MPSI / PCSI Correction du DS3 du 14 Décembre 2024

Q1 : $Q_p(p) = K_s U_s(p)$ (i)

$\frac{V_0}{B} p \Delta p(t) = Q_p(p) - Q_m(p)$ (ii)

$Q_m(p) = C_y \Omega_m(p)$ (iii) $\quad C_m(p) = C_y \Delta P(p)$

$J_{eq} p \Omega_m(p) + a \Omega_m(p) = C_m(p) - C_r(p)$ (v)

Q2 : $\boxed{H_1(p) = \frac{Q_p(p)}{U_s(p)}}$ $\boxed{H_2(p) = \frac{\Delta P(p)}{Q_p(p) - Q_m(p)}}$

$\boxed{H_3(p) = \frac{C_m(p)}{\Delta P(p)}}$ $\boxed{H_4(p) = \frac{Q_m}{\Omega_m}}$ $\boxed{H_5(p) = \frac{\Omega_m}{C_m - C_r}}$

Q3 $H_1(p) = K_s$ $\quad H_2(p) = \frac{B}{V_0} \times \frac{1}{p}$ $\quad H_3(p) = C_y$

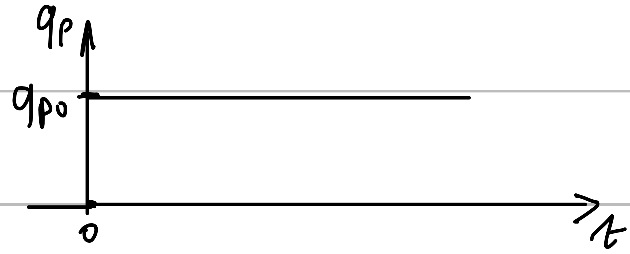
$\boxed{H_4(p) = C_y}$ $\boxed{H_5(p) = \frac{1}{J_{eq} p + a}}$

Q4 $$H(p) = \frac{\frac{B}{V_0} \times \frac{1}{p} \times C_y \times \frac{1}{J_{eq}p + a}}{1 + C_y \times \frac{B}{V_0} \times \frac{1}{p} \times C_y \times \frac{1}{J_{eq}p+a}} = \frac{B C_y}{V_0 p (J_{eq}p + a) + B C_y^2}$$

$$H(p) = \frac{B C_y}{V_0 J_{eq} p^2 + V_0 a p + B C_y^2} = \boxed{\frac{1/C_y}{1 + \frac{V_0 a}{B C_y^2} p + \frac{V_0 J_{eq}}{B C_y^2} p^2}}$$

Ordre 2 classe 0 et gain K_s/C_y

Q5



domaine temporel $e(t) = \begin{cases} q_{p0} & \text{si } t \geq 0 \\ 0 & \text{si } t < 0 \end{cases}$

domaine de Laplace :

$$\boxed{E(p) = \frac{q_{p0}\,(p)}{p}}$$

Théorème de la valeur finale

Q6

$$\omega_m(+\infty) = \lim_{t \to +\infty} \omega_m(t) = \lim_{p \to 0} p\,\Omega_m(p) = \lim_{p \to 0} \cancel{p} \times H(p)\,\frac{q_{p0}}{\cancel{p}}$$

$$= \frac{k_s}{C_y}\,q_{p0}$$

$$\text{erreur} = q_{p0} - \frac{k_s}{C_y}\,q_{p0} = q_{p0}\left(1 - \frac{k_s}{C_y}\right)$$

le système n'est pas précis

Q7 :

$$H_{cor} = \frac{\dfrac{K_p(1+\tau_i p)}{\tau_i p} \cdot \dfrac{\frac{1}{C_y}}{1 + \frac{aV_0}{BC_y^2}p + \frac{J_{eq}V_0}{BC_y^2}p^2}}{1 + \dfrac{\frac{K_p(1+\tau_i p)}{\tau_i p} \times \frac{1}{C_y}}{1 + \frac{aV_0}{BC_y^2}p + \frac{J_{eq}V_0}{BC_y^2}p^2}}$$

$$H_{cor} = \frac{\frac{K_p}{C_y} \times \frac{1+\tau_i p}{\tau_i p}}{1 + \frac{aV_0}{BC_y^2}p + \frac{J_{eq}V_0}{BC_y^2}p^2 + \frac{K_p(1+\tau_i p)}{\tau_i p} \times \frac{1}{C_y}}$$

