche n° 9. Dérivation

Réponses

9.1 a) $6x^2 + 2x - 11$

9.1 b) $5x^4 - 6x^2 + 4x - 15$

9.1 c) $(2x^2 - 2x + 10) \exp(2x)$

9.1 d) $(6x - 1) \ln(x - 2) + \frac{3x^2 - x}{x - 2}$

9.2 a) $5(x^2 - 5x)^4(2x - 5)$

9.2 b) $4(2x^3 + 4x - 1)(3x^2 + 2)$

9.2 c) $8 \cos^2(x) - 6 \cos(x) \sin(x) - 4$

9.2 d) $-3(3 \cos(x) - \sin(x))^2(3 \sin(x) + \cos(x))$

9.3 a) $\frac{2x}{x^2 + 1}$

9.3 b) $\frac{1}{x \ln(x)}$

9.3 c) $(-2x^2 + 3x - 1) \exp(x^2 + x)$

9.3 d) $6 \cos(2x) \exp(3 \sin(2x))$

9.4 a) $\frac{6x}{(x^2 + 1)^2} \cos\left(\frac{2x^2 - 1}{x^2 + 1}\right)$

9.4 b) $\frac{2x^2 + 2x - 8}{(x^2 + 4)^2} \sin\left(\frac{2x + 1}{x^2 + 4}\right)$

9.4 c) $\frac{\cos(x)}{2\sqrt{\sin(x)}}$

9.4 d) $\frac{\cos(\sqrt{x})}{2\sqrt{x}}$

9.5 a) $\frac{(2x + 3)(2 \sin(x) + 3) - (x^2 + 3x) \times 2 \cos(x)}{(2 \sin(x) + 3)^2}$

9.5 b) $\frac{2 - 3x}{2\sqrt{x}(3x + 2)^2}$

9.5 c) $-\frac{2(x^2 + 1) \sin(2x + 1) + x \cos(2x + 1)}{(x^2 + 1)^2}$

9.5 d) $\frac{(4x + 3) \ln(x) - 2x - 3}{(\ln(x))^2}$

9.6 a) $2x \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)$

9.6 b) $\frac{9}{(9 - x^2)\sqrt{9 - x^2}}$

9.6 c) $\frac{1}{1 - x^2}$

9.6 d) $\frac{x \cos(x) - \sin(x)}{x \sin(x)}$

9.7 a) $\frac{10x - 5}{(3 - x)^2(2 + x)^2}$

9.7 b) $\frac{2}{x + 1} \left(x + \frac{1 + \sqrt{3}}{2}\right) \left(x + \frac{1 - \sqrt{3}}{2}\right)$

9.7 c) $\frac{2x^2 + 2x + 5}{(x + 2)(x - 1)^2}$

9.7 d) $\frac{x^2}{(x + 1)^2}$

9.7 e) $\frac{2}{x(1 - \ln(x))^2}$

Fiche n° 10. Primitives

Réponses

| | | | |
|---------------|---|---------------|--|
| 10.1 a) | $\ln t + 1 $ | 10.5 c) | $-\ln \cos t $ |
| 10.1 b) | $-\frac{3}{t + 2}$ | 10.5 d) | $-\ln 1 - \sin t $ |
| 10.1 c) | $-\frac{3}{2(t + 2)^2}$ | 10.5 e) | $-2 \cos \sqrt{t}$ |
| 10.1 d) | $-\frac{\cos(4t)}{4}$ | 10.5 f) | $\frac{1}{\pi} \sin(\pi \ln t)$ |
| 10.2 a) | $\frac{2}{3}(1 + t)^{\frac{3}{2}} - \frac{3}{4}t^{\frac{4}{3}}$ | 10.5 g) | $\tan t - t$ |
| 10.2 b) | $\frac{1}{2}e^{2t+1}$ | 10.5 h) | $\frac{1}{2} \tan^2 t + \ln \cos t $ |
| 10.2 c) | $\frac{1}{2} \text{Arcsin}(2t)$ | 10.5 i) | $\frac{1}{4} \tan^4 t$ |
| 10.2 d) | $\frac{1}{3} \text{Arctan}(3t)$ | 10.5 j) | $2\sqrt{\tan t}$ |
| 10.3 a) | $\frac{2}{3} \ln 1 + t^3 $ | 10.5 k) | $\frac{1}{\tan t}$ |
| 10.3 b) | $\frac{1}{6}(1 + 2t^2)^{\frac{3}{2}}$ | 10.5 l) | $\frac{1}{2} \frac{1}{(1 - \sin t)^2}$ |
| 10.3 c) | $-\sqrt{1 - t^2}$ | 10.5 m) | $\frac{1}{2} \text{Arctan}(2t)$ |
| 10.3 d) | $\frac{3}{4}(1 + 7t^2)^{\frac{2}{3}}$ | 10.5 n) | $\text{Arctan}(e^t)$ |
| 10.3 e) | $\frac{1}{6} \ln(1 + 3t^2)$ | 10.5 o) | $\frac{1}{2} (\text{Arcsin}(t))^2$ |
| 10.3 f) | $-\frac{1}{(1 + 3t^2)^2}$ | 10.5 p) | $\ln \text{Arcsin}(t) $ |
| 10.4 a) | $\frac{1}{4} \ln^4 t$ | 10.6 a) | $\frac{t}{2} + \frac{\sin(2t)}{4}$ |
| 10.4 b) | $2\sqrt{\ln t}$ | 10.6 b) | $-\frac{\cos(4t)}{8} - \frac{\cos(2t)}{4}$ |
| 10.4 c) | $\frac{2}{(3 - e^{2t})^2}$ | 10.6 c) | $-\cos t + \frac{1}{3} \cos^3 t$ |
| 10.4 d) | $-\frac{2}{3t^{\frac{3}{2}}}$ | 10.6 d) | $\ln(1 + \sin^2 t)$ |
| 10.4 e) | $\ln 1 - e^{-t} + e^t $ | 10.6 e) | $\ln \tan t $ |
| 10.4 f) | $-e^{\frac{1}{t}}$ | 10.6 f) | $-\cot t + \tan t$ |
| 10.5 a) | $-\frac{1}{3} \cos^3 t$ | 10.6 g) | $\frac{1}{4} \ln \tan 2t $ |
| 10.5 b) | $e^{\sin t}$ | 10.7 a) | $t + \ln t - \frac{1}{t}$ |
| | | 10.7 b) | $\ln t - \frac{1}{2t^2}$ |

