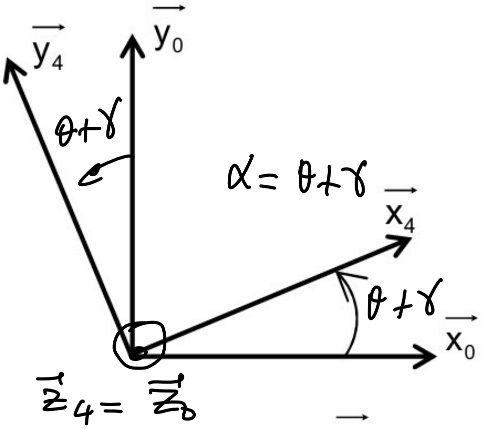
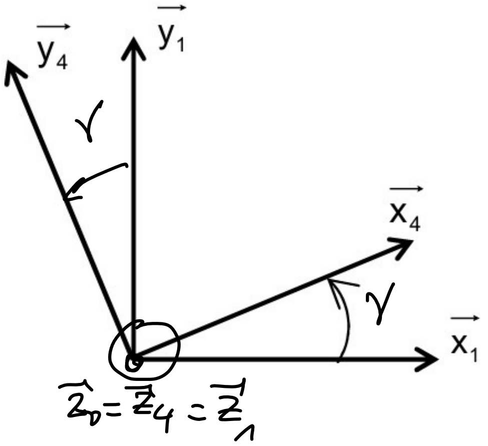
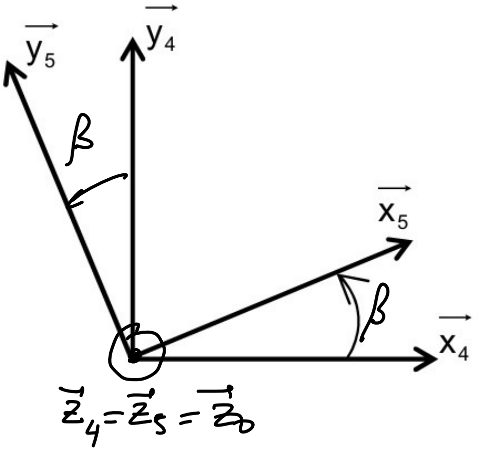
Correction du DM Hiver 2026 MPSI PCSI

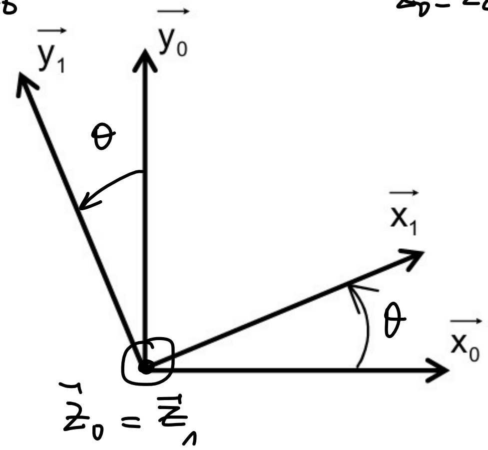
EXERCICE 1

Q1 : $(\vec{x}_0,\vec{x}_4)=(\vec{x}_0,\vec{x}_1)+(\vec{x}_1,\vec{x}_4)=\theta+\gamma$









$$\boxed{\vec{\Omega}_{4/1}=\dot{\gamma}\,\vec{z}_0\ ,\quad \vec{\Omega}_{5/4}=\dot{\beta}\,\vec{z}_0\ ,\quad \vec{\Omega}_{4/0}=(\dot{\theta}+\dot{\gamma})\,\vec{z}_0\ ,\quad \vec{\Omega}_{1/0}=\dot{\theta}\,\vec{z}_0}$$

Q2 : $\vec{V}_{E\in 4/1}=\underbrace{\vec{V}_{D\in 4/1}}_{=\vec{0}}+\vec{ED}\wedge\vec{\Omega}_{4/1}=-t\,\vec{y}_4\wedge\dot{\gamma}\,\vec{z}_0$

$= -t\,\dot{\gamma}\,\vec{x}_4$

Q3 : $\vec{V}_{E\in 1/0}=\vec{V}_{C\in 1/0}+\vec{EC}\wedge\vec{\Omega}_{1/0}=\sqrt{2gH}\,\vec{y}_0+(-t\,\vec{y}_4+d\,\vec{x}_1)\wedge\dot{\theta}\,\vec{z}_0$

$\vec{V}_{E\in 1/0}=\sqrt{2gH}\,\vec{y}_0-t\,\dot{\theta}\,\vec{x}_4-d\,\dot{\theta}\,\vec{y}_1$

Q4 : $\vec{V}_{E\in 4/0}=\vec{V}_{E\in 4/1}+\vec{V}_{E\in 1/0}=\sqrt{2gH}\,\vec{y}_0-t(\dot{\theta}+\dot{\gamma})\,\vec{x}_4-d\,\dot{\theta}\,\vec{y}_1$

