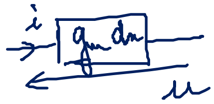
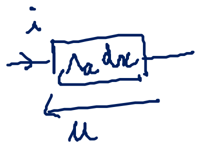
I. Axone

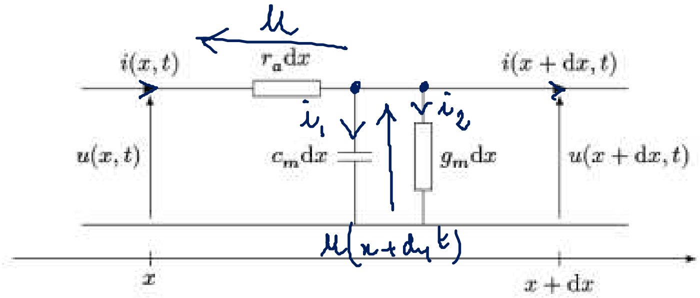


$i = g_m dx\, u$

Données: $r_a = 6,4.10^9\ \Omega.m^{-1}$, $g_m = 63\ mS.m^{-1}$, $c_m = 0,32\ \mu F.m^{-1}$



$u = r_a dx\, i$



$i_1 = c_m dx \frac{\partial u}{\partial t}(x+dx,t)$

$i_2 = g_m dx\, u(x+dx,t)$

1. Loi des noeuds: $i(x,t) = i_1 + i_2 + i(x+dx,t)$ avec

$$i(x+dx,t) - i(x,t) = \frac{\partial i}{\partial x}\times dx = -c_m dx \frac{\partial u}{\partial t}(x+\cancel{dx},t) - g_m dx\, u(x+\cancel{dx},t)$$

D.L. à l'ordre 1 en dx

$$\boxed{\frac{\partial i}{\partial x} = -c_m \frac{\partial u}{\partial t} - g_m u} \quad (1)$$

Loi des mailles:

$u(x+dx,t) + u = u(x,t)$ donc $u(x+dx,t) - u(x,t) = \frac{\partial u}{\partial x}dx = -r_a dx\, i$

avec $u = r_a dx\, i(x,t)$

$$\boxed{\frac{\partial u}{\partial x} = -r_a i} \quad (2)$$

2. Equation de propagation de $u(x,t)$:

$$\frac{\partial^2 u}{\partial x^2} \overset{(2)}{=} -r_a \frac{\partial i}{\partial x} \overset{(1)}{=} -r_a\left[-c_m \frac{\partial u}{\partial t} - g_m u\right] \Rightarrow \boxed{\frac{\partial^2 u}{\partial x^2} - r_a c_m \frac{\partial u}{\partial t} - r_a g_m u = 0}$$

Solution de la forme $\underline{u}(x,t) = u_0 e^{i(\omega t - \underline{k}x)}$.

3. Simplifier l'équation de propagation pour $\omega >> \omega_c = \frac{g_m}{c_m}$

$$\frac{\left|r_a c_m \frac{\partial u}{\partial t}\right|}{\left|r_a g_m \underline{u}\right|} = \frac{\left|r_a c_m i\omega \underline{u}\right|}{\left|r_a g_m \underline{u}\right|} = \frac{c_m \omega}{g_m} \gg 1 \Rightarrow \boxed{\omega \gg \frac{g_m}{c_m} = \omega_c}$$

on peut négliger $r_a g_m u$ devant $r_a c_m \frac{\partial u}{\partial t}$

$$\boxed{\frac{\partial^2 u}{\partial x^2} = r_a c_m \frac{\partial u}{\partial t}}$$

éq. de diffusion

4. Quel est le phénomène décrit? éq. de propagation :

diffusion de particules : $\frac{\partial n}{\partial t} = D\Delta n = D\frac{\partial^2 n}{\partial x^2}$

$n(x,t)$

diffusion thermique : $\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c}\frac{\partial^2 T}{\partial x^2}$

5. On pose $\delta = \sqrt{\dfrac{2}{r_a c_m \omega}}$.

$$\begin{cases} \underline{u} = u_0 e^{i(\omega t - \underline{k} x)} \\ \dfrac{\partial^2 \underline{u}}{\partial x^2} = r_a c_m \dfrac{\partial u}{\partial t} \end{cases} \Rightarrow -\underline{k}^2 \underline{u} = r_a c_m i\omega \underline{u}$$

$$\boxed{\underline{k}^2 = \underbrace{-i}_{e^{-i\pi/2}} r_a c_m \omega}$$

$$\underline{k}^2 = e^{-i\pi/2} r_a c_m \omega$$

$$\boxed{\underline{k} = e^{-i\pi/4}\sqrt{r_a c_m \omega} = \left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)\sqrt{r_a c_m \omega} = \frac{1-i}{\delta}}$$

$Re(\underline{k}) = \dfrac{1}{\delta} \neq 0 \Rightarrow$ il y a propagation

$Im(\underline{k}) = -\dfrac{1}{\delta} \neq 0 \Rightarrow$ il y a de l'atténuation

$$\underline{u} = u_0 e^{i\left(\omega t - \frac{1-i}{\delta} x\right)} = u_0 e^{-x/\delta} e^{i(\omega t - x/\delta)}$$

$$\boxed{u(x,t) = Re(\underline{u}(x,t)) = \underbrace{u_0 e^{-x/\delta}}_{\text{amplitude qui} \downarrow \text{ quand l'onde se propage}} \cos\underbrace{(\omega t - x/\delta)}_{\text{phase}}}$$

propagation selon $+Ox$

6. Vitesse phase:

$$\boxed{v_\varphi} = \frac{\omega}{Re(\underline{k})} = \frac{\omega}{1/\delta} = \delta\omega = \boxed{\sqrt{\frac{2\omega}{r_a c_m}}}$$

v_φ dépend de ω

il y a de la dispersion