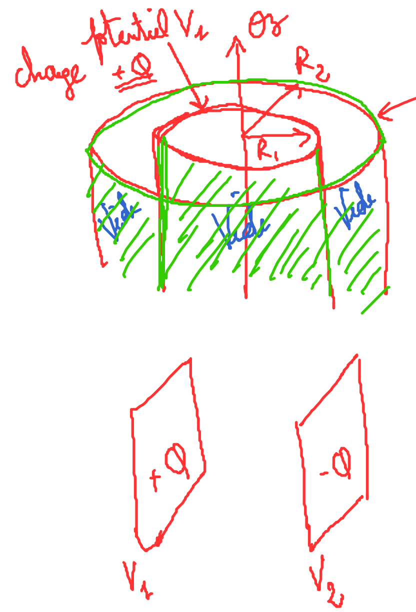
Exercice V · condensateur cylindrique

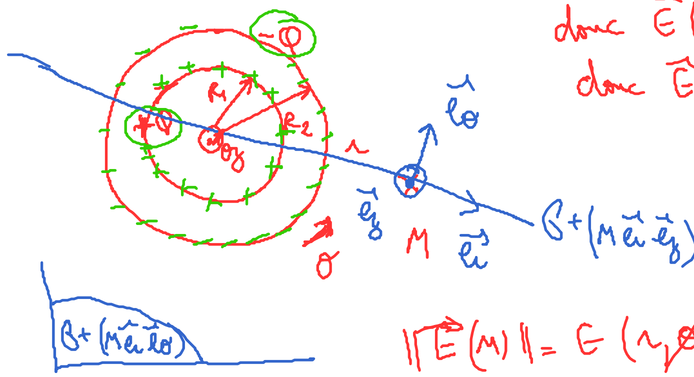


1) Etape 1 :

$M \in \mathcal{P}^+(M\vec{e_r}\vec{e_z})$ et $\mathcal{P}^+(M\vec{e_r}\vec{e_\theta})$

donc $\vec{E}(M) \in$ à ces 2 plans

donc $\vec{E}(M)$ est selon $\vec{e_r}$

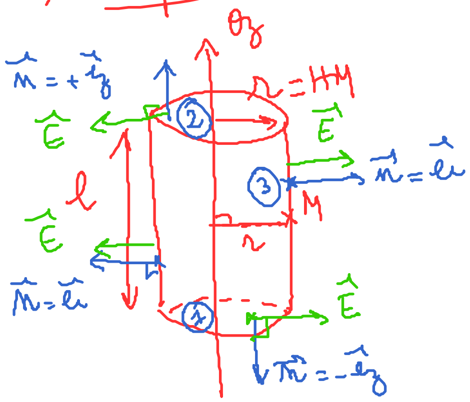


invariance par translation

$\|\vec{E}(M)\| = E(r, \cancel{\theta}, \cancel{z})$

invariance par rotation

2) Etape 2 .



$$\Phi = \oiint \vec{E}(M)\, dS\, \vec{n}(M) = \underbrace{\iint \cancel{E(r)\vec{e_r}\cdot dS\vec{e_z}}}_{1} + \underbrace{\iint \cancel{E(r)\vec{e_r} dS(-\vec{e_z})}}_{2} + \underbrace{\iint E(r)\vec{e_r}\, dS\, \vec{e_r}}_{3}$$