| | | | |
|---------------|---|---------------|---|
| 10.7 c) | $t + \frac{t^3}{3} + \frac{t^5}{5}$ | 10.8 h) .. | $-\frac{3t^2 - 2t - 3}{(t^2 + 1)^2}$ puis $\frac{3}{2} \ln(t^2 + 1) - \text{Arctan}(t)$ |
| 10.7 d) | $t - \frac{t^2}{2} + \frac{t^3}{3}$ | 10.8 i) | $\cos t(3 \cos^2 t - 2)$ puis $-\frac{1}{3} \cos^3 t$ |
| 10.7 e) | $t - 2 \ln t + 1 $ | 10.8 j) | $\sinh(t)^2 + \cosh^2(t)$ puis $\frac{1}{2} \sinh^2(t)$ |
| 10.7 f) | $t - \frac{t^2}{2} + \frac{t^3}{3} - \ln t + 1 $ | 10.8 k) | $-\frac{2t \sin \frac{1}{t} + \cos \frac{1}{t}}{t^4}$ puis $\cos \frac{1}{t}$ |
| 10.7 g) | $\frac{1}{2} \ln(1 + t^2) - \text{Arctan}(t)$ | 10.8 l) | $\frac{2e^t}{(2 + e^t)^2}$ puis $\ln(2 + e^t)$ |
| 10.7 h) | $\ln t + 1 + \frac{1}{t + 1}$ | 10.8 m) | $\frac{2 \cos t + 3}{(2 + 3 \cos t)^2}$ puis $-\frac{1}{3} \ln 2 + 3 \cos t $ |
| 10.8 a) | $2(t - 1)$ puis $\frac{1}{3}t^3 - t^2 + 5t$ | 10.8 n) | $\frac{1}{(1 - t^2)^{3/2}}$ puis $-\sqrt{1 - t^2}$ |
| 10.8 b) | $-\frac{1}{t^2} \left(\frac{2}{t} + 1 \right)$ puis $-\frac{1}{t} + \ln t $ | 10.8 o) | $2 \frac{3 \cos^2 t - 1}{(1 + \cos^2 t)^2}$ puis $-\ln(1 + \cos^2(t))$ |
| 10.8 c) | $\frac{1}{2\sqrt{t}} + \frac{3}{t^4}$ puis $\frac{2}{3}t^{\frac{3}{2}} + \frac{1}{2t^2}$ | 10.8 p) | $(1 - 2t^2)e^{-t^2}$ puis $-\frac{1}{2}e^{-t^2}$ |
| 10.8 d) | $-\frac{4}{t^5} - \frac{3}{2} \frac{1}{t^{5/2}}$ puis $-\frac{1}{3} \frac{1}{t^3} - \frac{2}{\sqrt{t}}$ | 10.8 q) | $\frac{\ln t - 2}{t^2}$ puis $\ln t - \frac{1}{2} \ln^2 t$ |
| 10.8 e) | $2e^{2t} - 3e^{-3t}$ puis $\frac{1}{2}e^{2t} - \frac{1}{3}e^{-3t}$ | 10.8 r) | $\frac{1 + \ln t}{t^2 \ln^2 t}$ puis $\ln \ln t $ |
| 10.8 f) | $3e^{3t-2}$ puis $\frac{1}{3}e^{3t-2}$ | 10.8 s) | $\frac{\cos \ln t - \sin \ln t}{t^2}$ puis $-\cos(\ln t)$ |
| 10.8 g) | $-\frac{t(t^3 + 2)}{(t - 1)^2(t^2 + t + 1)^2}$ puis $\frac{1}{3} \ln(t^3 - 1)$ | 10.8 t) | $-\frac{e^t(e^{2t} - 1)}{(1 + e^{2t})^2}$ puis $\text{Arctan}(e^t)$ |

Fiche n° 11. Calcul d'intégrales

Réponses

| | | | | | | | |
|--------------|-----------------|--------------|---|--------------|---|--------------|-----------------------------|
| 11.1 a)..... | Positif | 11.3 e)..... | $-\frac{1}{30}$ | 11.5 e)..... | 6 | 11.7 c)..... | e^2 |
| 11.1 b)..... | Négatif | 11.3 f)..... | $-\frac{2}{101}$ | 11.5 f)..... | $\frac{1}{2} - \frac{\sqrt{3}}{2}$ | 11.7 d)..... | $3e - 4$ |
| 11.1 c)..... | Positif | 11.4 a)..... | 0 | 11.6 a)..... | 0 | 11.7 e)..... | $-\frac{1}{3}$ |
| 11.2 a)..... | 14 | 11.4 b)..... | 1 | 11.6 b)..... | 0 | 11.7 f)..... | $\frac{5}{8}$ |
| 11.2 b)..... | 50 | 11.4 c)..... | $\frac{1}{2}$ | 11.6 c)..... | $\ln\left(\frac{2}{\sqrt{3}}\right)$ | 11.8 a)..... | 0 |
| 11.2 c)..... | $\frac{147}{2}$ | 11.4 d)..... | 18 | 11.6 d)..... | $\frac{1}{384}$ | 11.8 b)..... | $\frac{\pi}{4}$ |
| 11.2 d)..... | -54 | 11.4 e)..... | $e^2 - e^{-3}$ | 11.6 e)..... | $\frac{1}{2}\left(1 - \frac{1}{e}\right)$ | 11.8 c)..... | $\frac{99}{\ln 10}$ |
| 11.2 e)..... | 0 | 11.4 f)..... | $-\ln 3$ | 11.6 f)..... | $\frac{7}{48}$ | 11.8 d)..... | $\frac{e - \frac{1}{e}}{2}$ |
| 11.2 f)..... | $\frac{5}{2}$ | 11.5 a)..... | 78 | 11.7 a)..... | $\frac{1}{2} - \frac{1}{e+1}$ | 11.8 e)..... | $\frac{2}{3}$ |
| 11.3 a)..... | 8 | 11.5 b)..... | $2(e^3 - 1)$ | 11.7 b)..... | $\frac{17}{2}$ | 11.8 f)..... | $\frac{2\pi}{9}$ |
| 11.3 b)..... | -2 | 11.5 c)..... | $\frac{1}{\pi} \ln\left(1 + \frac{\pi}{2}\right)$ | | | | |
| 11.3 c)..... | $\frac{8}{3}$ | 11.5 d)..... | $\frac{\sqrt{2}}{6}$ | | | | |
| 11.3 d)..... | 0 | | | | | | |

Fiche n° 12. Intégration par parties

Réponses

12.1 a) $\frac{\pi}{2} - 1$

12.1 b) $\frac{5}{2}\text{ch}(2) - \frac{1}{2}\text{sh}(2) - \frac{3}{2}$

12.1 c) 4

12.1 d) $\frac{(\ln(2))^2 2^{\ln(2)} - 2 \ln(2) - 2^{\ln(2)} + 2}{(\ln(2))^2}$

12.1 e) 1

12.1 f) $2 \ln 2 - \frac{3}{4}$

12.1 g) $\ln(2) - 2 + \frac{\pi}{2}$

12.1 h) $\frac{\pi}{4} - \frac{1}{2}$

12.1 i) $\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$

12.1 j) $-\frac{2\sqrt{2}}{3} + \frac{4}{3}$

12.1 k) $\frac{4}{3}\sqrt{2}\ln(2) - \frac{8}{9}\sqrt{2} + \frac{4}{9}$

12.1 l) $\frac{\pi}{4} - \frac{1}{2}\ln 2 - \frac{\pi^2}{32}$

12.2 a) $\begin{cases} \mathbb{R} \rightarrow \mathbb{R} \\ x \mapsto (-x+2)e^x \end{cases}$

12.2 b) $\begin{cases} \mathbb{R}_+^* \rightarrow \mathbb{R} \\ x \mapsto -\frac{1+\ln x}{x} \end{cases}$

12.2 c) $\begin{cases} \mathbb{R} \rightarrow \mathbb{R} \\ x \mapsto x \arctan(x) - \frac{1}{2} \ln(1+x^2) \end{cases}$

12.2 d) $\begin{cases} \mathbb{R} \rightarrow \mathbb{R} \\ x \mapsto x \text{sh}(x) - \text{ch}(x) \end{cases}$

12.3 a) $\frac{5}{2} - e^2$

12.3 b) $\frac{e^{\frac{\pi}{2}} + 1}{2}$

12.4 a) ... $\begin{cases} \mathbb{R} \rightarrow \mathbb{R} \\ x \mapsto \frac{1}{2}(-\cos(x)\text{sh}(x) + \sin(x)\text{ch}(x)) \end{cases}$

12.4 b) $\begin{cases} \mathbb{R}_+^* \rightarrow \mathbb{R} \\ x \mapsto x \ln^2 x - 2x \ln x + 2x \end{cases}$

12.4 c) $\begin{cases} \mathbb{R}_+^* \rightarrow \mathbb{R} \\ x \mapsto x^3 \left(\frac{1}{3} \ln^2 x - \frac{2}{9} \ln x + \frac{2}{27} \right) \end{cases}$

12.4 d) .. $\begin{cases}]-1, 1[\rightarrow \mathbb{R} \\ x \mapsto \frac{1}{2} e^{\arccos(x)} (x - \sqrt{1-x^2}) \end{cases}$