En posant $\alpha = \theta + \gamma$

Q5 : $\vec{V}_{E\in 4/0} \cdot \vec{y}_5 = \sqrt{2gH}\, \vec{y}_0 \cdot \vec{y}_5 - t(\dot\theta + \dot\gamma)\vec{x}_4 \cdot \vec{y}_5 - d\dot\theta\, \vec{y}_1 \cdot \vec{y}_5$

$= \sqrt{2gH}\, \vec{y}_0 \cdot \left(-\sin(\alpha + \beta)\vec{x}_0 + \cos(\alpha + \beta)\vec{y}_0\right)$

$- t(\dot\theta + \dot\gamma)\, \vec{x}_4 \cdot (-\sin\beta\, \vec{x}_4 + \cos\beta\, \vec{y}_4) - d\dot\theta \left(\vec{y}_1 \cdot -\sin(\gamma+\beta)\vec{x}_1 + \cos(\gamma+\beta)\vec{y}_1\right) \cdot \vec{y}_1$

$= \sqrt{2gH}\, \cos(\alpha + \beta) + \sin\beta\, t(\dot\theta + \dot\gamma) - d\dot\theta \cos(\gamma + \beta)$

$= \sqrt{2gH}\left(\cos\alpha \cos\beta - \sin\alpha \sin\beta\right) + \sin\beta\, t(\dot\theta + \dot\gamma) - d\dot\theta\left(\cos\gamma\cos\beta - \sin\gamma\sin\beta\right)$

$= \sin\beta\left(d\dot\theta \sin\gamma - \sqrt{2gH}\sin\alpha\right) + \sin\beta\, t(\dot\theta + \dot\gamma) + \cos\beta\left(\sqrt{2gH}\cos\alpha - d\dot\theta\cos\gamma\right)$

Q6 $\alpha = \gamma = \frac{\pi}{4}$ $\quad \beta = \theta = 0$ $\quad \dot\theta = 0{,}01\ \text{rad}\,s^{-1}$ $\quad \dot\gamma = 0{,}01\ \text{rad}\,s^{-1}$

$$\boxed{\vec{V}_{E\in 4/0} \cdot \vec{y}_5 = \left(\sqrt{2 \times 9{,}81 \times 0{,}5} \times \frac{\sqrt{2}}{2} - 0{,}35 \times 0{,}01 \times \frac{\sqrt{2}}{2}\right) = 2{,}23\ m\,s^{-1}}$$

EXERCICE 2

Q1 : $\vec{V}_{O_1\in 1/0} = \frac{d}{dt}\vec{O_0O_1}\Big|_0 = \frac{d}{dt}\left(\lambda(t)\vec{x}_0 + R\vec{y}_0\right) = \dot\lambda(t)\vec{x}_0$

$\vec{\Omega}_{1/0} = \vec{0}$: l'avion est en translation au sol.

$$\boxed{\left\{\mathcal{V}_{1/0}\right\}_{O_1} = \left\{\begin{matrix}\vec{0} \\ \dot\lambda(t)\vec{x}_0\end{matrix}\right\}_{O_1}}$$

Q2 : $\vec{V}_{O_1\in 2/0} = \underbrace{\vec{V}_{O_1\in 2/1}}_{\vec{0}} + \vec{V}_{O_1\in 1/0} = \dot\lambda(t)\,\vec{x}_0$ (Composition des vitesses et 1 en translation)

$\vec{\Omega}_{2/0} = \vec{\Omega}_{2/1} + \vec{\Omega}_{1/0} = \dot\alpha\,\vec{y}_0 + \vec{0}$

$$\boxed{\left\{\mathcal{V}_{2/0}\right\}_{O_1} = \left\{\begin{matrix}\dot\alpha\,\vec{y}_0 \\ \dot\lambda(t)\vec{x}_0\end{matrix}\right\}_{O_1}}$$

Alors $\vec{V}_{G\in 2/0} = \vec{V}_{O_1\in 2/0} + \vec{GO_1} \wedge \dot{\alpha}\vec{y}_0$

$= \dot{\lambda}(t)\vec{x}_0 + \frac{L}{2}\vec{x}_2 \wedge \dot{\alpha}\vec{y}_0$ en posant $d(G,O_1)=L$

$$\boxed{\vec{V}_{G\in 2/0} = \dot{\lambda}(t)\vec{x}_0 + \frac{L}{2}\dot{\alpha}\vec{z}_2}$$

Q3 : $\vec{V}_{O_3\in 3/0} = \underbrace{\vec{V}_{O_3\in 3/2}}_{=\vec{0}} + \vec{V}_{O_3\in 2/0} = \vec{V}_{O_1\in 2/0} + \vec{O_3O_1}\wedge\vec{\Omega}_{2/0}$

$= \dot{\lambda}(t)\vec{x}_0 + L\vec{x}_2 \wedge \dot{\alpha}\vec{y}_0$

$= \dot{\lambda}(t)\vec{x}_0 + L\dot{\alpha}\vec{z}_2$

$\vec{\Omega}_{3/0} = \vec{\Omega}_{3/2} + \vec{\Omega}_{2/0} = \dot{\beta}\vec{z}_3 + \dot{\alpha}\vec{y}_0$