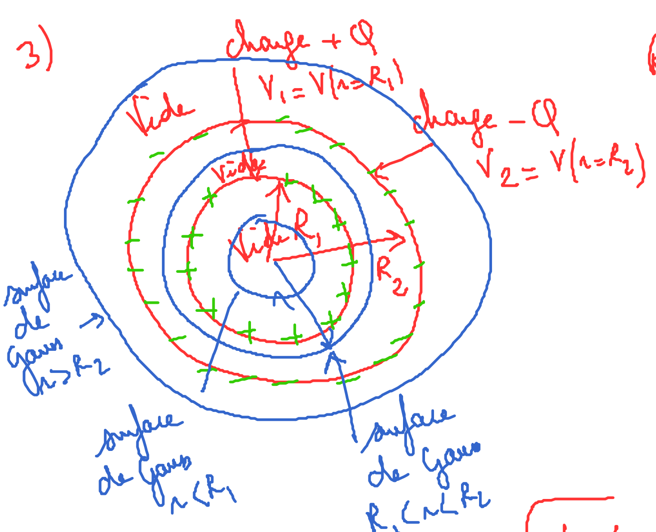
$$\Phi = E(r) \iint dS$$

surface latérale $= 2\pi r l$

$$\boxed{\Phi = E(r)\, 2\pi r l}$$

$E(r) > 0$

3)



$R_1 < r < R_2$: $q_{int} = +Q$

$$\Phi = E(r)\, 2\pi r l = \frac{Q}{\varepsilon}$$

$$\boxed{\vec{E}(r) = \frac{Q}{2\pi r l \varepsilon}\vec{e_r}}$$

$\vec{OM} = r\vec{e_r} + z\vec{e_z}$

$$V_1 - V_2 = V(r=R_1) - V(r=R_2) = \int_{r=R_1}^{r=R_2} \vec{E}(M)\, d\vec{OM}$$

$$\boxed{V_1 - V_2 = \int_{R_1}^{R_2} \frac{Q}{2\pi r l \varepsilon}\vec{e_r}} \left(dr\,\vec{e_r} + \dots \vec{e_\theta} + \dots \vec{e_z}\right) = \int_{R_1}^{R_2} \frac{Q}{2\pi r l \varepsilon} dr = \boxed{\frac{Q}{2\pi \varepsilon l} \ln \frac{R_2}{R_1}}$$

Rq : $r < R_1$: $q_{int} = 0$ $\quad \Phi = E\, 2\pi r l = 0 \Rightarrow \boxed{\vec{E} = \vec{0}}$

$r > R_2$: $q_{int} = +Q - Q = 0 \Rightarrow \boxed{\vec{E} = \vec{0}}$

4) $\underbrace{V_1 - V_2}_{U} = \frac{Q}{2\pi\varepsilon l}\ln\frac{R_2}{R_1}$ $\qquad Q = CU \rightarrow \boxed{C = \frac{Q}{U} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{2\pi\varepsilon l}\ln\frac{R_2}{R_1}} = \frac{2\pi\varepsilon l}{\ln\frac{R_2}{R_1}}}$

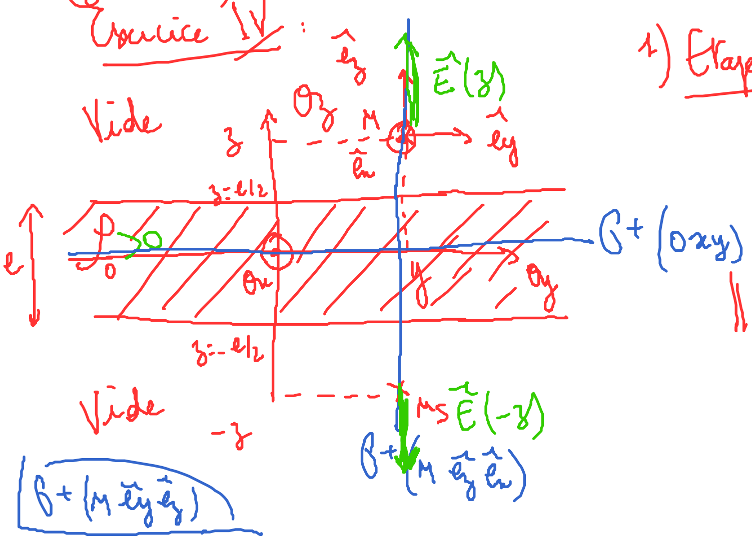
$[\varepsilon] = F.m^{-1}$ $\qquad [C] = [\varepsilon \times l] = F$

