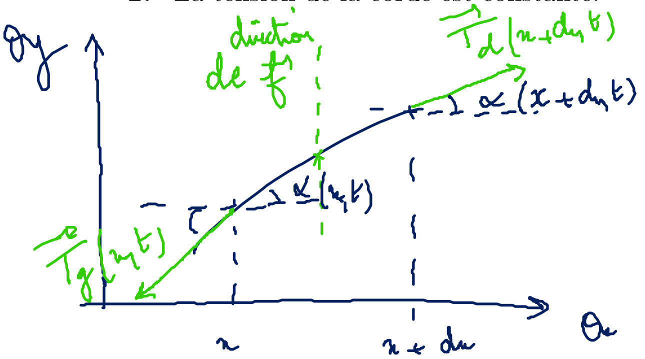
I. Corde vibrante dans un fluide visqueux

La portion de corde comprise entre x et $x+dx$ à l'instant t, subit la force $\overbrace{-hdx\vec{v}(x,t)}$.

1. Unité de h:

$$[h] = \left[\frac{F}{dx\, v}\right] = \frac{kg\ m s^{-2}}{m\ m s^{-1}} = kg\ m^{-1} s^{-1}$$

2. La tension de la corde est constante:



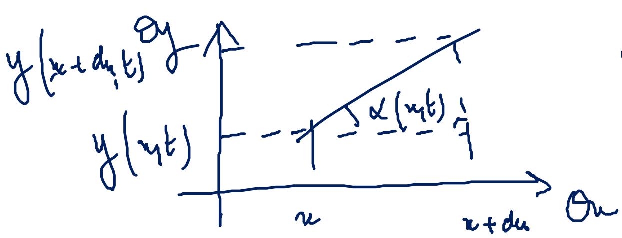
On applique la RFD au système infinitésimal entre x et $x+dx$:

$$\mu\, dx \frac{\partial^2 y}{\partial t^2}\vec{e}_y = \vec{T}_d(x+dx,t) + \vec{T}_g(x,t) - h\,dx\frac{\partial y}{\partial t}\vec{e}_y$$

On la projette sur (Ox) : $0 = \|\vec{T}_d(x+dx,t)\| \overbrace{\cos\alpha(x+dx,t)}^{\simeq 1} - \|\vec{T}_g(x,t)\| \overbrace{\cos\alpha(x,t)}^{\simeq 1}$

$$\|\vec{T}_d(x+dx,t)\| = \|\vec{T}_g(x,t)\| = T_0 \quad : \text{tension uniforme}$$

3. Exprimer $\alpha(x,t)$ en fonction d'une dérivée partielle de $y(x,t)$:



$$\tan\alpha(x,t) = \frac{\overbrace{y(x+dx,t) - y(x,t)}^{=\frac{\partial y}{\partial x}\times dx}}{dx} \simeq \boxed{\alpha(x,t) \simeq \frac{\partial y}{\partial x}(x,t)}$$

4. Equation de propagation:

On projette la RFD sur (Oy) : $\mu\, dx \frac{\partial^2 y}{\partial t^2} = T_0 \overbrace{\sin\alpha(x+dx,t)}^{\alpha(x+dx,t)} - T_0 \overbrace{\sin\alpha(x,t)}^{\alpha(x,t)} - h\,dx\frac{\partial y}{\partial t}$

$$\mu \cancel{dx} \frac{\partial^2 y}{\partial t^2} = T_0\left[\alpha(x+dx,t) - \alpha(x,t)\right] - h\,dx\frac{\partial y}{\partial t} = T_0\frac{\partial \alpha}{\partial x}\cancel{dx} - h\cancel{dx}\frac{\partial y}{\partial t}$$

$$\mu\frac{\partial^2 y}{\partial t^2} = T_0\frac{\partial^2 y}{\partial x^2} - h\frac{\partial y}{\partial t} \Rightarrow \boxed{\underbrace{\frac{\partial^2 y}{\partial x^2} - \frac{\mu}{T_0}\frac{\partial^2 y}{\partial t^2}}_{\text{éq. de d'Alembert avec } c=\sqrt{\mu/T_0}} - \frac{h}{T_0}\frac{\partial y}{\partial t} = 0}$$

terme supplémentaire lié aux frottements

5. Solutions de la forme $\underbrace{\underline{y}(x,t) = y_0 e^{i(\omega t - \underline{k}x)}}_{\text{OPPH}^+}$:

$$\frac{\partial^2 y}{\partial x^2} - \frac{\mu}{T_0}\frac{\partial^2 y}{\partial t^2} - \frac{h}{T_0}\frac{\partial y}{\partial t} = 0$$

$$-\underline{k}^2 \underline{y} - \frac{\mu_0}{T_0}(-\omega^2)\underline{y} - \frac{h}{T_0} i\omega \underline{y} = 0 \quad \text{d'où} \quad \boxed{\underline{k}^2 = \overbrace{\frac{\mu_0 \omega^2}{T_0}}^{\omega^2/c^2} - \frac{i\omega h}{T_0}}$$

6. Dans le cas où $h >> \mu\omega$: $\underline{k}^2 \simeq -i\omega \frac{h}{T_0} \simeq e^{-i\pi/2} \omega \frac{h}{T_0}$

$$\underline{k} = e^{-i\pi/4} \sqrt{\frac{\omega h}{T_0}} = \left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)\sqrt{\frac{\omega h}{T_0}} = \frac{1-i}{\delta} \quad \text{avec } \delta = \sqrt{\frac{2T_0}{\omega h}}$$

$Re(\underline{k}) = \frac{1}{\delta} \neq 0 \Rightarrow$ il y a propagation $\quad Im(\underline{k}) = -\frac{1}{\delta} \neq 0 \Rightarrow$ il y a absorption

$$\underline{y} = y_0 e^{i\left(\omega t - \frac{1-i}{\delta} x\right)} = y_0 e^{-x/\delta} e^{i(\omega t - x/\delta)}$$

$$\boxed{y(x,t) = Re\left(\underline{y}(x,t)\right) = \underbrace{y_0 e^{-x/\delta}}_{\text{amplitude qui } \downarrow \text{ quand l'onde avance}} \cos\big(\underbrace{\omega t - x/\delta}_{\text{phase}}\big)}$$

(OPPH+)

7. Exprimer la vitesse de phase. Conclure.

$$\boxed{v_\varphi = \frac{\omega}{Re(\underline{k})} = \delta\omega = \sqrt{\frac{2T_0\omega}{h}}}$$

v_φ dépend de ω

il y a dispersion