$$H_{cor} = \frac{\frac{K_p}{C_y}(1+\tau_i p)}{\tau_i p\left(1 + \frac{aV_0}{BC_y^2}p + \frac{J_{eq}V_0}{BC_y^2}p^2\right) + \frac{K_p}{C_y} \times (1+\tau_i p)}$$

dans ce cas de figure $\lim_{t \to +\infty} \omega_m(t) = \lim_{p \to 0} p\,\Omega_m(p)$

$$= \lim_{p \to 0} \cancel{p}\,H_{corr}(p)\,\frac{U_{s0}}{\cancel{p}} = \underline{U_{s0}}$$

le système est précis

Q8 :

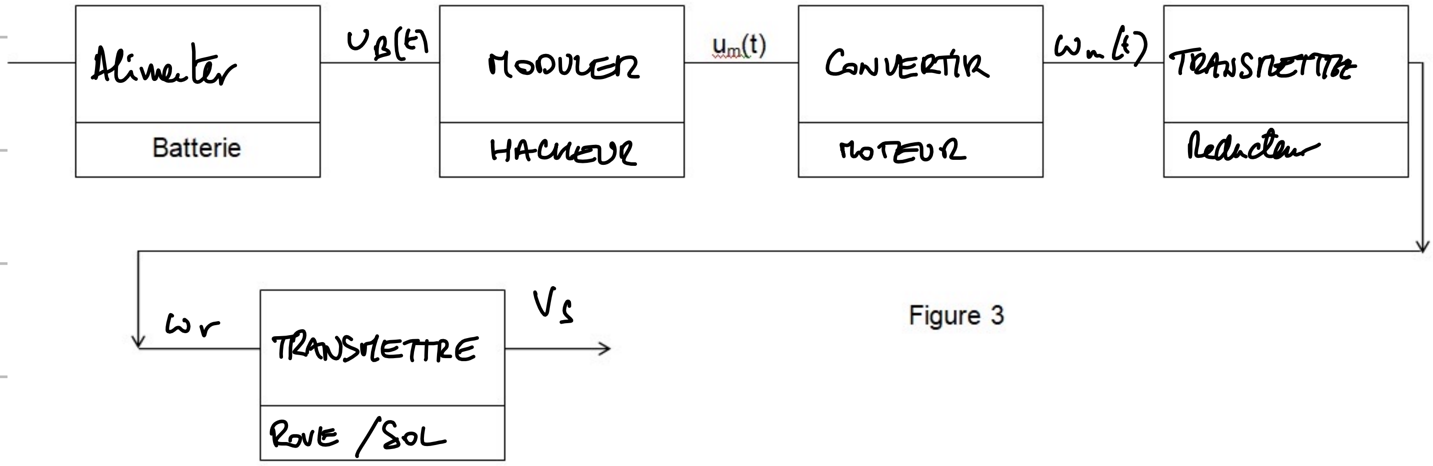


Figure 3

Q9 $\quad \dfrac{\omega_r}{\omega_m} = \dfrac{1}{r} \qquad \dfrac{V_s}{\omega_r} = \dfrac{D_{roue}}{2} \Rightarrow \boxed{\dfrac{V_s}{\omega_m} = \dfrac{D_{roue}}{2r}}$

A.N. $\quad i = \dfrac{0{,}4}{50 \times 2} = 0{,}004 \text{ m}$

Q10 $\quad \underset{[m/s]}{V_s} = \dfrac{D_{roue}}{2r} \times \omega_m \Rightarrow \underset{km/h}{V_s} = \omega_m \times \dfrac{D_{roue}}{2r} \times \dfrac{3600}{1000}$

$$= 1{,}8\, \omega_m \times \frac{D_{roue}}{r}$$

d'où $\quad \omega_m = \dfrac{r V_s}{1{,}8\, D_{roue}} \Rightarrow N_m = \dfrac{\omega_m}{2\pi} \times 60 = \dfrac{30\, \omega_m}{\pi}$

$$N_m = \frac{30}{\pi} \times \frac{r V_s}{1{,}8\, D_{roue}} = \boxed{\frac{30\, r V_s}{1{,}8 \pi\, D_{roue}} = N_m}$$

