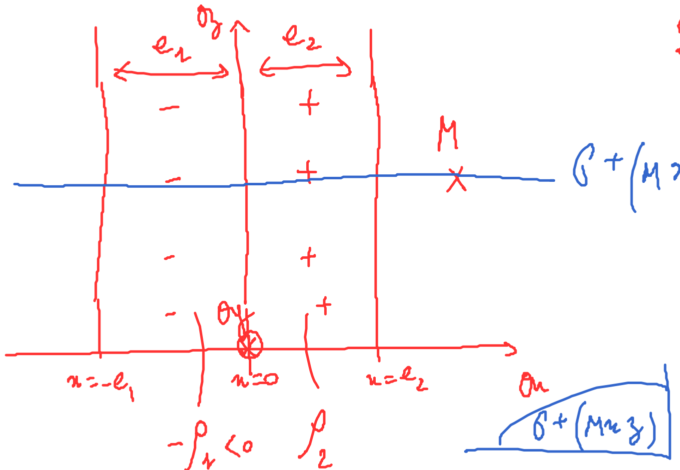
Correction DM 8

1)



Conseil : ne pas faire des dessins en 3D

M est un point quelconque

2) $M \in \Pi^+(M x y)$ et $\Pi^+(M x z)$

donc $\vec{E}(M) \in$ à ces 2 plans

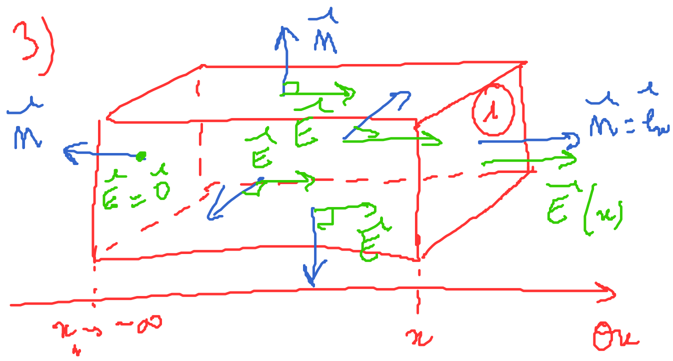
donc $\vec{E}$ est selon (Ox)

$\|\vec{E}(M)\| = E(x, \cancel{y}, \cancel{z})$

invariance par translation selon Oy et Oz

$$\boxed{\vec{E}(M) = E(x)\,\vec{e}_x}$$

3)



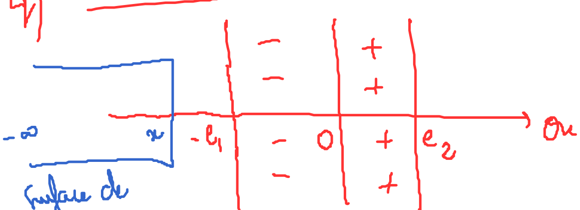
$\phi = \oiint \vec{E}(M) \cdot dS\,\vec{n}(M)$

$= \iint_{1} E(x)\vec{e}_x \cdot dS\,\vec{e}_x + 0$

↳ en $x_1 = -\infty$ le champ est nul

$$\boxed{\phi = E(x)\iint_1 dS = E(x)S}$$

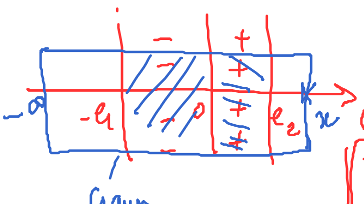
4) $x < -e_1$:



$q_{int} = 0 \Rightarrow \phi = E(x)S = 0$

donc $\boxed{E = 0}$

$x > e_2$:



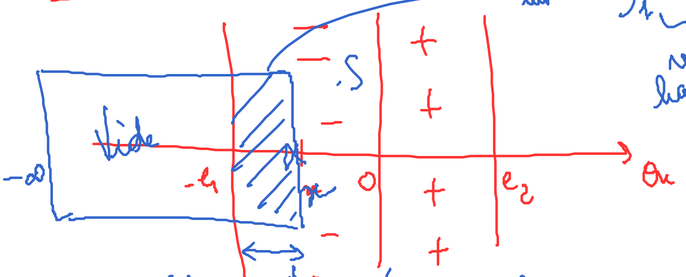
$q_{int} = 0 = +\overset{>0}{Q_1} + \overset{<0}{Q_2}$

$\phi = E(x)S = 0$

$$\boxed{\begin{aligned}\vec{E}(x > e_2) &= \vec{0}\\ \vec{E}(x < -e_1) &= \vec{0}\end{aligned}}$$

$-e_1 < x < 0$:

