Recherche d'un équivalent ou d'une limite

Limite en 0 de $\frac{2}{sin^2(x)} - \frac{1}{1-\cos(x)}$.

$\frac{2}{sin^2(x)} \sim_0 \frac{2}{x^2}$ et $\frac{1}{1-\cos(x)} \sim_0 \frac{1}{\frac{x^2}{2}} = \frac{2}{x^2}$. Donc, je ne peux pas conclure.

$$\frac{2}{sin^2(x)} - \frac{1}{1-\cos(x)} = \frac{2}{\left(x-\frac{x^3}{6}+o_0(x^4)\right)^2} - \frac{1}{\frac{x^2}{2}-\frac{x^4}{24}+o_0(x^4)} = \frac{2}{x^2-\frac{x^4}{3}o_0(x^4)} - \frac{2}{x^2-\frac{x^4}{12}+o_0(x^4)} = \frac{2}{x^2}\left(\frac{1}{1-\frac{x^2}{3}o_0(x^2)} - \frac{1}{1-\frac{x^2}{12}+o_0(x^2)}\right)$$

$$= \frac{2}{x^2}\left(\left(1+\frac{x^2}{3}o_0(x^2)\right) - \left(1+\frac{x^2}{12}+o_0(x^2)\right)\right)$$

$$= \frac{2}{x^2}\left(\frac{x^2}{4}+o_0(x^2)\right) \sim_0 \frac{1}{2}.$$ Donc, $\lim_{x\to 0} \frac{2}{sin^2(x)} - \frac{1}{1-\cos(x)} = \frac{1}{2}$

Equivalent en 1 de $f(x) = x^x - x$.

$f(x) = x^x - x = f(x) = x(x^{x-1}-1) \sim_1 x^{x-1} - 1 = e^{(x-1)\ln(x)} - 1 \sim_1 (x-1)\ln(x) \sim_1 (x-1)^2$.

Equivalent en 0 de $f(x) = sin(ln(1+x)) - ln(1+sin(x))$.

$$f(x) = sin(ln(1+x)) - ln(1+sin(x)) = sin\left(x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+o_0(x^4)\right) - ln\left(1+x-\frac{x^3}{6}+o_0(x^4)\right)$$

$$f(x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}-\frac{x^3}{6}+\frac{x^4}{6}+\frac{x^4}{12}-\left[x-\frac{x^3}{6}-\frac{x^2}{2}+\frac{x^4}{6}+\frac{x^3}{3}-\frac{x^4}{4}\right]+o_0(x^4) = \frac{x^4}{12}+o_0(x^4) \sim_0 \frac{x^4}{12}.$$

Equivalent en 0 de $f(x) = x^x - sin(x)^{sin(x)}$.

$$f(x) = x^x - \sin(x)^{\sin(x)} = e^{xln(x)} - e^{\sin(x)\ln(\sin(x))} = e^{xln(x)} - e^{(x-\frac{x^3}{6}+o_0(x^3))\ln\left(x-\frac{x^3}{6}+o_0(x^3)\right)}$$

$$= e^{xln(x)} - e^{(x-\frac{x^3}{6}+o_0(x^3))\ln\left(x\left(1-\frac{x^2}{6}+o_0(x^2)\right)\right)} = e^{xln(x)} - e^{\left(x-\frac{x^3}{6}+o_0(x^3)\right)\left[ln(x)+\ln\left(1-\frac{x^2}{6}+o_0(x^2)\right)\right]}$$

$$= e^{xln(x)} - e^{\left(x-\frac{x^3}{6}+o_0(x^3)\right)\left[ln(x)-\frac{x^2}{6}+o_0(x^2)\right)} = e^{xln(x)} - e^{xln(x)-\frac{x^3}{6}ln(x)+o_0\left(\frac{x^3}{6}ln(x)\right)}$$

$$= e^{xln(x)}\left(1-e^{-\frac{x^3}{6}\ln(x)+o_0\left(\frac{x^3}{6}ln(x)\right)}\right)$$

$$\underbrace{\sim_0}_{car\ \lim_{x\to 0} e^{xln(x)}=1.} 1-e^{-\frac{x^3}{6}\ln(x)+o_0\left(\frac{x^3}{6}ln(x)\right)}$$

$$\underbrace{\sim_0}_{\substack{car\ \lim_{x\to 0} -\frac{x^3}{6}\ln(x)+o_0\left(\frac{x^3}{6}ln(x)\right)=0 \\ et\ e^u-1\sim_0 u}} -\frac{x^3}{6}\ln(x)+o_0\left(\frac{x^3}{6}ln(x)\right)$$

$$\sim_0 -\frac{x^3}{6}\ln(x).$$

Equivalent en +∞ de $f(x) = (ln(x))^2\left[sin\left(\frac{1}{ln(x)}\right) - sin\left(\frac{1}{ln(x+1)}\right)\right]$.

$$f(x) = (ln(x))^2\left[sin\left(\frac{1}{ln(x)}\right) - sin\left(\frac{1}{ln(x+1)}\right)\right] = (ln(x))^2\left[2\sin\left(\frac{1}{2ln(x)}-\frac{1}{2ln(x+1)}\right)\cos\left(\frac{1}{2ln(x)}+\frac{1}{2ln(x+1)}\right)\right]$$

