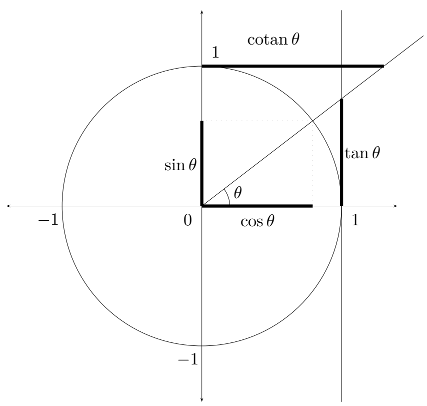
PC Lycée Pierre Corneille 2025/2026

Formulaire trigonométrie circulaire

Cercle trigonométrique



Formules élémentaires

$$\cos^2 x + \sin^2 x = 1 \qquad \tan x = \frac{\sin x}{\cos x}$$
$$1 + \tan^2 x = \frac{1}{\cos^2 x}$$

Ces formules sont valables lorsque leurs éléments sont définis

Formules "qui se voient"

$\cos(-x) = \cos x$
$\sin(-x) = -\sin x$
$\tan(-x) = -\tan x$
$\cos(\pi/2 - x) = \sin x$
$\sin(\pi/2 - x) = \cos x$
$\cos(x + \pi/2) = -\sin x$
$\sin(x + \pi/2) = \cos x$

$\cos(\pi - x) = -\cos x$
$\sin(\pi - x) = \sin x$
$\tan(\pi - x) = -\tan x$
$\cos(x + \pi) = -\cos x$
$\sin(x + \pi) = -\sin x$
$\tan(x + \pi) = \tan x$

Formules d'addition

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$
$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$
$$\sin(a+b) = \sin a \cos b + \sin b \cos a$$
$$\sin(a-b) = \sin a \cos b - \sin b \cos a$$
$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$
$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

Formules de duplication

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$
$$\sin 2x = 2 \sin x \cos x$$
$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Formules de linéarisation

$$\cos^2 x = \frac{1 + \cos 2x}{2} \qquad \sin^2 x = \frac{1 - \cos 2x}{2}$$

Transformation sommes en produits (à retrouver)

$$\cos p + \cos q = 2\cos\frac{p+q}{2}\cos\frac{p-q}{2}$$
$$\cos p - \cos q = -2\sin\frac{p+q}{2}\sin\frac{p-q}{2}$$
$$\sin p + \sin q = 2\sin\frac{p+q}{2}\cos\frac{p-q}{2}$$
$$\sin p - \sin q = 2\cos\frac{p+q}{2}\sin\frac{p-q}{2}$$

Transformation produits en sommes (à retrouver)

$$\cos a \cos b = \tfrac{1}{2}(\cos(a-b) + \cos(a+b))$$
$$\sin a \sin b = \tfrac{1}{2}(\cos(a-b) - \cos(a+b))$$
$$\sin a \cos b = \tfrac{1}{2}(\sin(a-b) + \sin(a+b))$$

Équations trigonométriques

$$\begin{aligned}
\cos x = \cos a &\Leftrightarrow x = a\ [2\pi]\ \textbf{ou}\ \ x = -a\ [2\pi] \\
&\Leftrightarrow \exists k \in \mathbb{Z} \,/\, x = a + 2k\pi\ \textbf{ou}\ x = -a + 2k\pi \\
\sin x = \sin a &\Leftrightarrow x = a\ [2\pi]\ \textbf{ou}\ \ x = \pi - a\ [2\pi] \\
&\Leftrightarrow \exists k \in \mathbb{Z} \,/\, x = a + 2k\pi\ \textbf{ou}\ x = \pi - a + 2k\pi \\
\tan x = \tan a &\Leftrightarrow x = a\ [\pi], \quad (\text{pour } a \neq \tfrac{\pi}{2} + n\pi,\ n \in \mathbb{Z}) \\
&\Leftrightarrow \exists k \in \mathbb{Z} \,/\, x = a + k\pi,
\end{aligned}$$

Formules en $t = \tan\frac{\theta}{2}$

On pose $t = \tan\left(\frac{\theta}{2}\right)$

$$\cos\theta = \frac{1-t^2}{1+t^2}$$
$$\sin\theta = \frac{2t}{1+t^2}$$
$$\tan\theta = \frac{2t}{1-t^2}$$

Angles remarquables

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\times$

Formules d'Euler et de De Moivre

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$
$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$
$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i \sin n\theta$$