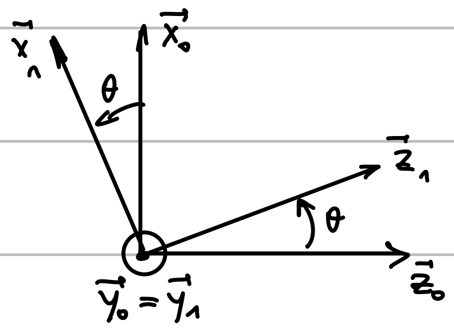
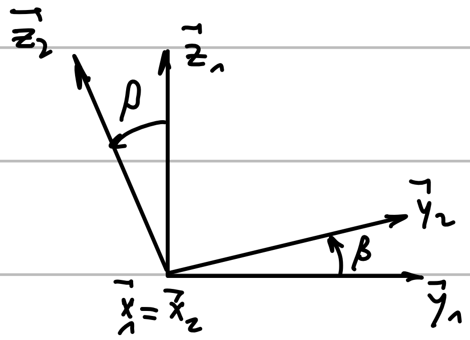
PCSI MPSI Correction D.L. vacances d'hiver 2025

Exercice 1

Q1





Q2 : $\vec{\Omega}_{2/1} = \dot{\beta}\,\vec{x}_{12}$ $\quad\vec{\Omega}_{1/0} = \dot{\theta}\,\vec{y}_{01}$ $\quad\vec{\Omega}_{2/0} = \vec{\Omega}_{2/1} + \vec{\Omega}_{1/0}$

$$= \dot{\beta}\,\vec{x}_{12} + \dot{\theta}\,\vec{y}_{01}$$

Exercice 2 :

Q1 : $\boxed{\vec{\Omega}_{2/1} = \omega_{2/1}\,\vec{z}_{12}}$

Q2 : On raisonne par rapport au solide 1

on a une transmission poulies/chaîne, donc :

$$\frac{\omega_{2/1}}{\omega_{0/1}} = \frac{R_0}{R_1} \quad \text{or } \omega_{0/1} = -\omega_{1/0}$$

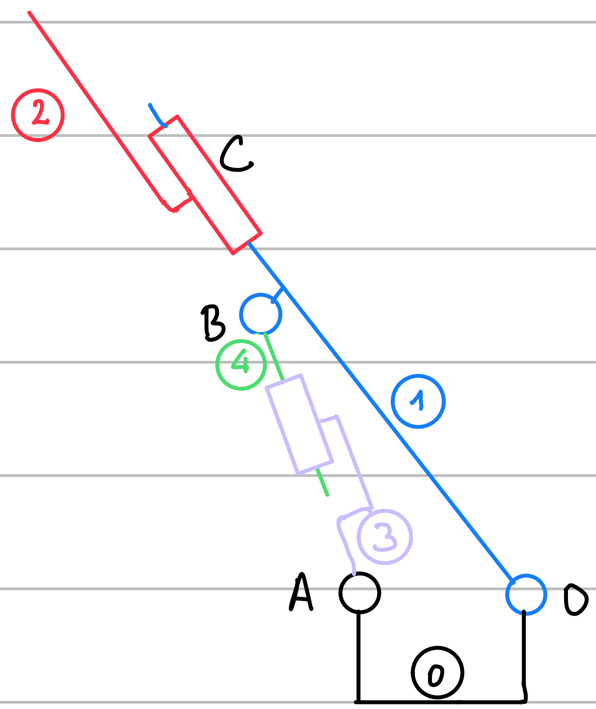
$$\text{d'où } \frac{\omega_{2/1}}{\omega_{1/0}} = -\frac{R_0}{R_1}$$

$$\boxed{\omega_{1/0} = -\frac{R_1}{R_0}\,\omega_{2/1}}$$

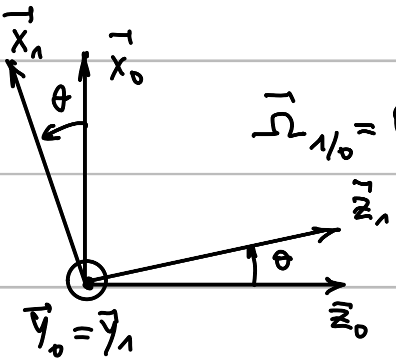
Q3 : $\boxed{\vec{\Omega}_{1/0} = -\frac{R_1}{R_0}\,\omega_{2/1}\,\vec{z}_{01}}$