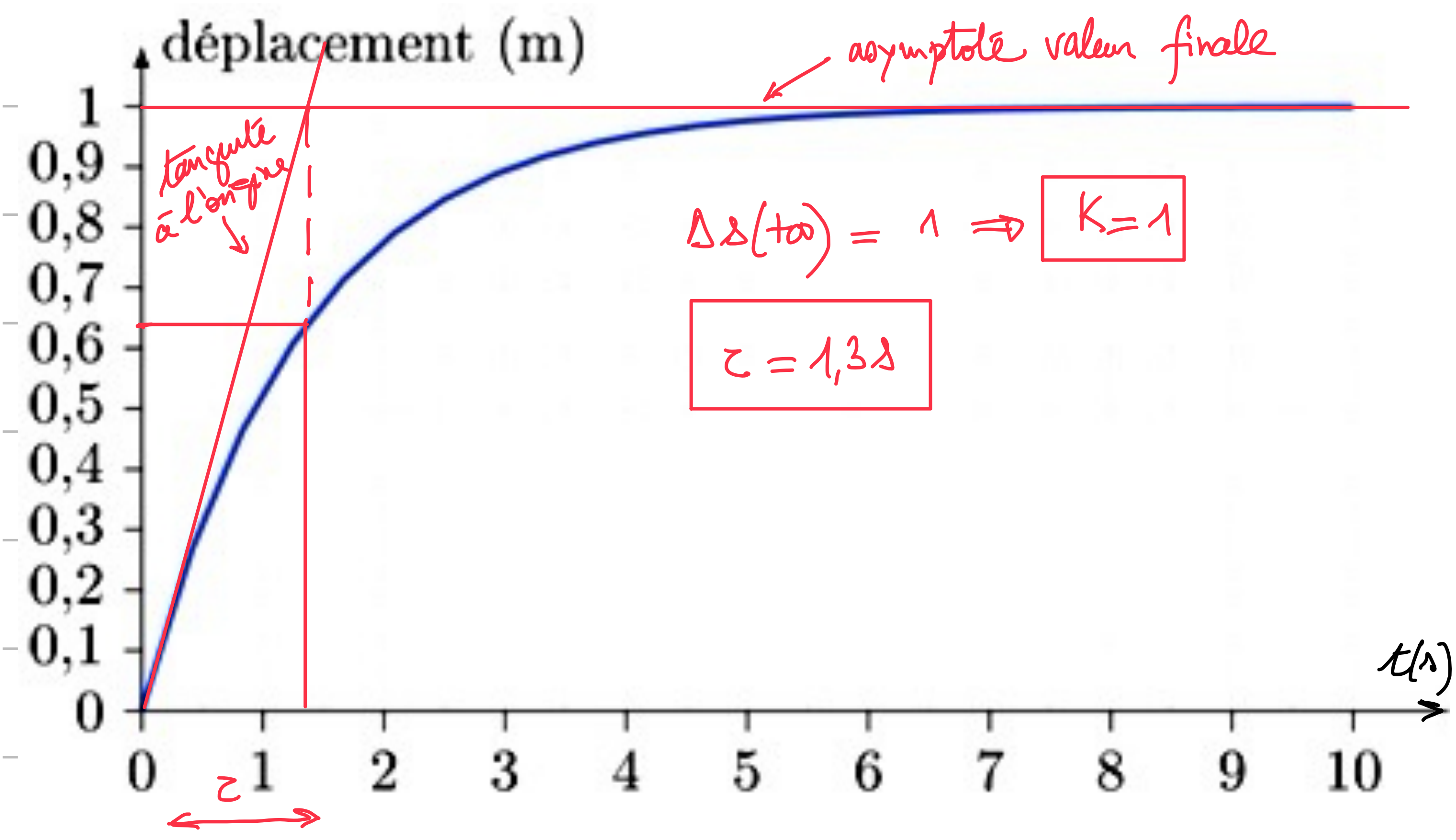
A.N. $\quad N_m = \dfrac{30 \times 50 \times 5}{1{,}8\, \pi \times 0{,}4} = 3315 \text{ tr/min}$

Q11 : $V_s = \frac{dx_s}{dt}$ $V_s(p) = p X_s(p) \Rightarrow \boxed{\frac{X_s(p)}{V_s(p)} = \frac{1}{p}}$