Fiche n° 13. Changements de variable

Réponses

| | | | |
|--------------|--|--------------|--|
| 13.1 a)..... | $\frac{\pi}{2}$ | 13.2 e)..... | $\frac{\pi}{12}$ |
| 13.1 b)..... | $\frac{\pi}{6}$ | 13.2 f)..... | $\frac{1}{2} \ln \frac{5}{2}$ |
| 13.1 c)..... | $2 \arctan(e) - \frac{\pi}{2}$ | 13.3 a)..... | $2e^2$ |
| 13.1 d)..... | $\frac{1}{4}$ | 13.3 b)..... | $-2((\sqrt{3}-1) \ln(\sqrt{3}-1) - 4 + 2\sqrt{3})$ |
| 13.1 e)..... | $\frac{1}{12}$ | 13.4 a)..... | $\left\{ \begin{array}{l}]0, \frac{\pi}{2}[\rightarrow \mathbb{R} \\ x \mapsto \tan x + \ln \tan(x) \end{array} \right.$ |
| 13.1 f)..... | $2 \ln\left(\frac{3}{2}\right)$ | 13.4 b)..... | $\left\{ \begin{array}{l} \mathbb{R} \rightarrow \mathbb{R} \\ x \mapsto \frac{x}{2} - \frac{e^{-2x}}{4} \end{array} \right.$ |
| 13.2 a)..... | $\frac{\pi}{3\sqrt{3}}$ | 13.4 c)..... | $\left\{ \begin{array}{l} \mathbb{R}_+^* \rightarrow \mathbb{R} \\ x \mapsto 2 \arctan(\sqrt{e^x - 1}) \end{array} \right.$ |
| 13.2 b)..... | $\frac{1}{2} \ln\left(\frac{2e+1}{3}\right)$ | 13.4 d)..... | $\left\{ \begin{array}{l} \mathbb{R}_+^* \rightarrow \mathbb{R} \\ x \mapsto \frac{3}{2} \ln(x^{\frac{2}{3}} + 1) \end{array} \right.$ |
| 13.2 c)..... | $\frac{\pi}{2}$ | 13.4 e)..... | $\left\{ \begin{array}{l}]1, +\infty[\rightarrow \mathbb{R} \\ x \mapsto \arctan \sqrt{x^2 - 1} \end{array} \right.$ |
| 13.2 d)..... | $\frac{1}{4} + \frac{\pi}{8}$ | | |

Fiche n° 14. Intégration des fractions rationnelles

Réponses

14.1 a) $\ln\left(\frac{3}{2}\right)$

14.1 b) $\frac{1}{2}\ln\left(\frac{5}{3}\right)$

14.2 a) $2\ln\frac{9}{10}$

14.2 b) $\ln(a+1)$

14.3 a) $\frac{3}{2} + \ln(2) - \ln(3)$

14.3 b) $-\frac{1}{48} + \frac{51}{64}\ln\frac{21}{19}$

14.4 a) $\ln\left(\frac{7}{3}\right)$

14.4 b) $\ln\frac{33}{28}$

14.5 a) $\ln\left(2\sqrt{\sqrt{2}-1}\right)$

14.5 b) $\frac{1}{2a}\ln\left(\frac{a+1}{2}\right)$

14.6 a) 1 et 2

14.6 b) $A = -1$ et $B = 1$

14.6 c) $2\ln\frac{4}{3}$

14.7 a) $\ln\frac{1}{3}$

14.7 b) $2\ln\frac{4}{3}$

14.7 c) $\frac{1}{2}\ln\frac{3}{2}$

14.7 d) $\frac{1}{4}\ln\frac{1}{5}$

14.8 $\frac{1}{2\sqrt{a}}\ln\left(\frac{\sqrt{a}-a}{a+\sqrt{a}}\right)$

14.9 a) $\frac{a}{a^2+x^2}$

14.9 b) $\frac{1}{a}\arctan\left(\frac{x}{a}\right)$

14.10 a) $\frac{\pi}{4}$

14.10 b) $\frac{\pi}{6\sqrt{3}}$

14.11 $\frac{\pi}{2\sqrt{2}}$

14.12 a) $\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$

14.12 b) $2\left(x - \frac{3}{4}\right)^2 - \frac{1}{8}$

14.12 c) .. $\sqrt{2}\left(x + \frac{1}{4}\right)^2 + \sqrt{2}\frac{15}{16}$

14.12 d) $a\left(x + \frac{a}{2}\right)^2 + \frac{3a^3}{4}$

14.13 a) $\frac{1}{2}$

14.13 b) $\frac{2\pi}{3\sqrt{3}}$

14.13 c) $\frac{2\pi}{3\sqrt{3}}$

14.13 d) $\ln(2)$

14.14 a) $\frac{\pi}{12}$

14.14 b) $\ln\left(\frac{a^2}{a^2-1}\right)$

14.15 $\frac{1}{3}\left(\ln(2) + \frac{\pi}{\sqrt{3}}\right)$

Fiche n° 15. Systèmes linéaires

Réponses

15.1 a) $\{(3, 1)\}$

15.1 b) $\{(7, 2)\}$

15.1 c) $\left\{\left(\frac{1}{3}, \frac{2}{3}\right)\right\}$

15.1 d) $\left\{\left(\frac{\sqrt{2}}{3}, \frac{\sqrt{2}}{2}\right)\right\}$

15.2 a) $\left\{\left(1 - \frac{a}{4}, \frac{-1}{2} + \frac{3}{8}a\right)\right\}$

15.2 b) $(2, -3)$

15.2 c) $\left\{\left(\frac{1}{13}a + \frac{5}{13}a^2, \frac{2}{13}a - \frac{3}{13}a^2\right)\right\}$

15.2 d) $(a - 2a^2, a + a^2)$

15.3 a) $\{(1 + z, -z, z); z \in \mathbb{R}\}$

15.3 b) $\{(1, y, 3 + 2y); y \in \mathbb{R}\}$

15.3 c) $\left\{\left(\frac{13}{6} - \frac{5}{3}z, -\frac{1}{3} + \frac{4}{3}z, z\right); z \in \mathbb{R}\right\}$

15.3 d) $\left\{\left(x, \frac{-5}{12} - \frac{3}{2}x, \frac{-25}{24} - \frac{7}{4}x\right); x \in \mathbb{R}\right\}$

15.4 a) $\{(2, -1, 3)\}$

15.4 b) $\{(-1, 4, 2)\}$

15.4 c) \emptyset

15.4 d) $\left\{\left(-\frac{2}{7} - z, \frac{-3}{7}, z\right); z \in \mathbb{R}\right\}$

15.5 a) $\{(1, 1/2, 1/2)\}$

15.5 b) \emptyset

15.5 c) $\{(5z, 1 - 4z, z); z \in \mathbb{R}\}$

15.5 d) $\left\{\left(1, \frac{1}{a+2}, \frac{1}{a+2}\right)\right\}$

15.6 a) $\{(5, 3, -1)\}$

15.6 b) \emptyset

15.6 c) $\left\{\left(\frac{a^2 + a - 1}{a^3 - 1}c, \frac{a^2 - a - 1}{a^3 - 1}c, \frac{-a^2 + a + 1}{a^3 - 1}c\right)\right\}$

15.7 a) $\{(0, 0, 0)\}$

15.7 b) $\{(x, y, -x - y); (x, y) \in \mathbb{R}^2\}$

15.7 c) $\{(x, x, x); x \in \mathbb{R}\}$

Fiche n° 16. Nombres complexes

Réponses

16.1 a) $4 + 32i$

16.1 b) $13 - i$

16.1 c) $7 - 24i$

16.1 d) 5

16.1 e) .. $-119 + 120i$

16.1 f) $\frac{3}{10} + \frac{1}{10}i$

16.1 g) $\frac{4}{29} - \frac{19}{29}i$

16.1 h) $\frac{1}{2} - \frac{\sqrt{3}}{2}i$

16.2 a) 12

16.2 b) $8e^{i\pi}$

16.2 c) $\sqrt{3}e^{i\frac{\pi}{2}}$

16.2 d) $2e^{-i\frac{\pi}{2}}$

16.2 e) $2e^{i\frac{8\pi}{5}}$

16.2 f) $5e^{-\frac{\pi}{4}i}$

16.2 g) $10e^{-\frac{2\pi}{3}i}$

16.2 h) $2 \cos\left(\frac{\pi}{12}\right) e^{i\frac{\pi}{4}}$

16.3 a) 1

16.3 b) ... $\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$

16.3 c) .. $-\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$

Fiche n° 17. Trigonométrie et nombres complexes

Réponses

17.1 a) $\frac{1}{4} \cos(3x) + \frac{3}{4} \cos(x)$

17.1 b) $-\frac{1}{4} \cos(4x) + \frac{1}{2} \cos(2x) - \frac{1}{4}$

17.1 c) ... $-\frac{1}{8} \cos(6x) + \frac{1}{4} \cos(4x) - \frac{3}{8} \cos(2x) + \frac{1}{4}$