Exercice 3 :

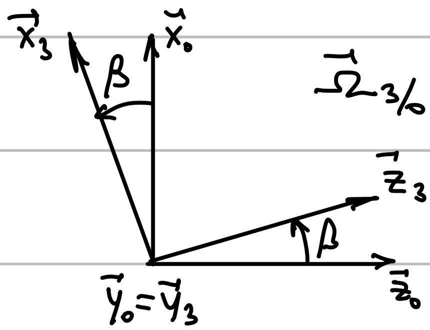
Q1 :



Q2



$$\vec{\Omega}_{1/0} = \dot{\theta}\, \vec{y}_{01}$$



$$\vec{\Omega}_{3/0} = \dot{\beta}\, \vec{y}_{03}$$

Q3 : On écrit une fermeture géométrique entre 0, A et B

$$\vec{F} = \vec{OA} + \vec{AB} + \vec{BO} = \vec{0}$$

$$\vec{F} = a\vec{x}_0 + \mu\vec{x}_3 - b\vec{x}_1 + c\vec{z}_1 = \vec{0}$$

$$\vec{F} = a\vec{x}_0 + \mu\left(-\sin\beta\,\vec{z}_0 + \cos\beta\,\vec{x}_0\right) - b\left(-\sin\theta\,\vec{z}_0 + \cos\theta\,\vec{x}_0\right) + c\left(\cos\theta\,\vec{z}_0 + \sin\theta\,\vec{x}_0\right) = \vec{0}$$

$$\vec{F}\cdot\vec{x}_0 = 0 = a + \mu\cos\beta - b\cos\theta + c\sin\theta \quad (1)$$

$$\vec{F}\cdot\vec{z}_0 = 0 = -\mu\sin\beta + b\sin\theta + c\cos\theta \quad (2)$$

(1) donne : $\mu \cos\beta = b\cos\theta - c\sin\theta - a$

$$\mu \sin\beta = b\sin\theta + c\cos\theta$$

d'où $\mu = \sqrt{(b\cos\theta - c\sin\theta - a)^2 + (b\sin\theta + c\cos\theta)^2}$

$$= \sqrt{b^2 - 2b\cos\theta(c\sin\theta + a) + (c\sin\theta + a)^2 + c^2\cos^2\theta + 2bc\sin\theta\cos\theta}$$

$$= \sqrt{b^2 \cancel{- 2bc\cos\theta\sin\theta} - 2ba\cos\theta + c^2\sin^2\theta + 2ac\sin\theta + a^2 + c^2\cos^2\theta \cancel{+ 2bc\sin\theta\cos\theta}}$$

$$\boxed{\mu = \sqrt{b^2 + a^2 + c^2 + 2a(c\sin\theta - b\cos\theta)}}$$

Q4 : $\vec{V}_{C\in 2/1} = \dfrac{d}{dt}\vec{OC}\Big|_1 = \dfrac{d\xi}{dt}\vec{x}_1 = \underline{\dot{\xi}\,\vec{x}_1}$

Q5 : $\vec{V}_{C\in 1/0} = \underbrace{\vec{V}_{O\in 1/0}}_{=\vec{0}} + \vec{CO}\wedge\vec{\Omega}_{1/0} = -\xi\,\vec{x}_1 \wedge \dot{\theta}\,\vec{y}_1 = \underline{-\xi\dot{\theta}\,\vec{z}_1}$

Q6 : $\vec{V}_{C\in 2/0} = \vec{V}_{C\in 2/1} + \vec{V}_{C\in 1/0} = \boxed{\dot{\xi}\,\vec{x}_1 - \xi\dot{\theta}\,\vec{z}_1}$