Q12 : Pas de dépassement, tangente à l'origine non nulle

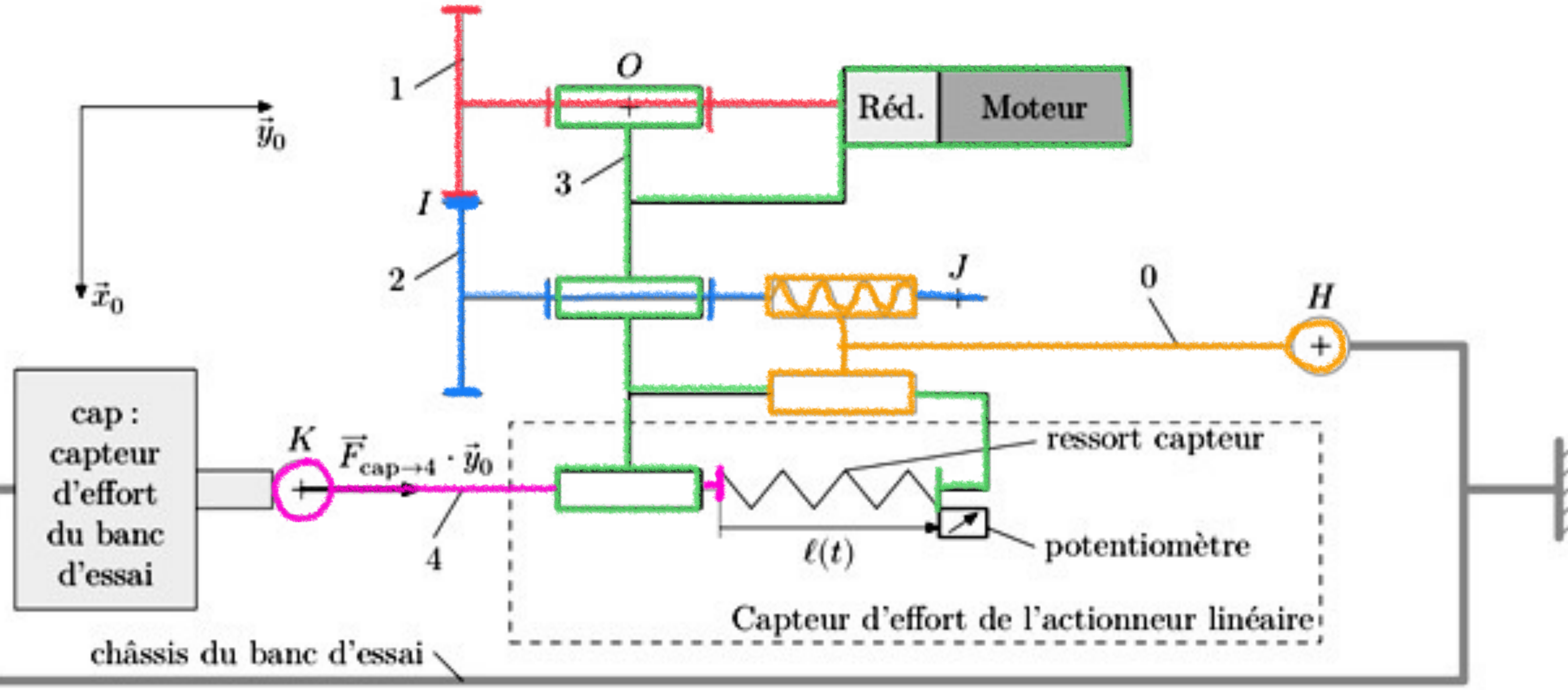
$\Rightarrow$ réponse du 1er ordre

Q13 :