17.1 d) ... $-\frac{\sin(9x)}{8} + \frac{3 \sin(5x)}{8} - \frac{\sin(3x)}{8} - \frac{3 \sin(x)}{8}$

17.1 e) $\frac{\cos(9x)}{8} + \frac{3 \cos(5x)}{8} + \frac{\cos(3x)}{8} + \frac{3 \cos(x)}{8}$

17.1 f) $-\frac{1}{4} \sin(11x) + \frac{1}{4} \sin(5x) + \frac{1}{2} \sin(3x)$

17.2 a) $2 \cos\left(\frac{\pi}{12}\right) e^{i \frac{\pi}{12}}$

17.2 b) $\left(-2 \cos\left(\frac{7\pi}{12}\right)\right) e^{-i \frac{5\pi}{12}}$

17.2 c) $2 \sin\left(\frac{\pi}{12}\right) e^{-\frac{7i\pi}{12}}$

17.2 d) $2 \cos\left(\frac{5\pi}{12}\right) e^{\frac{5i\pi}{12}}$

17.2 e) $2 \cos\left(\frac{\pi}{12}\right) e^{i \frac{13\pi}{12}}$

17.2 f) $2 \sin\left(\frac{\pi}{24}\right) e^{-i \frac{11\pi}{24}}$

17.2 g) $\frac{\cos\left(\frac{\pi}{12}\right)}{\sin\left(\frac{\pi}{24}\right)} e^{\frac{13i\pi}{24}}$

17.2 h) $2^{27} \cos^{27}\left(\frac{\pi}{12}\right) e^{i \frac{\pi}{4}}$

17.3 a) $2 \cos\left(\frac{\pi}{12}\right) e^{i \frac{5\pi}{12}}$

17.3 b) $2 \sin\left(\frac{\pi}{12}\right) e^{i \frac{11\pi}{12}}$

17.4 a) $4 \cos^3(x) - 3 \cos(x)$

17.4 b) $4 \cos^3(x) \sin(x) - 4 \cos(x) \sin^3(x)$

17.5 a) $2 \cos(2x) \cos(x)$

17.5 b) $2 \cos(4x) \sin(x)$

17.5 c) $2 \sin(x) \sin(2x)$

17.5 d) $2 \sin(4x) \cos(x)$

17.6 a) $\frac{\sin\left(\frac{3x}{2}\right) \sin(2x)}{\sin\left(\frac{x}{2}\right)}$

17.6 b) $\frac{\sin(8x)}{2 \sin(x)}$

17.6 c) 0

17.7 a) $\frac{e^\pi + 1}{2}$

17.7 b) $\frac{1}{5}(e^\pi - 2)$

Fiche n° 18. Sommes et produits

Réponses

18.1 a) $\boxed{n(n+2)}$

18.1 b) $\boxed{\frac{7(n+1)(n+4)}{2}}$

18.1 c) $\boxed{\frac{n(5n+1)}{2}}$

18.1 d) $\boxed{\frac{(n-2)(n-7)}{6}}$

18.2 a) $\boxed{\frac{n(n+1)(n+2)}{3}}$

18.2 b) ... $\boxed{n(n+1)(n^2+n+4)}$

18.2 c) $\boxed{\frac{9}{2}(3^{n-2}-1)}$

18.2 d) $\boxed{5^{n+1} \frac{1 - (\frac{2}{5})^{n+1}}{3}}$

18.2 e) ... $\boxed{\frac{7}{6}(7^n - 1) + n(n+4)}$

18.2 f) $\boxed{\frac{n+1}{2n}}$

18.3 a) $\boxed{2^{q-p+1}}$

18.3 b) $\boxed{3^{\frac{n(n+1)}{2}}}$

18.3 c) $\boxed{5^n (n!)^{\frac{3}{2}}}$

18.3 d) $\boxed{0}$

18.4 a) $\boxed{\frac{n(n+1)}{2}}$

18.4 b) $\boxed{0}$

18.4 c) $\boxed{n2^{n+1} + 2(1-2^n)}$

18.4 d) $\boxed{\frac{n^2(n+1)^2}{4}}$

18.5 a) $\boxed{(n+2)^3 - 2^3}$

18.5 b) $\boxed{\ln(n+1)}$

18.5 c) $\boxed{1 - \frac{1}{(n+1)!}}$

18.5 d) $\boxed{(n+1)! - 1}$

18.6 a) $\boxed{n+1}$

18.6 b) $\boxed{1-4n^2}$

18.6 c) $\boxed{\frac{1}{n}}$

18.6 d) $\boxed{\frac{n+1}{2n}}$

18.7 a) $\boxed{1 - \frac{1}{n+1}}$

18.7 b) $\boxed{\frac{1}{2} - \frac{1}{n+3}}$

18.8 a) $\boxed{2n^2 + n}$

18.8 b) $\boxed{\frac{n(3n+1)}{2}}$

18.9 a) $\boxed{\frac{n^2(n+1)}{2}}$

18.9 b) $\boxed{\frac{n(n+3)}{4}}$

18.9 c) $\boxed{\frac{n(n^2-1)}{2}}$

18.9 d) .. $\boxed{\frac{n(n+1)(7n^2+13n+4)}{12}}$

18.9 e) $\boxed{\frac{n(n+1)}{2} \ln(n!)}$

18.9 f) $\boxed{\frac{n(n+1)(4n-1)}{6}}$

Fiche n° 19. Coefficients binomiaux

Réponses

- 19.1 a)** $10\ 100$
19.1 b) 720
19.1 c) $\frac{1}{30}$
19.1 d) 15
19.1 e) 56
19.1 f) 140
19.2 a) $\frac{9!}{5!}$
19.2 b) $\binom{9}{4}$
19.2 c) $2^n \times n!$
19.2 d) $\frac{(2n+1)!}{2^n \times n!}$
19.3 a) $\frac{n(n-1)}{2}$
19.3 b) $\frac{n(n-1)(n-2)}{6}$
19.3 c) $\frac{k+1}{n-k}$
19.3 d) $(n+2)(n+1)$
19.3 e) $\frac{1}{(n+1)!}$
19.3 f) $\frac{n! \times (n-3)}{2^{2n+2}}$
19.4 a) $\frac{(n+1)^3}{n \times (n+2)!}$
19.4 b) $\frac{3(3n+2)(3n+1)}{a^3(n+1)^2}$
19.5 a) 3^n
19.5 b) 0
19.5 c) 6^n
19.5 d) 12×15^n
19.6 a) $2 \times \sum_{p=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n}{2p}$
19.6 b) 2^{n-1}
19.7 a) 2^n
19.7 b) $n2^{n-1}$
19.7 c) $n(n+1)2^{n-2}$
19.7 d) $\frac{2^{n+1} - 1}{n+1}$
19.8 a) $\binom{2n}{n}$
19.8 b) $\sum_{k=0}^n \binom{n}{k}^2$
19.8 c) $\binom{2n}{n}$