5) $u_e = \frac{\varepsilon E^2}{2}$ en $J m^{-3}$ $\qquad U_e = \iiint \frac{\varepsilon E^2}{2} d\tau = \iiint \frac{\varepsilon}{2}\left(\frac{Q}{2\pi\varepsilon l r}\right)^2 r\,dr\,d\theta\,dz$

$$U_e = \int_{R_1}^{R_2} \frac{\varepsilon}{2}\frac{Q^2}{4\pi^2\varepsilon^2 l^2 r}\, dr \times \underbrace{\int_0^{2\pi} d\theta}_{2\pi} \times \underbrace{\int_0^{l} dz}_{l} = \frac{Q^2}{8\pi^2\varepsilon l^2}\ln\frac{R_2}{R_1} \times 2\pi l = \underbrace{\frac{Q^2}{4\pi\varepsilon l}\ln\frac{R_2}{R_1}}_{\frac{1}{2}\frac{Q^2}{C}}$$

$U_e = \frac{1}{2}CU^2 = \frac{1}{2}\frac{Q^2}{C}$ $\qquad \Rightarrow \boxed{C = \frac{2\pi\varepsilon l}{\ln(R_2/R_1)}}$

Exercice IV :



1) Étape 1 : $M \in \Pi^+(M\vec{e_z}\vec{e_x})$ et $\Pi^+(M\vec{e_y}\vec{e_z})$

donc $\vec{E}(M) \in$ à ces 2 plans

donc $\vec{E}(M)$ selon (Oz)

$\|\vec{E}(M)\| = E(\cancel{x}, \cancel{y}, z)$

invariance par translation selon (Ox) et (Oy)

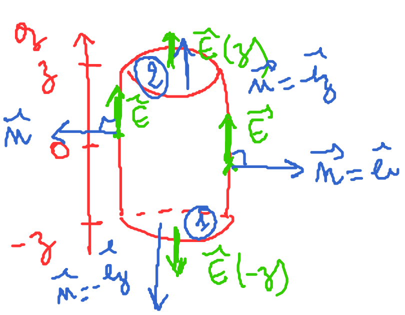
donc $\boxed{\vec{E}(z) = E(z)\vec{e_z}}$

$M(z)$ et $M_s(-z)$ sont symétriques / au plan $\Pi(Oxy)$ (est un plan Π^+)

donc $\vec{E}(z)$ et $\vec{E}(-z)$ sont symétriques / à ce plan

donc $\vec{E}(-z) = -\vec{E}(z)$ (d'après le schéma)

2) Étape 2 :

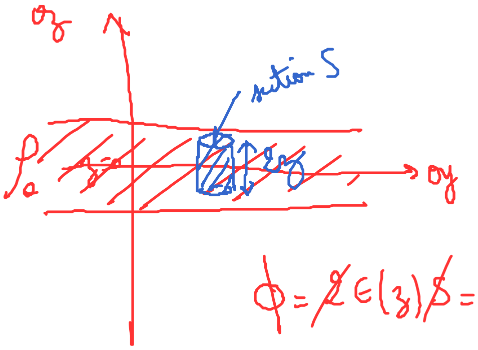


$$\Phi = \oiint \vec{E}(M)\, dS\, \vec{n}(M) = \underbrace{\iint_{①} \vec{E}(-z)\, dS(-\vec{e_z}) + \iint_{②} \vec{E}(+z)\, dS\, \vec{e_z}}_{\text{ces flux sont égaux}} + \underbrace{0}_{\text{surface latérale } \vec{n} \perp \vec{E}}$$

$$\boxed{\Phi = 2\iint_{(2)} E(+z)\,\vec{e}_z\, dS\,\vec{e}_z = 2E(z)\underbrace{\iint_{(2)} dS}_{S} = 2E(z)S}$$