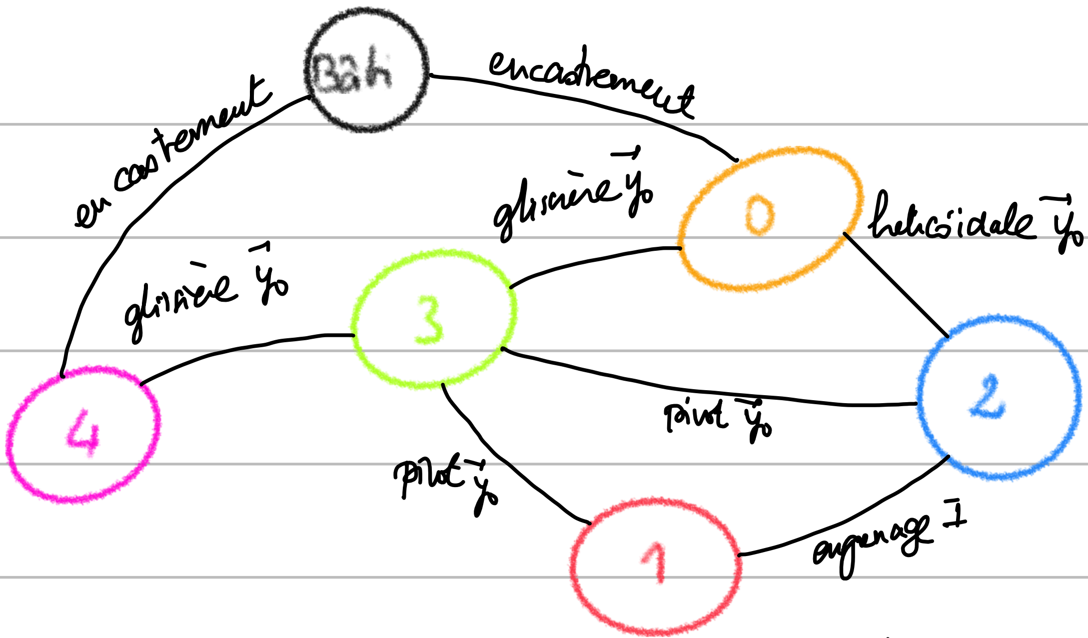


On a donc $\boxed{\frac{X_s(p)}{X_{s0}(p)} = \frac{1}{1+1,3\,p}}$

Q14 :



Q15 :



Q16 : Entre (1) et (3) on a un engrenage à contact extérieur
(3) et (0) on a système vis/écrou

Q17 :

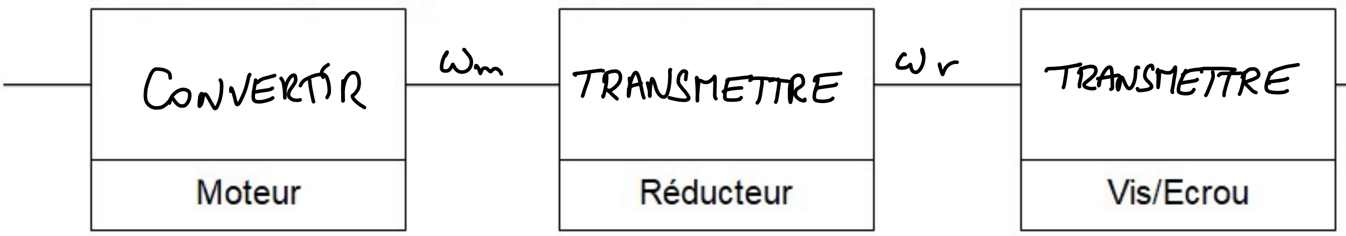


Figure 8

Q18 : $i_1 = \frac{\omega_r}{\omega_m} = \frac{1}{r}$ $\qquad \frac{V_{3/0}}{-\omega_r} = \frac{p}{2\pi}$ à l'entrée du système vis/écrou la vitesse ω_r a été inversée par les 2 roues dentées

$$i = i_1 i_2 = -\frac{1}{r} \times \frac{p}{2\pi} = \frac{V_{3/0}}{\omega_m}$$

or $\omega_m = \frac{N_m \times 2\pi}{60} = \frac{\pi N_m}{30}$

donc $i = -\frac{p}{2\cancel{\pi} \times r} = \frac{V_{3/0}}{\cancel{\pi} N_m} \times 30$

Finalement $\boxed{\frac{V_{3/0}}{N_m} = -\frac{p}{60 r}}$

Q19 Si $V_{3/0} = 1\ mm/s$ $\Rightarrow N_m = -\frac{60 r \times V_{3/0}}{p}$

$$\boxed{N_m = -\frac{60 \times 50 \times 10^{-3}}{10^{-3}} = 3000\ tr/min}$$

Q20 $\eta = \frac{P_s}{P_e} = \frac{F \times V}{C\omega} = \frac{F}{C} \times -\frac{p}{2\pi r}$

A.N. $\eta = \frac{\overset{10}{\cancel{500}}}{2} \times -\frac{10^{-3}}{2\pi \times \cancel{50}} = 7{,}9 \times 10^{-4}$

on peut dire que l'actionneur est un vérin électrique