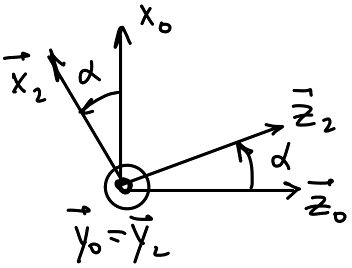
$$\boxed{\left\{\mathcal{V}_{3/0}\right\}_{O_3} = \left\{\begin{matrix}\dot{\beta}\vec{z}_3 + \dot{\alpha}\vec{y}_0 \\ \dot{\lambda}(t)\vec{x}_0 + L\dot{\alpha}\vec{z}_2\end{matrix}\right\}_{O_3}}$$

Q4 : $\vec{V}_{A\in 3/0} = \vec{V}_{O_3\in 3/0} + \vec{AO_3}\wedge\vec{\Omega}_{3/0}$ (formule de Varignon)

$= \dot{\lambda}(t)\vec{x}_0 + L\dot{\alpha}\vec{z}_2 + R\vec{y}_0 \wedge (\dot{\beta}\vec{z}_3 + \dot{\alpha}\vec{y}_0)$

$= \dot{\lambda}(t)\vec{x}_0 + L\dot{\alpha}\vec{z}_2 + R\dot{\beta}(\vec{y}_0 \wedge \vec{z}_3)$

$= \dot{\lambda}(t)\vec{x}_0 + L\dot{\alpha}\vec{z}_2 + R\dot{\beta}\vec{x}_2$



d'où $\vec{V}_{A\in 3/0} = \dot{\lambda}(t)\vec{x}_0 + L\dot{\alpha}(\cos\alpha\,\vec{z}_0 + \sin\alpha\,\vec{x}_0)$
$+ R\dot{\beta}(-\sin\alpha\,\vec{z}_0 + \cos\alpha\,\vec{x}_0) = \vec{0}$

Roulement sans glissement

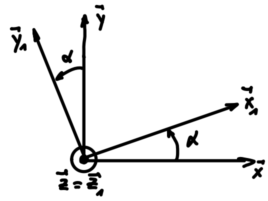
$$\begin{cases}\dot{\lambda}(t) + L\dot{\alpha}\sin\alpha + R\dot{\beta}\cos\alpha = 0 \quad (1)\\ L\dot{\alpha}\cos\alpha - R\dot{\beta}\sin\alpha = 0 \quad (2)\end{cases}$$

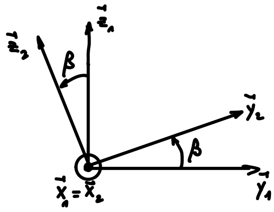
(2) donne $L\dot{\alpha}\cos\alpha = R\dot{\beta}\sin\alpha$

$$\frac{L\dot{\alpha}\cos\alpha}{\sin\alpha} = R\dot{\beta} \iff \boxed{\frac{\dot{\alpha}}{\tan\alpha} = \frac{R}{L}\dot{\beta}}$$

EXERCICE 3 :

Q1 :





Q2 : $\vec{\Omega}_{R_2/R} = \vec{\Omega}_{R_2/R_1} + \vec{\Omega}_{R_1/R} = \boxed{\dot{\beta}\vec{x}_1 + \dot{\alpha}\vec{z}_1 = \vec{\Omega}_{R_2/R}}$

Q3 : $\vec{V}_{A\in 2/R} = \vec{V}_{A\in 2/R_1} + \vec{V}_{A\in 1/R}$ (Composition des vitesses)

$$= \underbrace{\vec{V}_{C\in R_2/R_1}}_{\vec{0}} + \vec{AC}\wedge\vec{\Omega}_{2/1} + \underbrace{\vec{V}_{O\in 1/R}}_{\vec{0}} + \vec{AO}\wedge\vec{\Omega}_{1/R}$$

$$= -b\vec{z}_2\wedge\dot{\beta}\vec{x}_2 + (-b\vec{z}_2 - a\vec{x}_2)\wedge\dot{\alpha}\vec{z}_1$$

$$= -b\dot{\beta}\vec{y}_2 - b(-\sin\beta\vec{y}_1 + \cos\beta\vec{z}_1)\dot{\alpha}\vec{z}_1 + a\dot{\alpha}\vec{y}_2$$

$$\boxed{\vec{V}_{A\in 2/R} = -b\dot{\beta}\vec{y}_2 + b\sin\beta\,\dot{\alpha}\,\vec{x}_1 + a\dot{\alpha}\vec{y}_2}$$

Q4 : $\vec{x}_1 = \vec{x}_2$ d'où $\boxed{\|\vec{V}_{A\in 2/R}\| = \sqrt{(b\dot{\alpha}\sin\beta)^2 + (a\dot{\alpha} - b\dot{\beta})^2}}$

Si le vent est établi $\alpha =$ cte $\Rightarrow \dot{\alpha} = 0$ alors

$$\boxed{\|\vec{V}_{A\in 2/R}\| = b\dot{\beta}}$$

FIN