3) Etape 3 : ici $z > 0$

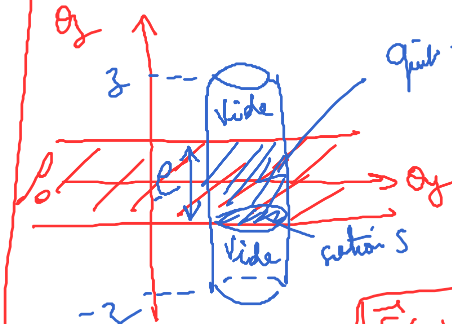
Cas 1 : $z < \frac{e}{2}$: $q_{int} = \rho_0 \times \underbrace{(S\, 2z)}_{\text{volume du cylindre de Gauss}}$



$$\Phi = \cancel{2}E(z)\cancel{S} = \frac{\rho_0 \cancel{S}\, \cancel{2}z}{\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{\rho_0 z}{\varepsilon_0}\vec{e}_z} \quad 0 < z < \frac{e}{2}$$

Cas 2 $z > \frac{e}{2}$.



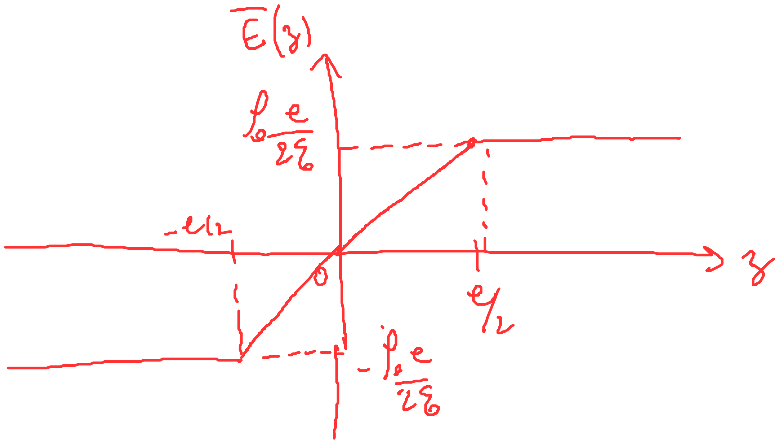
$q_{int} = \rho_0 \times \overbrace{eS}^{\text{volume hachuré}}$

$$\Phi = 2E(z)\cancel{S} = \frac{\rho_0 e \cancel{S}}{\varepsilon_0}$$

$$\boxed{\vec{E}(z) = \frac{\rho_0 e}{2\varepsilon_0}\vec{e}_z} \quad z > \frac{e}{2}$$

$$\vec{E}(z) = \underbrace{E(z)}_{>0 \text{ ou } <0}\vec{e}_z$$

$\vec{E}(-z) = -\vec{E}(z) \Rightarrow E(z)$ est impaire



4) $E\left(z > \frac{e}{2}\right) = \frac{\rho_0 e}{2\varepsilon_0} = -\frac{dV}{dz} \Rightarrow V = -\frac{\rho_0 e z}{2\varepsilon_0} + A$

$E\left(z < -\frac{e}{2}\right) = -\frac{\rho_0 e}{2\varepsilon_0} = -\frac{dV}{dz} \Rightarrow V = \frac{\rho_0 e z}{2\varepsilon_0} + B$

$E\left(-\frac{e}{2} < z < \frac{e}{2}\right) = \frac{\rho_0 z}{2\varepsilon_0} = -\frac{dV}{dz} \Rightarrow V = -\frac{\rho_0 z^2}{4\varepsilon_0} + C$ or $V(z=0) = 0$

$\Rightarrow C = 0$

$\vec{E} = -\vec{\text{grad}}\, V = -\frac{dV}{dz}\vec{e}_z \Rightarrow E = -\frac{dV}{dz}$

$V\left(z = \frac{e}{2}^-\right) = V\left(z = \frac{e}{2}^+\right)$