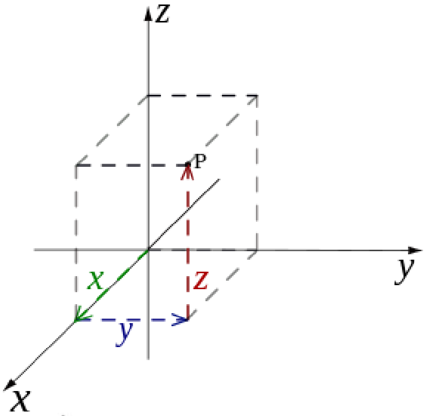
Systèmes de coordonnées

Coordonnées cartésiennes (à savoir)



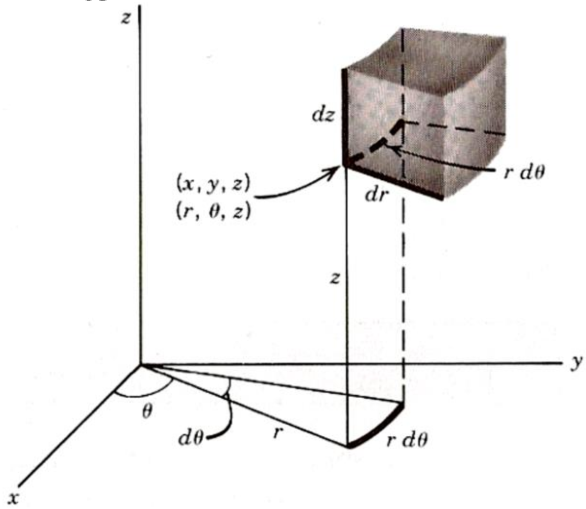
$$\overrightarrow{OM} = x\,\overrightarrow{u_x} + y\,\overrightarrow{u_y} + z\,\overrightarrow{u_z}$$

$$\overrightarrow{dl} = dx\,\overrightarrow{u_x} + dy\,\overrightarrow{u_y} + dz\,\overrightarrow{u_z}$$

$$\overrightarrow{d^2S_x} = dy.dz\,\overrightarrow{u_x}$$

$$d^3V = dx.dy.dz$$

Coordonnées cylindriques



$$\overrightarrow{OM} = r\,\overrightarrow{u_r} + z\,\overrightarrow{u_z}$$

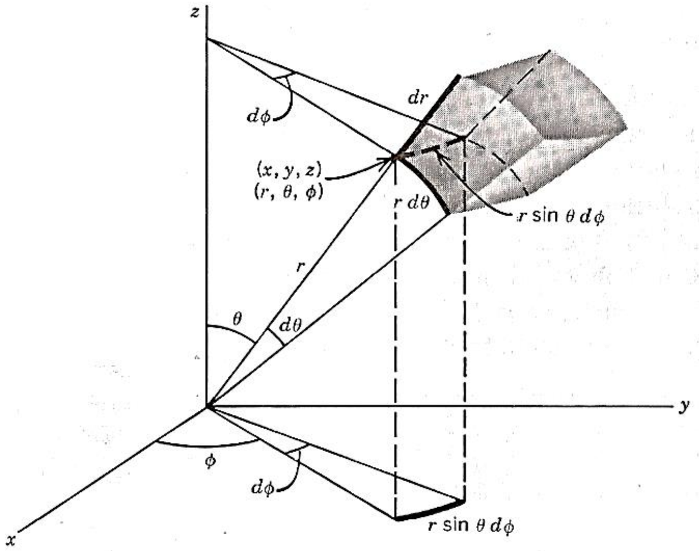
$$\overrightarrow{dl} = dr\,\overrightarrow{u_r} + r\,d\theta\,\overrightarrow{u_\theta} + dz\,\overrightarrow{u_z}$$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases} \quad \text{et} \quad \begin{cases} r = \sqrt{x^2+y^2} \\ \theta = \arctan\dfrac{y}{x}\ [\pi] \\ z = z \end{cases}$$

$$\overrightarrow{d^2S_r} = r.d\theta.dz\,\overrightarrow{u_r}$$

$$d^3V = r.dr.d\theta.dz$$

Coordonnées sphériques



$$\overrightarrow{OM} = r\overrightarrow{u_r}$$

$$\overrightarrow{dl} = dr\,\overrightarrow{u_r} + rd\theta\,\overrightarrow{u_\theta} + r\sin\theta\,d\varphi\,\overrightarrow{u_\varphi}$$

$$\begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{cases} \quad \text{et} \quad \begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \theta = \arctan\left(\dfrac{\sqrt{x^2+y^2}}{z}\right)\ [\pi]\ (*) \\ \varphi = \arctan\left(\dfrac{y}{x}\right)\ [\pi] \end{cases}$$

$$\overrightarrow{d^2S_r} = r^2.\sin\theta\,.d\theta.d\varphi\,\overrightarrow{u_r}$$

$$d^3V = r^2.\sin\theta\,.dr.d\theta.d\varphi$$

(*) Pas indispensable !