Fiche n° 20. Manipulation des fonctions usuelles

Réponses

| | | | | | |
|---------------|--------------------------|--------------|--|----------------|---|
| 20.1 a) | $\frac{\pi}{6}$ | 20.4 d)..... | $\frac{\ln(4)}{\ln(20/3)}$ | 20.7 e)..... | $[\ln(3 + \sqrt{10}), [$ |
| 20.1 b) | 2 | 20.5 a)..... | $\frac{\ln\left(\frac{\sqrt{17}-1}{2}\right)}{\ln(2)}$ | 20.7 f) | $]-\infty, \frac{1}{2} \ln(3)]$ |
| 20.1 c)..... | $\frac{\pi}{4}$ | 20.5 b)..... | $\left\{0; \frac{1}{2}\right\}$ | 20.8 a) ... | $x \mapsto \ln(2) \times 2^x + 2x$ |
| 20.1 d) | $\frac{\pi}{6}$ | 20.5 c)..... | $1 - \frac{\ln(2)}{\ln(3)}$ | 20.8 b). | $x \mapsto \frac{15^x \ln(3/5) + 3^x \ln(3)}{(5^x + 1)^2}$ |
| 20.1 e)..... | $\frac{\pi}{4}$ | 20.5 d)..... | $\frac{\ln\left(\frac{\sqrt{5}-1}{2}\right)}{\ln(3)}$ | 20.8 c)..... | $x \mapsto (\ln(x) + 1)x^x$ |
| 20.1 f) | $\frac{\pi}{3}$ | 20.6 a)..... | 1 | 20.8 d) . | $x \mapsto \frac{\pi}{2\sqrt{1-x^2} \arccos(x)^2}$ |
| 20.2 a)..... | 1 | 20.6 b)..... | 0 | 20.9 a) | $x \mapsto 2x \frac{1}{\sqrt{1-x^4}}$ |
| 20.2 b)..... | 0 | 20.6 c)..... | $\left\{\frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\}$ | 20.9 b) ... | $x \mapsto \operatorname{ch}^2(x) + \operatorname{sh}^2(x)$ |
| 20.2 c)..... | $\frac{5}{4}$ | 20.6 d) . | $\left\{\frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}\right\} \cup \left\{\frac{2\pi}{3} + 2k\pi, k \in \mathbb{Z}\right\}$ | 20.9 c)..... | $x \mapsto \frac{1 - \operatorname{th}^2(x)}{1 + \operatorname{th}^2(x)}$ |
| 20.2 d)..... | $\frac{4}{3}$ | 20.6 e) | $\left\{\frac{1}{3} + 2k\pi, k \in \mathbb{Z}\right\} \cup \left\{\pi - \frac{1}{3} + 2k\pi, k \in \mathbb{Z}\right\}$ | 20.9 d) | $x \mapsto \operatorname{sh}(x)\operatorname{ch}(\operatorname{ch}(x))$ |
| 20.2 e)..... | $\frac{13}{12}$ | 20.6 f)..... | 1 | 20.10 a) | $x \mapsto 0$ |
| 20.2 f) | $\frac{3}{5}$ | 20.7 a) . | $\{\ln(\sqrt{5}-2); \ln(\sqrt{5}+2)\}$ | 20.10 b) | $x \mapsto 0$ |
| 20.3 a)..... | $\operatorname{sh}(x+y)$ | 20.7 b)..... | $\ln(1 + \sqrt{2})$ | 20.11 a) | $x \mapsto (\ln(x) + 1)x^x e^{-x^{2x}}$ |
| 20.3 b)..... | $\operatorname{ch}(x+y)$ | 20.7 c)..... | $\frac{1}{2} \ln(2)$ | 20.11 b). | $x \mapsto \frac{\operatorname{sh}(x)}{\operatorname{ch}(x)^2} \frac{1}{2\sqrt{\ln(\operatorname{ch}(x))}}$ |
| 20.4 a)..... | $\frac{\ln(2)}{\ln(3)}$ | 20.7 d) . | $[-\ln(4+\sqrt{15}), \ln(4+\sqrt{15})]$ | 20.11 c) | $x \mapsto \arcsin(x)$ |
| 20.4 b) | 1 | | | 20.11 d)..... | $x \mapsto \arctan(x)$ |
| 20.4 c) | $-\frac{\ln(3)}{\ln(2)}$ | | | | |

Fiche n° 21. Suites numériques

Réponses

| | | | | | |
|--------------|------------------------------------|--------------|---------------------|---------------|---|
| 21.1 a)..... | $\frac{12}{5}$ | 21.6 a)..... | 21 | 21.9 a)..... | $\frac{\pi\sqrt{5}}{5}$ |
| 21.1 b)..... | 8 | 21.6 b)..... | 10 000 | 21.9 b)..... | $\frac{11\sqrt{5}}{25}$ |
| 21.1 c)..... | $\frac{(2n+5) \cdot 2^{n+3}}{5}$ | 21.6 c)..... | 2 001 | 21.10 a)..... | $3^n + (-2)^n$ |
| 21.1 d)..... | $\frac{3(2n+1) \cdot 2^{3n+2}}{5}$ | 21.6 d)..... | 10 201 | 21.10 b)..... | 211 |
| 21.2 a)..... | 13 | 21.7 a)..... | $\frac{17}{24}$ | 21.11 a) .. | $\frac{(1+\sqrt{2})^n - (1-\sqrt{2})^n}{2}$ |
| 21.2 b)..... | 29 | 21.7 b)..... | $\frac{1}{24}$ | 21.11 b)..... | $2\sqrt{2}$ |
| 21.3 a)..... | $2^{\frac{1}{8}}$ | 21.8 a)..... | $\frac{3}{512}$ | 21.12 a)..... | 257 |
| 21.3 b)..... | $2^{\frac{1}{64}}$ | 21.8 b)..... | $\frac{3069}{512}$ | 21.12 b)..... | 65 537 |
| 21.4 a)..... | 2 | 21.8 c)..... | $\frac{3}{1\ 024}$ | 21.12 c)..... | F_n |
| 21.4 b)..... | 2 | 21.8 d)..... | $\frac{6141}{1024}$ | 21.12 d)..... | $F_{n+1} - 2$ |
| 21.5 a)..... | $2n \ln(n)$ | | | 21.12 e)..... | $F_{n+1} + 2^{2^n+1}$ |
| 21.5 b)..... | $4n \ln(2n)$ | | | 21.12 f)..... | F_{n+2} |

Fiche n° 22. Développements limités

Réponses

22.1 a) $3x - x^2 + \frac{x^3}{2} - \frac{x^4}{2} + o_{x \rightarrow 0}(x^4)$

22.1 b) $x - \frac{3}{2}x^2 + \frac{11}{6}x^3 - \frac{25}{12}x^4 + o_{x \rightarrow 0}(x^4)$

22.1 c) $\frac{x^3}{2} - \frac{x^5}{24} + o_{x \rightarrow 0}(x^6)$

22.1 d) $x + x^2 + \frac{x^3}{3} - \frac{x^5}{30} - \frac{x^6}{90} + o_{x \rightarrow 0}(x^6)$

22.2 a) $e - \frac{ex}{2} + \frac{11ex^2}{24} - \frac{7ex^3}{16} + \frac{2447ex^4}{5760} + o_{x \rightarrow 0}(x^5)$

22.2 b) $1 - \frac{1}{4}x^2 - \frac{1}{96}x^4 - \frac{19}{5760}x^6 + o_{x \rightarrow 0}(x^7)$

22.2 c) $e \left(1 + ix - x^2 - \frac{5}{6}ix^3 \right) + o_{x \rightarrow 0}(x^3)$

22.2 d) $1 - x + \frac{3}{2}(x-1)^2 + o_{x \rightarrow 1}((x-1)^2)$

22.3 a) $1 - \frac{3\pi^2}{8} \left(x - \frac{\pi}{3} \right)^2 + o_{x \rightarrow \frac{\pi}{3}} \left(\left(x - \frac{\pi}{3} \right)^2 \right)$

22.3 b) $1 + 2 \left(x - \frac{\pi}{4} \right) + 2 \left(x - \frac{\pi}{4} \right)^2 + \frac{8}{3} \left(x - \frac{\pi}{4} \right)^3 + o_{x \rightarrow \frac{\pi}{4}} \left(\left(x - \frac{\pi}{4} \right)^4 \right)$

22.3 c) $-1 + \frac{\pi^2}{8} \left(x - \frac{\pi}{2} \right)^4 - \frac{\pi^2}{48} \left(x - \frac{\pi}{2} \right)^6 + o_{x \rightarrow \frac{\pi}{2}} \left(\left(x - \frac{\pi}{2} \right)^7 \right)$

22.4 a) $-\frac{1}{2x} + \frac{1}{12} - \frac{1}{720}x^2 + o_{x \rightarrow 0}(x^2)$

22.4 b) $\frac{1}{x^2} - \frac{1}{x^3} + \frac{5}{6x^4} - \frac{5}{6x^5} + o_{x \rightarrow +\infty} \left(\frac{1}{x^6} \right)$

22.4 c) $-\ln(x) + 1 - \frac{1}{2x} + \frac{1}{3x^2} - \frac{1}{4x^3} + o_{x \rightarrow +\infty} \left(\frac{1}{x^3} \right)$

22.4 d) $e^{-\frac{1}{2}} \left(e^x + \frac{e^x}{3x} - \frac{7e^x}{36x^2} \right) + o_{x \rightarrow +\infty} \left(\frac{e^x}{x^2} \right)$