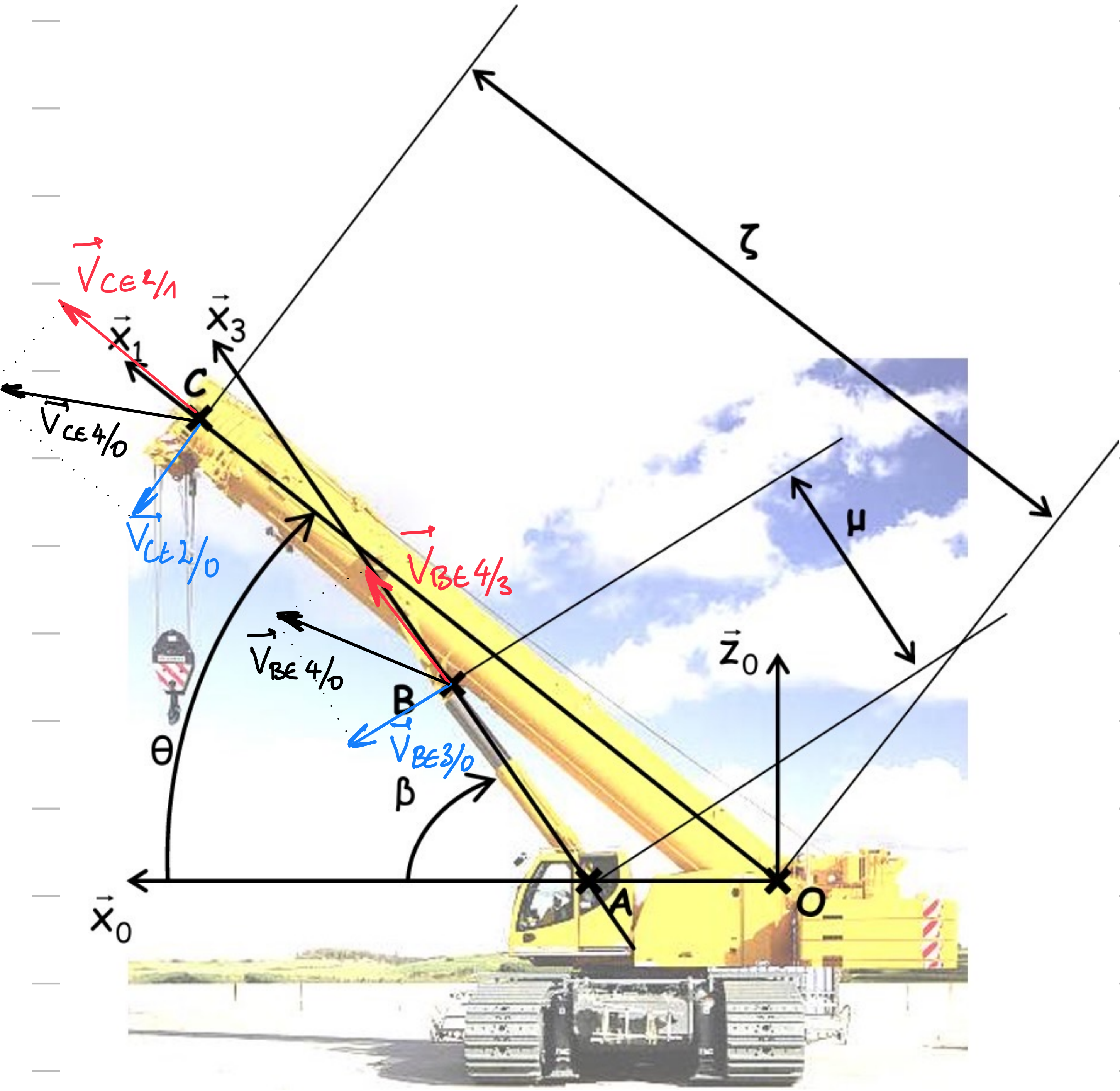
Q7 : $\vec{V}_{B\in 4/3} = \dfrac{d}{dt}\vec{AB}\Big|_3 = \dfrac{d}{dt}(\mu\,\vec{x}_3)\Big|_3 = \underline{\dot{\mu}\,\vec{x}_3}$

Q8 : $\vec{V}_{B\in 3/0} = \underbrace{\vec{V}_{A\in 3/0}}_{=\vec{0}} + \vec{BA} \wedge \vec{\Omega}_{3/0} = -\mu\,\vec{x}_3 \wedge \dot{\beta}\,\vec{y}_3 = -\mu\dot{\beta}\,\vec{z}_3$

Q9 : $\vec{V}_{B\in 4/0} = \vec{V}_{B\in 4/3} + \vec{V}_{B\in 3/0} = \boxed{\dot{\mu}\,\vec{x}_3 - \mu\dot{\beta}\,\vec{z}_3}$

Q10 : $\vec{V}_{B\in 4/1} = \vec{0}$

Q11 :



FIN