Formulaire d'analyse vectorielle

Coordonnées cartésiennes (à savoir)

$$\overrightarrow{\text{grad}}\, V = \vec{\nabla} V = \frac{\partial V}{\partial x}\,\vec{u}_x + \frac{\partial V}{\partial y}\,\vec{u}_y + \frac{\partial V}{\partial z}\,\vec{u}_z$$

$$\text{div}\,\vec{A} = \vec{\nabla}.\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\overrightarrow{\text{rot}}\,\vec{A} = \vec{\nabla} \wedge \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\vec{u}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\vec{u}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\vec{u}_z$$

$$\Delta V = \left(\vec{\nabla}\right)^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\Delta \vec{A} = \left(\vec{\nabla}\right)^2 \vec{A} = \frac{\partial^2 \vec{A}}{\partial x^2} + \frac{\partial^2 \vec{A}}{\partial y^2} + \frac{\partial^2 \vec{A}}{\partial z^2}$$

$$\left(\vec{A}.\overrightarrow{\text{grad}}\right)\vec{B} = \left(\vec{A}\cdot\vec{\nabla}\right)\vec{B} = \left(A_x \frac{\partial}{\partial x} + A_y \frac{\partial}{\partial y} + A_z \frac{\partial}{\partial z}\right).\left(B_x\vec{u}_x + B_y\vec{u}_y + B_z\vec{u}_z\right)$$

Coordonnées cylindriques

$$\overrightarrow{\text{grad}}\, V = \frac{\partial V}{\partial r}\,\vec{u}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\,\vec{u}_\theta + \frac{\partial V}{\partial z}\,\vec{u}_z$$

$$\text{div}\,\vec{A} = \frac{1}{r}\frac{\partial (r\,A_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$$

$$\overrightarrow{\text{rot}}\,\vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right)\vec{u}_r + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\vec{u}_\theta + \frac{1}{r}\left(\frac{\partial (r\,A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)\vec{u}_z$$

$$\Delta V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\Delta \vec{A} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \vec{A}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \vec{A}}{\partial \theta^2} + \frac{\partial^2 \vec{A}}{\partial z^2}$$

$$\left(\vec{A}.\overrightarrow{\text{grad}}\right)\vec{B} = \left(A_r \frac{\partial}{\partial r} + \frac{A_\theta}{r}\frac{\partial}{\partial \theta} + A_z \frac{\partial}{\partial z}\right).\left(B_r\vec{u}_r + B_\theta\vec{u}_\theta + B_z\vec{u}_z\right)$$

Coordonnées sphériques

$$\overrightarrow{\text{grad}}\, V = \frac{\partial V}{\partial r}\,\vec{u}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\,\vec{u}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi}\,\vec{u}_\varphi$$

$$\text{div}\,\vec{A} = \frac{1}{r^2}\frac{\partial (r^2\,A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial\,(\sin\theta\,A_\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial \varphi}$$

$$\overrightarrow{\text{rot}}\,\vec{A} = \left(\frac{1}{r}\frac{\partial \sin\theta\,A_\varphi}{\partial \theta} - \frac{\partial A_\theta}{\partial \varphi}\right)\vec{u}_r + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{\partial\, rA_\varphi}{\partial r}\right)\vec{u}_\theta + \frac{1}{r}\left(\frac{\partial (r\,A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)\vec{u}_\varphi$$

$$\Delta V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \varphi^2}$$

$$\Delta \vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \vec{A}}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial \vec{A}}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 \vec{A}}{\partial \varphi^2}$$

$$\left(\vec{A}.\overrightarrow{\text{grad}}\right)\vec{B} = \left(A_r \frac{\partial}{\partial r} + \frac{A_\theta}{r}\frac{\partial}{\partial \theta} + \frac{A_\varphi}{r\sin\theta}\frac{\partial}{\partial \varphi}\right).\left(B_r\vec{u}_r + B_\theta\vec{u}_\theta + B_\varphi\vec{u}_\varphi\right)$$

Calcul vectoriel

$$\overrightarrow{\text{rot}}\,\overrightarrow{\text{grad}}\, V = \vec{0}$$

$$\text{div}\,\overrightarrow{\text{rot}}\,\vec{A} = 0$$

$$\text{div}\,\overrightarrow{\text{grad}}\, V = \Delta\, V$$

$$\overrightarrow{\text{rot}}\,\overrightarrow{\text{rot}}\,\vec{A} = \overrightarrow{\text{grad}}\,\text{div}\,\vec{A} - \Delta\vec{A}$$

$$\overrightarrow{\text{grad}}\,(V_1\,.V_2) = V_1\,\overrightarrow{\text{grad}}\,V_2 + V_2\overrightarrow{\text{grad}}\,V_1$$

$$\overrightarrow{\text{rot}}\,(V\,\vec{A}) = V\,\overrightarrow{\text{rot}}\,\vec{A} + \overrightarrow{\text{grad}}\,V \wedge \vec{A}$$

$$\text{div}\,(V\,\vec{A}) = V\,\text{div}\,\vec{A} + \overrightarrow{\text{grad}}\,V.\vec{A}$$

$$\text{div}\,(\overrightarrow{A_1} \wedge \overrightarrow{A_2}) = \overrightarrow{A_2}\,.\overrightarrow{\text{rot}}\,\vec{A}_1 - \overrightarrow{A_1}.\overrightarrow{\text{rot}}\,\vec{A}_2$$

Théorèmes de Green-Ostrogradski (pas à savoir)

$\vec{S}$ étant une surface fermée, orientée vers l'extérieur et τ le volume intérieur à S	$\oiint_S \vec{A}\,.d\vec{S} = \iiint_\tau (\text{div}\,\vec{A})d\tau$

Théorème de Stokes-Ampère

C étant une courbe fermée, orientée (règle du tirebouchon) et $\vec{S}$ la surface intérieure à C	$\oint_C \vec{A}.d\vec{\ell} = \iint_S (\overrightarrow{\text{rot}}\,\vec{A}).d\vec{S}$