Fiche n° 23. Arithmétique

Réponses

23.1 a)..... $(6, 7)$

23.1 b)..... $(-7, 2)$

23.1 c)..... $(-6, 7)$

23.1 d)..... $(7, 2)$

23.2 a)..... 20

23.2 b)..... 4

23.3 a)..... 2

23.3 b)..... 4

23.4 1

23.5 a) 154

23.5 b)..... $\frac{65}{18}$

23.5 c)..... $29\ 160$

23.5 d)..... $\frac{1}{29\ 160}$

23.6 a)..... $(9, 8)$

23.6 b) $(12, 30)$

23.7 a) $(-5, 2)$

23.7 b) .. $8 \pmod{13}$

23.7 c).. $11 \pmod{13}$

23.8 5

23.8 $(2023, 6406)$

23.9 a)... $2 \times 3 \times 337$

23.9 b)..... 7×17^2

23.9 c)..... 43×47

23.9 d). il est premier

23.10 a) 67

23.10 b) 7

23.11 a) 1

23.11 b) 1

23.11 c)..... 6

23.11 d) 5

23.11 e) 66

23.11 f)..... 2

Fiche n° 24. Polynômes

Réponses

- 24.1 a)
$$\begin{matrix} Q = X^2 + 2X + 1 \\ R = 2 \end{matrix}$$
- 24.1 b)
$$\begin{matrix} Q = X^2 - 4X + 7 \\ R = -3X - 8 \end{matrix}$$
- 24.1 c)
$$\begin{matrix} Q = X^2 - 1 \\ R = -X^2 + X + 1 \end{matrix}$$
- 24.1 d)
$$\begin{matrix} Q = 13X + \frac{25}{2} \\ R = \frac{1}{2}(29X^2 - 5X - 23) \end{matrix}$$
- 24.2 a) $R = 1$
- 24.2 b) $R = 0$
- 24.2 c) $R = -2nX + 2n - 1$
- 24.2 d) $R = X^2 + X - 1$
- 24.3 a) $R = 2X - 3$
- 24.3 b) $R = -2X^3 - 3X^2 + 1$
- 24.3 c) $R = -8X^3 + 21X^2 - 20X + 5$
- 24.3 d) $R = -29X^3 + 11X^2 + 2X - 1$
- 24.4 a) $R = -36X + 64$
- 24.4 b) $64 - 36i$
- 24.5 a) $R = -108X - 110$
- 24.5 b) $-110 - 108\sqrt{2}$
- 24.6 a) $116 - 92\sqrt{2}$
- 24.6 b) $48 - 206i$

Fiche n° 25. Décomposition en éléments simples

Réponses

25.1 a)..... $X - 3 - \frac{1}{X} + \frac{1}{X+1} + \frac{7}{X+2}$

25.1 b)..... $1 - \frac{2}{X} + \frac{1}{2(X+1)} + \frac{3}{2(X-1)}$

25.1 c)..... $1 + \frac{\pi}{2(X-\pi)} - \frac{\pi}{2(X+\pi)}$

25.2 a)..... $\frac{e-1}{(e-2)(X+e)} + \frac{1}{(2-e)(X+2)}$

25.2 b)..... $\frac{3}{2(X-1)} - \frac{1+i}{4(X-i)} - \frac{1-i}{4(X+i)}$

25.2 c)..... $1 - \frac{5}{(\sqrt{2}+\sqrt{3})(x+\sqrt{3})} - \frac{4}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-X)}$

25.3 a)..... $\frac{-3}{X-2} + \frac{1}{X-3} + \frac{2}{X-1} + \frac{1}{(X-1)^2}$

25.3 b)..... $\frac{2}{X} + \frac{2}{X^2} - \frac{11}{4(X-1)} + \frac{3}{2(X-1)^2} + \frac{3}{4(X+1)}$

25.3 c)..... $\frac{1}{\pi^2 X} - \frac{1}{\pi^2(X+\pi)} - \frac{1+\pi}{\pi(X+\pi)^2}$

25.3 d)..... $\frac{2}{X-i} + \frac{1}{(X-i)^2} - \frac{2}{X-(1+i)} + \frac{1}{(X-(1+i))^2}$

25.4 a)..... $\frac{1}{X+1} - \frac{1}{2(X-1)} - \frac{1+3i}{4(X-i)} - \frac{1-3i}{4(X+i)}$

25.4 b)..... $\frac{1}{2X} + \frac{5}{6(X+2)} + \frac{2}{3(X-1)} + \frac{1}{(X-1)^2}$

25.5 a)..... $\frac{1}{2(n+1)} - \frac{1}{2n} + \frac{1}{4}$

25.5 b)..... $-\frac{2}{n+2} + \frac{1}{n} - \frac{1}{3}$

25.6 a)..... $\frac{2}{X+1} + \frac{1}{(X+1)^2} + \frac{1-2X}{X^2+1}$

25.6 b)..... $\frac{1}{2(X-1)} - \frac{3}{2(X+1)} + \frac{X-1}{X^2+X+1}$

25.7 a)..... $1 - 2\ln(3)$

25.7 b)..... $-\frac{1}{2}\ln(3) + \frac{2}{3}\ln(2)$

25.7 c)..... $\frac{2}{3} - 4\ln(2) + 2\ln(3)$

25.7 d)..... $\frac{1}{18} - \frac{1}{9}\ln(5) + \frac{2}{9}\ln(2)$

25.7 e)..... $\frac{\pi}{8}$

25.7 f)..... $\frac{1}{2}\ln(2) - \frac{1}{4}\ln(3)$

25.8 a)..... $\frac{1}{2}\ln\left|\frac{x-1}{1+x}\right|$

25.8 b)..... $x \mapsto \frac{1}{4(1-2x)^2}$

25.8 c)..... $\frac{1}{\sqrt{2}}\arctan\left(\frac{x}{\sqrt{2}}\right)$

25.8 d)..... $\frac{\sqrt{3}}{2}\arctan\left(\frac{2}{\sqrt{3}}X + \frac{1}{\sqrt{3}}\right)$

25.8 e)..... 2

25.8 f)..... $\frac{x^2}{2} + 2x + \frac{1}{6}\ln|x+1| - \frac{1}{2}\ln|x-1| + \frac{16}{3}\ln|x-2|$

25.8 g)..... $x \mapsto \frac{1}{6}\ln(x^2+2) - \frac{1}{3}\ln|x+1| + \frac{\sqrt{2}}{3}\arctan\left(\frac{x}{\sqrt{2}}\right)$

25.8 h)..... $x \mapsto \frac{1}{2}\frac{2x-1}{x^2-1} + \frac{1}{2}\ln\left|\frac{1-x}{1+x}\right|$

Fiche n° 26. Calcul matriciel

Réponses

26.1 a) $\begin{pmatrix} 1 & -3 & -1 \\ 3 & 3 & 4 \\ 9 & -7 & 3 \end{pmatrix}$

26.1 b) $\begin{pmatrix} -2 & -6 & -5 \\ 15 & -1 & 11 \\ 18 & -26 & -1 \end{pmatrix}$

26.1 c) 17 (matrice 1×1)

26.1 d) $\begin{pmatrix} 1 & 7 & -2 \\ 2 & 14 & -4 \\ -1 & -7 & 2 \end{pmatrix}$

26.1 e) $\begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}$

26.1 f) $(-5 \quad 15 \quad 3)$

26.1 g) $\begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$

26.1 h) $\begin{pmatrix} 5 & 3 & -1 & 1 \\ 4 & 3 & 1 & 2 \end{pmatrix}$

26.1 i) $\begin{pmatrix} 1 & 7 & -2 \\ 7 & 49 & -14 \\ -2 & -14 & 4 \end{pmatrix}$

26.2 a) $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

26.2 b) $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$

26.2 c) $\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$

26.2 d) $\begin{pmatrix} 4 & 5 \\ 0 & 9 \end{pmatrix}$

26.2 e) $\begin{pmatrix} 8 & 19 \\ 0 & 27 \end{pmatrix}$

26.2 f) $\begin{pmatrix} 2^k & 3^k - 2^k \\ 0 & 3^k \end{pmatrix}$

26.2 g) $\begin{pmatrix} \cos(2\theta) & -\sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{pmatrix}$