$Q_{int} = -\rho_1 \underbrace{S(x+e_1)}_{\text{volume hachuré}}$



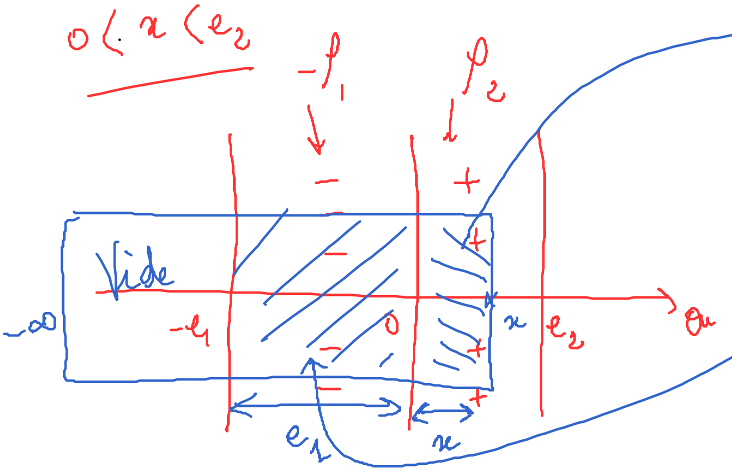
cette distance est $x - (-e_1) = x + e_1$

$\phi = E(x)\cancel{S} = \frac{q_{int}}{\varepsilon_0} = -\frac{\rho_1 \cancel{S}(x+e_1)}{\varepsilon_0}$

$$\boxed{\vec{E} = -\frac{\rho_1 (x+e_1)}{\varepsilon_0}\,\vec{e}_x}$$

⚠ $\vec{E}$ ne doit pas dépendre du choix de la surface de Gauss (ne dépend pas de S)

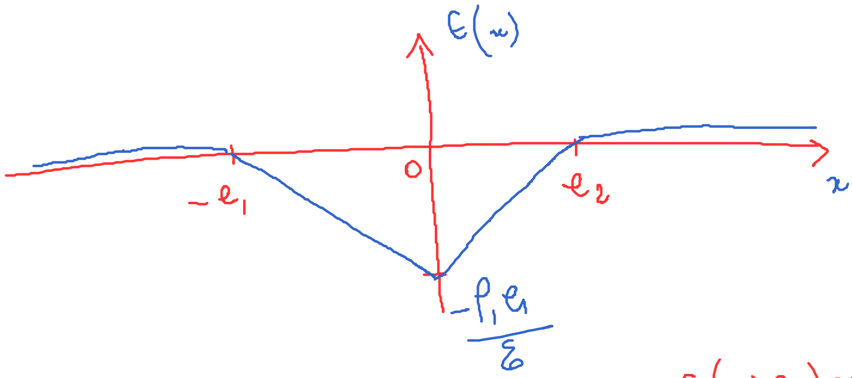
$0 < x < e_2$



$$Q_{int} = -\rho_1 \times (e_1 S) + \rho_2 \times (x S)$$

$$\phi = E(x)\cancel{S} = \frac{-\rho_1 e \cancel{S} + \rho_2 x \cancel{S}}{\varepsilon}$$

$$\boxed{\vec{E} = \frac{-\rho_1 e_1 + \rho_2 x}{\varepsilon}\,\vec{e}_x}$$



$E(x<-e_1)=0$

$E(x>e_2)=0$

$E(-e_1<x<0) = -\frac{\rho_1(x+e_1)}{\varepsilon}$

$E(0<x<e_2) = \frac{(-\rho_1 e_1 + \rho_2 x)}{\varepsilon}$

donc $E(x=-e_1)=0$

$E(x=0) = -\frac{\rho_1 e_1}{\varepsilon}$

donc $E(x=0) = -\frac{\rho_1 e_1}{\varepsilon}$

$E(x=e_2) = \frac{-\rho_1 e_1 + \rho_2 e_2}{\varepsilon} = 0$

5) On applique $\vec{E} = -\vec{grad}\, V = -\frac{dV}{dx}\vec{e}_x$

Pour $x<-e_1$: $E=0$ $\quad \frac{dV}{dx}=0 \quad V(x)=A$

Pour $-e_1<x<0$: $E = -\frac{\rho_1(x+e_1)}{\varepsilon} \quad \frac{dV}{dx} = +\frac{\rho_1(x+e_1)}{\varepsilon} \quad V(x) = +\frac{\rho_1 x^2}{2\varepsilon} + \frac{\rho_1 x}{\varepsilon} + B$

$V(x=0) = B = 0$

Pour $0<x<e_2$: $E = \frac{-\rho_1 e_1 + \rho_2 x}{\varepsilon} \quad \frac{dV}{dx} = \frac{+\rho_1 e_1 - \rho_2 x}{\varepsilon} \quad V(x) = +\frac{\rho_1 e_1 x}{\varepsilon} - \frac{\rho_2 x^2}{2\varepsilon} + C$

$V(x=0) = C = 0$

Pour $x > e_2$: $E = 0$ $\quad \frac{dV}{dx} = 0$ $\quad V(x) = D$

On trouve A et D avec la continuité du potentiel :

en $x = -e_1$: $V(x = -e_1^-) = V(x = -e_1^+)$

$$A = \rho_1 \frac{(-e_1)^2}{2\varepsilon_0} + \frac{\rho_1(-e_1)}{\varepsilon}$$

en $x = +e_2$: $V(x = e_2^-) = V(x = e_2^+)$

$$\frac{\rho_1 e_1 e_2}{\varepsilon} - \frac{\rho_2 e_2^2}{2\varepsilon_0} = D$$