$\sim_{+\infty} (ln(x))^2 2\sin\left(\underbrace{\frac{1}{2ln(x)}-\frac{1}{2ln(x+1)}}_{u(x)}\right)$ car $\lim_{x\to+\infty} \cos\left(\frac{1}{2ln(x)}+\frac{1}{2ln(x+1)}\right) = 1$. Or,

$$u(x) = \frac{1}{2ln(x)} - \frac{1}{2ln(x+1)} = \frac{\ln(x+1)-\ln(x)}{2ln(x)\ln(1+x)} = \frac{\ln\left(1+\frac{1}{x}\right)}{2ln(x)\ln\left(x\left(1+\frac{1}{x}\right)\right)} = \frac{\ln\left(1+\frac{1}{x}\right)}{2ln(x)\left[\ln(x)+\ln\left(1+\frac{1}{x}\right)\right]} \underbrace{\overbrace{\sim_{+\infty}}^{\substack{car\ \lim_{x\to+\infty}\frac{1}{x}=0 \\ et\ \ln(1+u)\sim_0 u.}}}_{\substack{car\ \lim_{x\to+\infty}\ln\left(1+\frac{1}{x}\right)=0 \\ et\ \lim_{x\to+\infty} ln(x)=+\infty \\ donc\ \ln(x)+\ln\left(1+\frac{1}{x}\right)\sim_{+\infty} ln(x).}} \frac{\frac{1}{x}}{2ln^2(x)} = \frac{1}{2xln^2(x)}.$$

En particulier, $\lim_{x\to+\infty} u(x) = 0$. Par conséquent, $\sin(u(x)) \sim_{+\infty} u(x) \sim_{+\infty} \frac{1}{2xln^2(x)}$. Et ainsi, $f(x) \sim_{+\infty} ln^2(x)\frac{1}{xln^2(x)} = \frac{1}{x}$.

Limite en +∞ de $\left(sin\left(\frac{1}{x}\right) + cos\left(\frac{1}{x}\right)\right)^x$.

$\left(sin\left(\frac{1}{x}\right)+cos\left(\frac{1}{x}\right)\right)^x = e^{xln\left(sin\left(\frac{1}{x}\right)+cos\left(\frac{1}{x}\right)\right)}$. Posons $t = \frac{1}{x}$ et $g(t) = f(x)$.

Alors $x=\frac{1}{t}$ et $g(t)=f\left(\frac{1}{t}\right)$ et $f(x)=g\left(\frac{1}{x}\right)$. Déterminons la limite de g en 0 .

$g(t)=e^{\frac{1}{t}\ln(sin(t)+cos(t))}=e^{\frac{1}{t}\ln(1+t+o_0(t)))}=e^{\frac{1}{t}(t+o_0(t))}=e^{1+o_0(1)}$. Donc, $\lim_{t\to 0} g(t)=e$ et ainsi, $\lim_{x\to+\infty} f(x)=e$.

Limite en $+\infty$ de $\left(\frac{ln(1+x)}{ln(x)}\right)^{xln(x)}$.

$\left(\frac{ln(1+x)}{ln(x)}\right)^{xln(x)}=e^{xln(x)ln\left(\frac{ln(1+x)}{ln(x)}\right)}=e^{xln(x)[ln(\ln(1+x))-\ln(\ln(x))]}$. Posons $t=\frac{1}{x}$ et $g(t)=f(x)$.
Alors $g(t)=e^{\frac{1}{t}ln\left(\frac{1}{t}\right)\left[ln\left(\ln\left(1+\frac{1}{t}\right)\right)-\ln\left(\ln\left(\frac{1}{t}\right)\right)\right]}$.Posons $h(t)=\frac{1}{t}ln\left(\frac{1}{t}\right)\left[ln\left(\ln\left(1+\frac{1}{t}\right)\right)-\ln\left(ln\left(\frac{1}{t}\right)\right)\right]$.
$h(t)=\frac{1}{t}ln\left(\frac{1}{t}\right)\left[ln\left(\ln\left(1+\frac{1}{t}\right)\right)-\ln\left(\ln\left(\frac{1}{t}\right)\right)\right]=-\frac{1}{t}\ln(t)\,[ln(\ln(1+t))-\ln(t))-\ln(-\ln(t))]$
$h(t)=-\frac{1}{t}\ln(t)\left[\ln(-ln(t))+\ln\left(1-\frac{ln(1+t)}{ln(t)}\right)-\ln(-\ln(t))\right]$
$=-\frac{1}{t}\ln(t)\ln\left(1-\frac{ln(1+t)}{ln(t)}\right)\underbrace{\sim_0}_{\substack{car\ \lim_{t\to 0}-\frac{ln(1+t)}{ln(t)}=0\\ et\ \ln(1+u)\sim_0 u}}-\frac{1}{t}\ln(t)\left(-\frac{ln(1+t)}{ln(t)}\right)=\frac{ln(1+t)}{t}\sim_0 1.$

Donc, $\lim_{t\to 0} g(t)=e$ et ainsi, $\lim_{x\to+\infty} f(x)=e.$