26.2 h) $\begin{pmatrix} \cos(3\theta) & -\sin(3\theta) \\ \sin(3\theta) & \cos(3\theta) \end{pmatrix}$

26.2 i) $\begin{pmatrix} \cos(k\theta) & -\sin(k\theta) \\ \sin(k\theta) & \cos(k\theta) \end{pmatrix}$

26.2 j) $\begin{pmatrix} n & \cdots & n \\ \vdots & (n) & \vdots \\ n & \cdots & n \end{pmatrix}$

26.2 k) $\begin{pmatrix} n^2 & \cdots & n^2 \\ \vdots & (n^2) & \vdots \\ n^2 & \cdots & n^2 \end{pmatrix}$

26.2 l) $n^{k-1}D$

26.3 a) $2 \times 3^{j-i} \times 5^{i-1}$

26.3 b) $2^{i+1}3^{j-i}(2^n - 1)$

26.3 c) $2 \times 3^{i+j} \left(1 - \left(\frac{2}{3}\right)^n\right)$

26.3 d) $\binom{i-1}{j} + \binom{i-1}{j-2}$

26.4 a) $2^{i-j} \binom{i-1}{j-1}$

26.4 b) $(1 - \delta_{i,1})(\delta_{i-1,j+1} + \delta_{i,j}) + (1 - \delta_{i,n})(\delta_{i,j} + \delta_{i+1,j-1})$

26.5 a) $\frac{1}{2(\pi - e)} \begin{pmatrix} 2 & -e \\ -2 & \pi \end{pmatrix}$

26.5 b) $\frac{1}{3} \begin{pmatrix} 1 & -1 - 2i \\ 1 & -1 + i \end{pmatrix}$

26.5 c) $\frac{1}{2} \begin{pmatrix} 5 & 2 & -1 \\ 3 & 2 & -1 \\ -6 & -2 & 2 \end{pmatrix}$

26.5 d) $\frac{1}{4\pi} \begin{pmatrix} 0 & 4 & 0 \\ 0 & -2 & -2 \\ 2 & -1 & 1 \end{pmatrix}$

26.5 e) $\frac{1}{8} \begin{pmatrix} 8 & 4 & -2 \\ -16 & -6 & 7 \\ 0 & -2 & 1 \end{pmatrix}$

26.5 f) $\frac{1}{6} \begin{pmatrix} -2 & 2 & 2 \\ 1 & -1 & 2 \\ 4 & 2 & -4 \end{pmatrix}$

26.5 g) $\frac{1}{2} \begin{pmatrix} 4 & -2 & 2 & 0 \\ 8 & -6 & 4 & 2 \\ -7 & 5 & -3 & -1 \\ -5 & 3 & -1 & -1 \end{pmatrix}$

26.5 h) Non inversible!

26.5 i) $\frac{1}{2} \begin{pmatrix} 0 & -1 & 0 & -1 \\ 1 & 1 & 0 & 0 \\ -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$

26.6 a) $\lambda \neq 1$

26.6 b) $\frac{1}{1-\lambda} \begin{pmatrix} -4 & -1 & 3 \\ 2\lambda+2 & \lambda & -2\lambda-1 \\ \lambda-1 & 0 & 1-\lambda \end{pmatrix}$

26.6 c) $\lambda \neq 1$

26.6 d) $\frac{1}{1-\lambda} \begin{pmatrix} -1-\lambda+\lambda^2 & 1-\lambda & 2-\lambda \\ 1 & 0 & -1 \\ 1-\lambda^2 & \lambda-1 & \lambda-1 \end{pmatrix}$

Fiche n° 27. Algèbre linéaire

Réponses

27.1 a)..... $(3, -1)$

27.1 b)..... $(-1, 3)$

27.1 c)..... $(9/11, 2/11)$

27.1 d)..... $(-2, 4/5, 11/5)$

27.1 e)..... $(-1, 1/2, 1/2)$

27.1 f)..... $(0, 2, 4, 1)$

27.1 g)..... $(1/2, -\sqrt{3}/2)$

27.2 a)..... 2

27.2 b)..... 1

27.2 c)..... 1

27.2 d)..... 2

27.2 e)..... 2

27.2 f)..... 1

27.3 a)..... 2

27.3 b)..... 2

27.3 c)..... 3

27.3 d)..... 4

27.4 a)..... $\begin{pmatrix} 1 & 1 \\ 3 & -5 \end{pmatrix}$

27.4 b)..... $\begin{pmatrix} -5 & 3 \\ 1 & 1 \end{pmatrix}$

27.4 c)..... $\frac{1}{2} \begin{pmatrix} -19 & -43 \\ 9 & 21 \end{pmatrix}$

27.4 d)..... $\begin{pmatrix} 1 & 0 & 1 \\ 3 & -1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

27.4 e)..... $\begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$

27.5 a)..... $\begin{pmatrix} -1 & -1 & 1 \\ 4 & 15 & 0 \end{pmatrix}$

27.5 b)..... $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Fiche n° 28. Équations différentielles

Réponses

28.1 a) $x \mapsto 56e^{12x}$

28.1 b) $x \mapsto 6e^x - 1$

28.1 c) $x \mapsto \frac{8e^{3x} - 5}{3}$

28.1 d) $x \mapsto 9e^{2x} - 6$

28.2 a) $x \mapsto e^{(6-x)/5}$

28.2 b) $x \mapsto 1 - 2e^{-2x/7+2}$

28.2 c) $x \mapsto \left(\frac{6}{\sqrt{5}} + \pi \right) e^{\sqrt{5}x} - \frac{6}{\sqrt{5}}$

28.2 d) $x \mapsto \left(12 + \frac{2e}{\pi} \right) e^{\pi x - \pi^2} - \frac{2e}{\pi}$

28.3 a) $x \mapsto e^{2x}$

28.3 b) $x \mapsto e^x$

28.3 c) $x \mapsto 2e^{2x} - e^x$

28.3 d) $x \mapsto (2 - 3i)e^x + (3i - 1)e^{2x}$

28.4 a) $x \mapsto e^x$

28.4 b) $x \mapsto 7e^{-x} - 5e^{-2x}$

28.4 c) $x \mapsto \frac{4}{3}e^x - \frac{1}{3}e^{-2x}$

28.4 d) $x \mapsto (2 - x)e^x$

28.4 e) $x \mapsto (2 - x)e^{2-2x}$

28.5 a) $x \mapsto \cos x + 2 \sin x$

28.5 b) $x \mapsto e^{-x/2} \left(\cos \frac{\sqrt{3}x}{2} - \frac{1}{\sqrt{3}} \sin \frac{\sqrt{3}x}{2} \right)$

28.5 c) $x \mapsto e^{-x} \sin(x)$

28.5 d) $x \mapsto e^x \left(\frac{-1 + i}{2} e^{2ix} + \frac{1 + i}{2} e^{-2ix} \right)$

Fiche n° 29. Séries numériques

Réponses

29.1 a).... $\boxed{\text{divergente}}$

29.1 b)..... $\boxed{2}$

29.1 c)..... $\boxed{\frac{2}{2-\sqrt{2}}}$

29.1 d)..... $\boxed{\frac{1}{2 \times 3^9}}$

29.2 a)..... \boxed{e}

29.2 b)..... $\boxed{e^2 - 3}$

29.2 c)..... $\boxed{e^{\frac{1}{2}}}$

29.3 a)..... $\boxed{\frac{\pi^2}{6}}$

29.3 b).... $\boxed{\text{divergente}}$

29.3 c).... $\boxed{\text{divergente}}$

29.3 d)..... $\boxed{\frac{7-49i}{35\sqrt{2}}}$

29.3 e).... $\boxed{\frac{-2-5\sqrt{2}i}{54}}$

29.4 a)..... $\boxed{1}$

29.4 b)..... $\boxed{\frac{1}{4}}$

29.4 c)..... $\boxed{\ln(2)}$

29.4 d)..... $\boxed{\frac{\pi}{4}}$

29.5 a)..... $\boxed{\frac{1}{12}}$

29.5 b)..... $\boxed{\frac{e}{e-1}}$

29.5 c).... $\boxed{\text{divergente}}$

29.5 d)..... $\boxed{4}$

29.6 a)..... $\boxed{2}$

29.6 b)..... $\boxed{\frac{11}{4}}$

29.6 c)..... $\boxed{16}$

29.6 d)..... $\boxed{\frac{2e^3}{(e-1)^3}}$

Fiche n° 30. Structures euclidiennes

Réponses

30.1 a)..... $4 \ln 2 - 2$

30.1 b)..... $\frac{7}{12}$

30.1 c)..... $2 \sin(1) + \cos(1) - 1$

30.1 d)..... $\frac{1}{2}(e^2 - 1)$

30.2 a)..... 11

30.2 b)..... 10

30.2 c)..... 0

30.3 a)..... $\frac{1}{6\sqrt{5}}$

30.3 b)..... $\frac{1}{5\sqrt{3}}$

30.3 c)..... $\frac{1}{3}$

30.4 a)..... $(1, 2\sqrt{3}(X - \frac{1}{2}))$

30.4 b)..... $(\sqrt{3}X, \sqrt{\frac{240}{43}}(X^2 - \frac{9}{4}X + 1))$

30.5 a)..... $\frac{1}{3} \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$

30.5 b)..... $\frac{1}{5} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & 4 \end{pmatrix}$

30.5 c)..... $\frac{1}{11} \begin{pmatrix} 9 & -6 & 2 \\ -6 & -7 & 6 \\ 2 & 6 & 9 \end{pmatrix}$

Fiche n° 31. Groupes symétriques

Réponses

- 31.1 a)** .. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 1 & 3 & 2 & 6 & 5 \end{pmatrix}$
- 31.1 b)** .. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 6 & 5 & 1 & 3 & 4 \end{pmatrix}$
- 31.1 c)** .. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 3 & 2 & 5 & 1 \end{pmatrix}$
- 31.1 d)** .. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 6 & 5 & 3 & 4 \end{pmatrix}$
- 31.1 e)** .. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 6 & 5 & 4 & 2 & 3 \end{pmatrix}$
- 31.1 f)** .. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 2 & 1 & 5 & 4 \end{pmatrix}$
- 31.2 a)** $(a b)$
- 31.2 b)** $(c b a)$
- 31.2 c)** $(7 2 5 3 1)$
- 31.2 d)** $(a c b)$
- 31.2 e)** $(2 1 5 4)$
- 31.2 f)** $(1 2 7 5 3)$
- 31.3 a)** $(1 7 4)(2 6 8 10)(3 9 5)$
- 31.3 b)** .. $(1 3 10 6 4)(5 7)(8 9)$
- 31.3 c)** $(1 7)(2 4 3 5 8)$
- 31.3 d)** $(1 2)(3 4)$
- 31.3 e)** $(1 4 6 2 3 5)$
- 31.4 a)** $(1 4 2)(5 6)$
- 31.4 b)** id
- 31.4 c)** $(1 2 6 5 3)$
- 31.4 d)** $(1 6 7 4)(2 5 3)$
- 31.5 a)** -1
- 31.5 b)** 1
- 31.5 c)** 1
- 31.5 d)** -1
- 31.5 e)** -1
- 31.5 f)** 1
- 31.6 a)** -1
- 31.6 b)** 1
- 31.6 c)** 1
- 31.6 d)** 1

Fiche n° 32. Déterminants

Réponses

- | | | | | | |
|---------------|------------|--------------|------------------|---------------|--------------------------|
| 32.1 a) | $-2a^2$ | 32.2 c)..... | $227/336$ | 32.4 b) | $6i - 12$ |
| 32.1 b) | 6 | 32.2 d)..... | $3\,919$ | 32.4 c)..... | $4/375$ |
| 32.1 c)..... | $-5 + 6i$ | 32.2 e)..... | $7\sqrt{2} + 13$ | 32.5 a).... | $x^3 + y^3 + z^3 - 3xyz$ |
| 32.1 d)..... | 20 | 32.3 a)..... | 0 | 32.5 b)..... | $-6 \ln^3(a)$ |
| 32.2 a)..... | -2 | 32.3 b)..... | -40 | 32.5 c)... | $(y - x)(z - y)(z - x)$ |
| 32.2 b)..... | $9 \ln(2)$ | 32.3 c)..... | 0 | 32.5 d)..... | 0 |
| | | 32.4 a)..... | -4 | | |

Fiche n° 33. Fonctions de deux variables

Réponses

- 33.1 a) $\{(x, y) \in \mathbb{R}^2, x - 1 \leq y \leq x + 1\}$
- 33.1 b) $]0, +\infty[\times]0, +\infty[$
- 33.1 c) $\{(x, y) \in \mathbb{R}^2, y \geq 0\} \setminus \{(0, 0)\}$
- 33.1 d) \emptyset
- 33.2 a) $\frac{\partial f}{\partial x}(x, y) = 2x + y$ et $\frac{\partial f}{\partial y}(x, y) = 5y^4 + x$
- 33.2 b) $\frac{\partial f}{\partial x}(x, y) = 2y \cos(2xy - y)$ et $\frac{\partial f}{\partial y}(x, y) = (2x - 1) \cos(2xy - y)$
- 33.2 c) $\frac{\partial f}{\partial x}(x, y) = (2xy, 2x)$ et $\frac{\partial f}{\partial y}(x, y) = (x^2, -2y)$
- 33.2 d) $\frac{\partial f}{\partial x}(x, y) = \frac{2}{1 + (2x + y)^2}$ et $\frac{\partial f}{\partial y}(x, y) = \frac{1}{1 + (2x + y)^2}$
- 33.3 a) $\frac{\partial f}{\partial x}(x, y) = -\sin(x - y)$ et $\frac{\partial f}{\partial y}(x, y) = \sin(x - y)$
- 33.3 b) $\frac{\partial f}{\partial x}(x, y) = \cos(e^{xy}) - xy \sin(e^{xy}) e^{xy}$ et $\frac{\partial f}{\partial y}(x, y) = -x^2 \sin(e^{xy}) e^{xy}$
- 33.3 c) $\frac{\partial f}{\partial x}(x, y) = y x^{y-1}$ et $\frac{\partial f}{\partial y}(x, y) = x^y \ln x$
- 33.3 d) $\frac{\partial f}{\partial x}(x, y) = \begin{cases} \frac{y^2(y^2 - x^2)}{(x^2 + y^2)^2} & \text{si } (x, y) \neq (0, 0) \\ 0 & \text{sinon} \end{cases}$ et $\frac{\partial f}{\partial y}(x, y) = \begin{cases} \frac{2x^3 y}{(x^2 + y^2)^2} & \text{si } (x, y) \neq (0, 0) \\ 0 & \text{sinon} \end{cases}$
- 33.4 a) $\sin(2t)$
- 33.4 b) $\frac{2e^{4t} + e^{-2t}}{\sqrt{e^{4t} - e^{-2t}}}$
- 33.4 c) $-72 \cos(4t) - 46 \sin(4t)$
- 33.5 a) $\frac{\partial(f \circ \varphi)}{\partial u}(u, v) = \frac{1}{2} \frac{\partial f}{\partial x} \left(\frac{u+v}{2}, \frac{v-u}{2c} \right) - \frac{1}{2c} \frac{\partial f}{\partial y} \left(\frac{u+v}{2}, \frac{v-u}{2c} \right)$
- 33.5 a) $\frac{\partial(f \circ \varphi)}{\partial v}(u, v) = \frac{1}{2} \frac{\partial f}{\partial x} \left(\frac{u+v}{2}, \frac{v-u}{2c} \right) + \frac{1}{2c} \frac{\partial f}{\partial y} \left(\frac{u+v}{2}, \frac{v-u}{2c} \right)$
- 33.5 b) $\frac{\partial(f \circ \varphi)}{\partial r}(r, \theta) = \cos \theta \frac{\partial f}{\partial x}(r \cos \theta, r \sin \theta) + \sin \theta \frac{\partial f}{\partial y}(r \cos \theta, r \sin \theta)$
- 33.5 b) $\frac{\partial(f \circ \varphi)}{\partial \theta}(r, \theta) = -r \sin \theta \frac{\partial f}{\partial x}(r \cos \theta, r \sin \theta) + r \cos \theta \frac{\partial f}{\partial y}(r \cos \theta, r \sin \theta)$